<u>Walden University</u>

COLLEGE OF MANAGEMENT AND TECHNOLOGY

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Alfred Sepulveda

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Abstract

Information-theoretic Metamodel of Organizational Evolution

by

Alfredo Sepulveda

MS Mathematics, University of Texas San Antonio, 1979
BS Mathematics, California Institute of Technology, 1977

Dissertation Submitted in Partial Fulfillment
of the Requirements for the Degree of
Doctor of Philosophy
Applied Management and Decision Science Information Systems Management

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Abstract

Social organizations are abstractly modeled by holarchies—self-similar connected networks—and intelligent complex adaptive multiagent systems—large networks of autonomous reasoning agents interacting via scaled processes. However, little is known of how information shapes evolution in such organizations, a gap that can lead to misleading analytics. The research problem addressed in this study was the ineffective manner in which classical model-predict-control methods used in business analytics attempt to define organization evolution. The purpose of the study was to construct an effective metamodel for organization evolution based on a proposed complex adaptive structure—the info-holarchy. Theoretical foundations of this study were holarchies, complex adaptive systems, evolutionary theory, and quantum mechanics, among other recently developed physical and information theories. Research questions addressed how information evolution patterns gleamed from the study's inductive metamodel more aptly explained volatility in organization. In this study, a hybrid grounded theory based on abstract inductive extensions of information theories was utilized as the research methodology. An overarching heuristic metamodel was framed from the theoretical analysis of the properties of these extension theories and applied to business, neural, and computational entities. This metamodel resulted in the synthesis of a metaphor for, and generalization of organization evolution, serving as the recommended and appropriate analytical tool to view business dynamics for future applications. This study may manifest positive social change through a fundamental understanding of complexity in business from general information theories, resulting in more effective management.

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Dedication

I wish to present this work for all those interested in the truth of what information is, what it creates, and its uncharacteristic ubiquity in all things physical and not.

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Chapter 1: Introduction to the Study

On the mesoscopic level of human existence, information streams supply the source for any input that eventually is crystallized into a perception of reality. This reality is only manifested, in the end, by individualized receivers—sensors that are mechanized devices, biological wetware or particle-sized matter. This study posited that from first principles, information as an abstract particle forms matter, energy, and physical fields, eventually leading to organization. As an instantiation, the prototypical information-based business was treated as an abstraction of a techno-socio-economic organization. More specifically, the evolution of organizations was constructed through the lens of adaptive game theory, unified physical principles of quantum gravity information, and an underlying organization model, the proposed info-holarchy.

From this model, as a robust alternative to classical model-predict-control business analytics, patterns of evolution and morphology of an organization will be tracked and guided in virtual reality dashboard-caves—immersive 3-D environments utilizing multisensorial control and feedback devices. In this chapter, historic and technical introductions will be made to physical and information theories and their post-modern extensions to fuzzy, quantum, and other generalized uncertainty notions. Linkage will be made to complexity, nonlinear dynamics, information physics, and networks. This will prepare the reader for a thorough review of modern mathematical information theory and its extensions thereby revealing some surprising connections to post-modern physical theories in chapter 2.

Background

Modeling and predicting natural and man-made phenomenon have always been the ultimate curiosity and goal of a sometimes maddening control-centric science, preceded only by the origin and evolution of matter in the universe. Following this tenet especially are the motivations from organization theories: the prediction and control of the dynamics of human organizations. Nonetheless, a unifying theory of organization, by its conspicuous absence, signals both a lack of ties to physical first principles and a fundamental misunderstanding about the limitations and impotence of predicting nature and human societies. Regardless of this warning, stepwise experiential and experimental feedback continue to lure scientists into devising the next great unified framework.

In the 18th century, Laplace proposed his metaphysical demon—the notion that utilizing Newton's equations of motion to their ultimate conclusion reduced the description of the universe to a perfectly mechanical and reversible clock device (Laplace, 1826/1995). Laplace, nonetheless, knew this proposition to be unobtainable and along with Cardano, de Fermat, Pascal, Bernoulli, Gauss, and others, helped create the early foundation for a probability theory in order to surmise an uncertain and incomplete dataset for the universe, albeit, with a penchant for more successful gambling (Hacking, 1999). Kolmogorov would axiomatize probability theory later in the early 20th century (Charpentier, Lesne, & Nikolski, 2007). Formal statistical science which utilized probabilistic models to endeavor to estimate, predict, model, and control phenomena would develop during this same period from the experimental and theoretical works of Fisher, Pearson, Box, and others (Salsburg, 2001).

Along with this shift in modeling came the extraordinary works of the non-Euclidean geometers, Minkowski (1907/1915), Lobachevski (1914), Bolyai (1831), and Riemann (1868), paving the way for Einstein's formulation of general relativity (Einstein, 1916). Topologists would follow geometers in describing invariant spaces – tools for describing the dynamics and morphogenesis of objects without regards to location in the universe. This mindset set the stage for the spacetime geometric theories of relativity and the robust structure of quantum probability, which along with Darwinian evolution, were the three most important paradigm shifts in science in the last two centuries. It would therefore be appropriate to address their respective usage in any unified theory of organization.

These mammoth shifts in scientific thinking collectively paved the way for the arrival of new frameworks for viewing and describing uncertainty in nature and human interpretation. Quantum mechanics postulated that uncertainty was part and parcel to the fabric of the universe itself. Einstein, as one of the three founders of quantum mechanics refused to believe in the indeterminacy of the universe most ostensibly through his famous original comment, "I, at any rate, am convinced that He does not throw dice" (Einstein, 1926, p. 91). Furthermore, Einstein tried to prove the determinancy of QM through a gedankenexperiment, the Einstein, Podolsky, Rosen (EPR) paradox, in which two quanta correlate only at superluminal speed, a direct refutation of special relativity (Einstein, Podolsky, & Rosen, 1935). Einstein's principle of locality in general relativity and realist approach to physics prevented a full acception of QM, though this was loosen somewhat during those last years endeavoring to develop the foundation for a grand unification theory (GUT).

It was Bohm who would rekindle this belief in a development of a general nonlocal hidden variables theory called quantum potential in which the equations of quantum mechanics were reformulated to display a deterministic relationship (Bohm & Hiley, 1993). Bell's theorem and inequality posited that a local version of Hidden Variables was all but impossible in describing a quantum mechanical world. Aspect and others showed by experiment that local hidden variables were essentially impossible, but did not rule out nonlocal versions (Bell, 1964). 't Hooft (2007) later posited the notion of superdeterminism in which the universe "knows" a-priori, all information at all times and hence is in no need of superluminal communication for all nonlocal correlations. Nonetheless, the predominant view of QM is probabilistic, especially since a probabilistic generalization to nonlocal hidden variables theories can be formulated. Correlations between widely distanced quanta are possible (supercorrelations), while superluminal information transfer between the two is not. This was a new kind of information causality and a generalization to the notion of "no-signaling" theories of physics (Pawlowski, et al., 2009). No-signaling simply means that a meaningful signal or parcel of information cannot be transmitted at greater than light speeds.

Analogous to the indeterminism of standard quantum mechanics was the notion of the second law of thermodynamics in statistical mechanics: Energy is conserved, remains constant in a closed environment. Heat and temperature were then axiomatized to relate to molecular movement and location in the form of Maxwell's demon, as further developed and discussed by Maxwell (2001) and Carnot (1897). Boltzmann (1909) would form the basis for the thermodynamic definition of entropy—the movement of heated gas (molecular density) as a probabilistic notion, eventually leading to the famous entropy law $S = K \log W$ where S is entropy, W is the probability of movement, and K is the Boltzmann constant. The idea of a thermodynamic entropic property as a measure of information was advanced by Szilárd (1929) and later by Brillouin (1959) and expanded to computation by Landauer (Plenio & Vitelli (2001)). Finally, entropic information was thrown into the realm of the quantum and later matured through the usage of properties of quanta such as entanglement and decoherence for quantum computation by Bennett & Wiesner (1992), Zurek (2003,2009), and Wootters & Zurek (2009). The discovery of the computational properties of quanta also elevated the role of the observer as a causal agent in the measureability of information.

Briefly returning to what would become the seminal motivating idea of information transfer, Maxwell's demon, these theoretical entities are capable of sifting through the energy and information measurement of every molecule in a closed container and hence of being able to order them into two subchambers, one consisting of the slowest moving particles and the other consisting of the fastest moving particles. Maxwell's demon would then be able to optimally separate Boltzmann entropic subgroups of the original set of particles, entropy being the notion of the possible number of microstates that can give rise to the resultant macrostate as in temperature or total energy of the particle groups. It would then be able to convert heat and energy at will. It would turn out that the important process was not conservation and/or conversion of heat or energy, but of storage of old information about the state of energy, the forgetting part. Macrostates are gardered at the expense of losing microstate information. Hence how one would store and interprete this information is more important, again an observer-dependent phenomena. Quantum mechanics is a probabilistic microtheory, while thermodynamics is a probabilistic mesoscopic theory, both observer-relevant processes.

General relativity (GR) shows that space and time are part of the same coordinate system and hence time could be disparate and not neatly ordered in the topology of regions. Hence GR is a macro theory in the scheme of anthropoids and once again as with QM and Thermodynamics, observer-relevant. In all three paradigms, observers take center stage and are mainly probabilistic in interpretation, given that Bohmian Mechanics and nonlocal QM theories can be generalized to degenerate probabilistic versions. In this dissertation, the commonality between these models is the notion of information and its form in their respective development. Information is thus first treated as observer relevant and second as the primary source for the creation of matter and energy in quanta. Secondly, the information model posited in this dissertation, the info-holarchy, based on these three paradigms, and generalized for nonclassical logics, is used to develop a calculus for building general organization, including natural and man-made cohorts of organization.

These new maths and physical theories propagated into the depths of all sciences including biology and the social sciences. Strict reductionism was to be supplanted by the holism of quantum uncertainty and relativistic geometry. Subsequent to these changes the age of electronics erupted, creating the field of information theory, control theory, and automata as born out by Shannon (1948), Weiner (1948), and von Neumann (1966), creating frameworks for the mathematical formulation of information in objective communications, cybernetics, and intelligent computation respectively. Others were more concerned with the interpretation of signals. Any creditable and complete theory of information would have to include not only a study of signal content and bandwidth, but also of observer interpretation. A quantum theory of information would bear this proposition out.

Adjacent to this mathematical information movement, a physical spacetime theory would give life to the notion that information could be shaped in part by a geometric field. Field theories hold quantum, electromagnetic, string, and dbranes accountable for ensemble flows. If information is created by a physical event, an information ensemble should be described by a complementary information field. This statement begs the ultimate question concerning information: Does a physical presence emanate information or does information create physical observables? The conventional wisdom is that of the former. However, a new wave of theorists has conjectured about and worked towards the development of a theory of an informationally discrete universe—digital physics.

Wheeler (1990) was the most influential initiator of the digital physics movement, and coined along with other famous physics terminology, the prophetic term, "it from bit". This mantra led to the general thesis of approaching physics from an information-theoretic point of view. Even before this development, Feynman (1982) had proposed the notion of a digital universal quantum simulator as a Turing machine (i.e., a quantum computer). Disciples of both Wheeler and Feynman, along with others followed suit by forming ideas about the physical importance of information-theoretic concepts in the various major models of reality, including thermodynamics, super string/m theory, loop quantum gravity, general relativity and cosmology, and natural philosophy (Bekenstein, 1973; Cliche & Kempf, 2009; Deutsch, 1985; Duff, 2010; Fredkin, 2000; Lloyd, 1996: Terno, 2006; Vedral, 2010; Von Baeyer, 2001, 2004; Zizzi, 2005b). Their general premise is that at the quantum and subquantum levels approaching the Planck scale, and extending to the large-scale cosmos, every aspect of energy-mass depends on information, its generation, transfer, reception, and utilization.

In particular, a qubit, the quantum extension of an information bit, could represent any aspect of a quanta, such as its spin. Multidimensional arrays of qubits could then represent any cosmological object, most notably, the surface of a black hole horizon which contains the projection of all information contained in the volume of that cosmological entity. This information-theoretic notion is the so-called holographic principle (Bekenstein, 2003; Schumacher, 1995). It can be generalized to any object (Bousso, 2002). The natural extension of this idea was to describe the universe and all its processes as a special, *the special* computation machine (Deutsch, 1997; Lloyd, 2006a). One of the most radical extensions of information as a model for the universe was put forth by Tegmark (2007) in his theory of the universe as a mathematical machine – the universe is equivalent to all of mathematics, discovered and constructed (i.e., a mathematical universe).

One of the most intriguing areas of study in the complexity sciences is that of emergent behavior that presents itself in complex multiagent systems. Emergence is a sister concept to that of information. As with the use of the word information, emergence similarly defies a singular and clear definition. They are, as it were, concomitant and intertwined concepts. Combining the processes of information and emergence, organisms and complex organizations exhibit collective emergent information flows and self-creation. Complex adaptive multiagent systems (CAMSs) are ubiquitous examples of these types of mass organization. While escaping a simple definition, fundamentally, a CAMS is a large network of reasoning autonomous entities (agents) that collaborate with each other (or with subsets of neighbors) and with a containing environment (containers) using rules of engagement while manifesting emergent group behavior not explicitly possible from individual members or smaller subgroups thereof (Amaral & Ottino, 2004).

Emergence may be exhibited at different scales or levels of a system in varying ways and in time-dependent fashions (i.e., dynamically). Through these rules of engagement, groups of agents may show the characteristics of self-awareness, self-organization, self-creation, and acquired intelligence. Reasoning is approximated and iteratively improved by utilizing these algorithms. This definition does not preclude the possibility that meaningful subgroups of agents within a CAMS, also exhibit the emergent behavior from self-similarity. Indeed, it is worthwhile to investigate these substructures of CAMSs that are themselves CAMSs. Holonic structures or holarchies, pseudo-heirarchical networks of holons, succinctly describe this type of nested self-similar complexity.

The holon was a term first coined by Koestler (1967/1990) to describe an entity that simultaneously possesses the properties of wholeness and partness of other similar but possibly heterogeneous structures of entities. This description of a holon has the flavor of self-similarity in organizations and networks. Koestler also suggested that holons are capable of self-creation through an evolution of processes. Structures that are comprised of holons as interconnected complex adaptive agents are called holarchies. One may then impart intelligence to these structures by attaching rules of engagement through the exchange of and reflection upon information. These organisms will be labeled as intelligent holarchies. By imposing an information field theory and information particle model - the informaton, upon these intelligent holonic organisms through the use of nonclassical physical theories and logic systems, one begins to arrive at the notion of info-holarchies in this study.

Invariably CAMSs are artificially glued together in a premeditative weave using apriori rules of engagement and reflection. This rule set or decision space is described microscopically between holons or agents, macroscopically between larger homogenous groups, and mesoscopically, in a bridge between these scales by control processes. For example, in the formation of coalitions, a subclass of holarchies that have degrees of self-centeredness and isolation, game theoretic structures are imposed in order to shape group dynamics and form (Ray, 2007). Smaller subcoalitions may exhibit self-awareness and eventually break out through evolutionary processes. In the meantime, these subcoalitions are held together to keep the large coalition in place through a binding entropic rule. In the quagmire in between these events is the mesoscopic bonding between potential renegades and team players. What all these scales of dynamics have in common is the exchange of information within and outside of each boundary.

Information is created, shared, changed, or isolated. This information dynamic is the central tenant of the metamodel process of this study. In this study, a novel model for information will be presented that utilizes information fields (a generalization of Bayesian statistical field theory) in the tradition of physical fields and a dynamic evolutional model for information particles. This metamodel will be the basis for a calculus of info-holarchies.

Info-holarchies may span vast spacetime regions or the smallest of known abstract particles. As such, they are subject to both relativistic and quantum effects. The theory of quantum gravity (QG) endeavors to melt these two physical paradigms. However, no mathematical framework for QG has been validated such as those of loop quantum gravity (LQG), string (superstring) m-theory, algebraic and Euclidean quantum theories, Penrose's twistor theory, and Lisi's E_8 model to name some currently popular conceptual frameworks (Halvorson & Mueger, 2006; Lisi, 2007; Penrose, 1967; Rovelli, 2008; Witten, 1998).

One of the most profound problems to this unification is the idea of a theory of uncertainty, such as that in quantum mechanics (QM), that respects an invariance of time, such as in general relativity (GR). The consensus of physicists is that any creditable unification theory must embrace both QM and GR (Woodard, 2009). The present limitation of experimental proof of the full correctness of any quantum gravity theory notwithstanding, the notion of relativistic gravitational effects at quantum Planck scales is one of the cornerstones of LQG and any quantum gravity theory.

To the point of the role of information in these physical theories, recently, an information-theoretic model in physics, the holographic principle, that reduces the theoretical amount of information necessary to completely represent a volume of matter in the universe to a proportional amount on its surface area, was used to show from first principles how information forms the classical Newtonian gravitation laws, GR, and a possible plausible explanation for the density of dark energy—so called entropic (information-based) gravity (Verlinde, 2010). Quantum mechanics is already an information-theoretic model. Hence, information is treated as a unifier for quantum gravity theory and generalized organization in this study. Every force (field carrier) is assumed by most theoreticians, to move particles of some sort, including gravity which is posited to move gravitons – theoretical spin-2 massless particles (Misner, Thorne, & Wheeler, 1973). Consistent with this construct, in this study, abstract information particles – informatons, are constructed for the purposes of acting as fundamental and simultaneous physical carriers and receivers of information that a field (the information field) move. This fundamental scheme for information particle-field spacetime is posited to construct all information for physical particles and fields and to be the conduit for information creation and organization.

The causaloid is a novel abstract mathematical mechanism that attempts to generalize probabilistic physical theories for QG or any causal-probabilistic notion of spacetime. In this notion, an operational definition of physicalism reduces the space of probabilistic representations of spacetime to smaller spaces of linear operators that overcome the prospect of an indefinite correlational or causal ordering through time

(Hardy, 2008). Quantum mechanics requires a causal time ordering via the quantum state relationship in the Schrödinger wave equation using an observable unitary operator, *U*:

$$\left|\psi(t)\right\rangle = U(t)\left|\psi(0)\right\rangle \tag{2.1}$$

General relativity (GR), on the other hand, requires a deterministic structure without a possible temporal ordering. Additionally, GR shapes a time surrogate through geometry. Statistical thermodynamics has a role in a new concept that replaces our conventional definition of time with state independent thermal flow in a relativistic notion of geometric spacetime horizons around the observer (Connes & Rovelli, 2008). Horizons are the theoretical limits of what an observer could causally affect within luminal constraints. Time may therefore be created from or replaced by the geometry and topology of the universe and the thermal flow sensed by an observer connected to that structure - the shape of things around us and their temperature. Information is no exception to this paradigm being thermodynamic and entropic. Could information and information flow replace time's arrow?

Any complete theory of information must subsume the physical theories of quantum mechanics, general relativity, and eventually that of a viable theory of quantum gravity (QG) in order to have a chance at conciliation with energy-matter. The field of QG endeavors to consistently unify the linear probabilistic principles of quantum mechanics at microlevels with the deterministic, but nonlinear principles of GR at macrolevels. In the reduction of the Schrödinger wave function, the microlevel fine-grained dynamics may then decohere from chains of successively more deterministic and less probabilistic models (their probability distributions become less uniform) in order to reduce to the course-grained deterministic macrolevel results. Mesoscopic thermodynamical levels may be transitional between these two regimes. This notion of wave reduction that attempts to reconcile the quantum-ness of the very small to the determinism of the large, is a variant of the decoherence formalism of Gell-Mann and Hartle’s modified version of Everett’s Many Worlds model of simultaneously existing histories adapted to quantum gravity (Gell-Mann & Hartle, 1994; Omnes, 2005; Siegfried, 2010). The various attempts at QG must also contend with quantum entanglement—a super-correlative concept of causality in which two (or more) entities (particles) when separated, can exactly determine the other’s properties upon their respective measuremen (i.e., only one measurement of one particle’s properties is needed to have a measurement of the multiparticle system’s properties, regardless of the separating distances).

If quanta (generalized particles) are pure information, as this study posits, then this conciliation from a quantum gravity theory must come to fruition. An effective information field theory and information particle model that neither requires determinism or definite causal ordering may describe the informational flow in a QG setting. This framework qualifies as a general information flow model that promulgates at both the quantum and general relativistic scales. In this dissertation, a field theory will be developed that complements a new particle model for information, the informaton, which consists of an event generator-observer pair exhibiting both quantum behavior and spacetime indefiniteness and invariance in general relativity.

With this goal in mind, causaloids—a generalized probabilistic-casual framework for plausible physical spacetimes (i.e., event-horizon regions), were utilized in forming the causality mechanism in this study's information models (Hardy, 2005). The notion of a generalized theory of uncertainty (GTU), Zadeh (2005), which can generate more expressive uncertainty operators, replace traditional probability operators in quantum state calculations in the original causaloid framework. GTU operators will make it possible to substitute other notions of uncertainty into the quantum state definition. In this sense, quantum probability is generalized in this study. In a similar manner, an uncertainty operator will be utilized in the macroprocess operators that will be bridged to the microprocesses of each informaton. The mechanism to be used to delivery this bridged concept is from the field of macroinfodynamics.

Macroinfodynamics is an approach to model and bridge stochastic macro and microprocesses through control functions (Lerner, 2003). In this study's approach, macroinfodynamics are generalized for the case described by GTU operator Itô stochastic diffusion equations which are posited to dictate informaton exchanges at the micro level, while generalized Shannon entropy measures blend in (ensemble average) the macro behavior of the formed large-scale structures. Control-theoretic (mesoscopic scaled) functions bridge these two levels in the scale boundary between the microscopic and macroscopic regimes. The concept of a statistical description (filtering) of turbulence, a phenomena that manifests itself in the indeterminate definition of turbulence in the nontrivial boundary between micro scaled air molecules and macro scaled air flow currents, serves as an analogy for this process (Majda, Harlim, & Gershgorin, 2010).

Information is posited, in this study, to be a flow emanating and generated from all scales relative to the source (event generator) and receiver (observer).

Uncertainty in information has been described in many ways including (a) probabilistic, (b) veristic, (c) belief, (d) fuzzy, (e) bimodal, (f) group, and (g) usuality. This notion of generalizing uncertainty concepts has been developed by Zadeh (2005) in an attempt to unify them into a single framework—the generalized theory of uncertainty (GTU). Central to this endeavor is the idea of a generalized constraint variable given by the form: $X\ is(r)\ R$ where X is a constrained variable, r is an index symbol that represents the semantics of the constraint, and R is a nonbivalent relationship. Associated with a generalized constraint will be a test-score function, $ts(u)$, that measures the degree of the object u satisfying the constraint. This definition of a general constraint for uncertainty is a generalization of the idea of the membership function in fuzzy logic. Generalized constraints will be expanded upon in more detail in chapters 2 and 4 of this study.

Quantum uncertainty as pertains to the condition implied by Heisenberg is a specialization of a probabilistic measure of uncertainty since quantum states are described as linear combinations (superpositions) of probability distributions of possible particle paths (histories).

Generalizations to Shannon entropy, von Neumann (1955) quantum entropy, and thermodynamic descriptions of information have been formed, most notably the α-Renyi entropy family (Hu & Ye, 2006; Rényi, 1961):

$$S_{\alpha}(\rho) = (1-\alpha)^{-1} \log Tr(\rho^{\alpha}) \tag{2.2}$$

which converges in the limit, $\alpha \to 1^{+}$ to the von Neumann entropy,

$$S(\rho) = -Tr(\rho)\log\rho \tag{2.3}$$

Here ρ is the probability density operator (matrix) for the quantum system. These definitions of information entropy depend on a probabilistic notion of particle distribution. The most general forms of quantum and general uncertainty entropies will be given and developed in chapter 2 and 4.

Generalizing this probability density operator utilizing the GTU further enhances a description for information flow (i.e., an agenda for generalized information entropy). However, these entropic measures do not consider the symbiotic nature of information - the syntactic, semantic, and pragmatic dimensions. Symbiotics is a general theory of information representation in terms of signs and symbols (tokens). Syntax is the relational nature between such symbols or tokens. Semantics is the study of the interpretive space that a receiving agent has of observed information. Formally, Korzybski (1994) developed general semantics as a means of differentiating human expressiveness of observation and reality. His work most famously expanded the universality of the "to be" verb of any expression and the reality behind the condition of the described object. Finally, pragmatics is the study of the action space of possible repercussions of these interpretations (Joslyn, 2002). Symbiotics contributes to a more appropriate and powerful definition of information that expands its application to human cognition and general intelligent interpretation.

While the ubiquitous term *information* is used in a flippant manner to mean a general relevancy of data and knowledge propagation and retrieval, the apparatus for its structure and flow between event and observer is ambiguous. The measurement problem in Quantum Mechanics points to this uncertainty in what information can be objectively measured and indeed if the collapse of a generalized Schrödinger wave equation dictates such measurement or is done separately by nature (Thaheld, 2007). Everett (1957) and his many-worlds interpretation of quantum mechanics posits otherwise.

In the many-worlds interpretation of quantum mechanics, parallel universes exist simultaneously. Each universe assumes a uniquely different history. No collapse of the wave equation ensues because every known particle is in one such universe and hence, all conceivable histories exist (Everett, 1957; Tegmark, 2003). From an information-theoretic viewpoint, all potential information exists. If each possible particle state exists in this parallel scenario, then each possible information state exists. Information has been theorized to propagate from physical systems alone. In other words, no information exists without a physical phenomenon. An earlier attempt at formulating an information field theory based on classical field techniques assumed this very reductionist approach (Enßlin, Frommert, & Kitaura, 2008).

Quantum mechanics further supplements this condition by implying that no information may exist if both an observer and a physical event do not exist. Here information will be introduced as an epiphenomenon. Information in the form of abstract particles that each consist of an observer and event-generator entity pair will be constructed. It will be hypothesized that abstract subquark particles such as the helon and preon models can be represented by these information particle systems. Helons and preons are theorized topologically defined subcomponents of quarks, leptons, gauge bosons, and fermions and hence of most physical particles in the standard model (Bilson-Thompson, 2008). These information particles will be called *informatons*.

In the tradition of physical field-theoretic methods, a new generalized information field theory will be constructed utilizing the structure of the informaton and its holonic nature in building organization clusters, the things of organized energy-matter. These clusters will then be used to construct classes of complex organization, the complex adaptive multiagent system (CAMS) and its holonic cousin, the holonic multiagent system (HMAS). Next, the info-holarchy, a novel synthesis of these organization types via the proposed informaton model will be constructed. The abstract mathematical structure of such informaton-based holonic systems will be investigated utilizing the new generalized information field theory, the informaton particle model, and their descriptions using topoi, a category-theoretic mathematical structure that generalizes the notions of set-points, metrics, topologies, and geometries.

In this way, a very general physico-mathematical information framework will be developed that takes into account the most promising of proposed unification theories, i.e., theories of everything (TOE), such as quantum gravity in the form of loop quantum gravity (LQG) and related field theories. LQG was founded in principle by Asketar and expanded quickly as a bonafide theoretical formulation of quantum gravity and a unification of general relativity and quantum mechanics (Ashtekar, 1986, 1987; Rovelli, 2008). LQG posits that spacetime consists of a connected network of *spinfoams* that are of themselves topological structures that are consistent with physical measurements of particles and forces. LQG is preferred to other unification models because it has a cleaner and more simplified topological structure and is background independent – a crucial pre-requisite for physical unification frameworks (Markopoulou, 2007). In this study, information is conjectured to follow LQG in a special way by utilizing evolutional microscopic, macroscopic, and bridging mesoscopic metamodels of GTU processes that generalize Itô processes—generalized info-dynamics. Furthermore, utilizing the proposed informaton model, information manifests physical entities (i.e., the universe is a complex holarchical network of information particles interacting in a manner consistent with LQG and a generalized physical probability framework—GTU causaloids). This proposal will then have implications for a new paradigm of information-laden organizations (e.g., a general organization that is codependent on information technology is simply a special version of a holarchical network of informatons—the prototypical info-holarchy).

While the info-holarchy model is utilized in this study to represent the evolutional dynamics and morphology of complex adaptive multiagent systems (e. g., CAMs) its application will be be steered to frame general inference machines, particularly mammalian neuronal dynamics and to complex business organizations leading to a novel visualization of their evolutional development—holographic, multisenory performance dashboard-caves. The concept of a cave is described by a 3-D immersive visualization utilizing multi-sensorial apparatus for feedback and control. The content of this visualization is a class of evolutional patterns of the organization that subsume ordinary analytical devices such as graphs, tables, and classical statistical methodologies. Finally, the info-holarchy is grounded on first principles of information emanating from particle and large-scale cosmological physics, complexity, and evolutional studies—lending credence to the physical nature of information in forming general organization. Information, as is purported in this study, is not a second cousin to energy-matter in the nature of things organized, but is a first-order building block of reality.

Problem Statement

The problem researched in this study was the inadequate, indirect, and incomplete manner in which classical organization information paradigms through the use of business analytics and intelligence applications attempt to model the evolution of organizations. This shortcoming is based on the overarching and simplified assumptions made by business analytics and the classical statistical and optimization approaches used in such paradigms. Davenport, Harris and Morison (2010) point to several situations which business analytics and hence classical business intelligence systems do not address or mislead. Among them are (1) real-time analysis, (2) small probability events, otherwise known as long-tail events, (3) past history models which mislead, so-called black swan events (Taleb, 2007), (4) classical statistical and causal algorithms, along with pseudo-dynamic analysis from neural network or other architecture building modelers do not adequately handle human decision-making heuristics, and (5) variables that are in some analytic models unmeasurable. Moreover, results from such business analysis are oftentimes not causally connected to business phenomena in an accurate, adaptive, and timely manner (Adrian, 2009).

More philosophically, the clash of cultures between the paradigms of classical statistical and optimization methodologies and algorithm learning/simulation-based research has prompted the question of why predictive modeling is, at best, a learned phenomenon, regardless of the amount of prior history and Bayesian analysis applied to a situation (Breiman, 2001). This study purports to go one step further in eliminating classical prediction methodology, instead invoking "evolution patternization" through information dynamics as a superior means of prognostication. Evolution patternization is a methodology of visualizing real-time patterns of behavior through the dynamic flow of information in different scales and levels in an organization in order to make an inference on its characteristic profile. The morphology of a pattern life of an organization replaces the folly of gleaming from a statistical profile based on overly restrictive assumptions and misleading model paradigms. Algorithmic (machine) learning helps in this quest, but concentrates instead on singular characteristics of an organization. This study endeavors to capture an abstract approach to the visualization of evolution of organizations, not merely a single or limited thread of activities.

Parallel to this development is the way in which information is treated informally in such theories. Information is handled as a secondhand characteristic not based on first principles of physics. Specifically, state-space models based on classical physics do not explain emergent behavior in organisms whose dynamicism may be nonlinear, chaotic, quantum-entangled, or have nonclassical probabilistic behavior such as higher order fuzziness or unknown adaptive mechanisms. Information will be treated as a first principle in this methodology. This proposal will be established by constructing a general information-theoretic metamodel for matter and processes based on ideas from contemporary physics, complexity studies, and network and organization theories, and represented through topos theory, a set-theoretic generalization from mathematical category theory—a metamathematical framework. Information represents the life-blood of any material in the universe. Its dynamicism is the most important process in forming matter and organisms on all scales. To this effect, the physics of business organizations endeavors to introduce rigorous and emergent physical models to better patternize business organization. It involves the abstraction of physical theories applied to the processes and structures of business organizations. Business objects are viewed as organisms with adaptive life cycles that follow fundamental emergent laws inherent in our best physical theories.

No longer may classical theories of organization and management that utilize statistical modeling and nonadaptive processes adequately describe business realism. Additionally, Newtonian mechanics serving as the paradigm for command-and-control type organization dynamics extrapolated from the Industrial Age to the Information Age, severely limits the current business-sphere in ecological, societal, and technical ways (Engdahl, 2005). This classical business paradigm is no longer self-sustaining nor is it appropriate for the highly adaptive and complex nature of the noösphere—the collective conscious and intellect of humankind (de Chardin, 2008; Vernadsky, 1945). As the major dynamical structural tool developed in this study, holarchies are sets of self-similar whole-part holon nodes that form practical networks for adaptive organizations (Koestler, 1967/1990). These are antithetical to hierarchies. When these holons are endowed with complexity, adaptability, reasoning, and self-awareness they take on the characteristics of CAMSs, thus serving as prototypical computational complex organisms. Other instances include computational devices, social constructions, cosmological components, and subatomic particle systems.

Unfortunately, CAMSs and their holonic flavors, holonic multiagent adaptive systems (HMASs) are difficult to model and predict with. They are more adequately used as patternizers of organizations. Nonetheless, HMASs and CAMSs hold the promise of adequately representing the dynamics, phenotype, and morphology of natural systems. Information systems theories (ISTs) portend to model information flow in simple systems such as finite feedback discrete networks with minimal exogenous time-dependent input. However, a more general emergent model of IST adapting general uncertainty features such as quantum, fuzzy, probabilistic, beliefs, and evolutional processes may better serve to represent CAMSs and natural complex adaptive systems (Snooks, 2008). The lack of a comprehensive and unified physical model of information makes the task of constructing complex systems based on microscopic information-based processes difficult and disconnected. In addition, a-priori macroscopic processes such as rule-based game theoretic dynamics are imposed on CAMSs without regard to any natural evolutionary microprocess or mesoscopic bridge between and within agent entities.

To this end, a hybrid research methodology will be applied to this study as a means of constructing the abstract metamodel for an information-based organization theory. This methodology focuses on finding generalizations to currently researched and sometimes universally accepted paradigms in abstract mathematics and mathematical physical theories and not on data per se (Brown & Porter, 2004). Rather than form models from data, as is done in traditional social science grounded theory, this hybrid method, rooted more in mathematical research as generalization and induction, is based on constructing general patterns from prior theories of modeling, then building more powerful metaphors and metamodels, rather than apriori, presenting a model and then testing it against data generated from sampling. In this study, these generalizations are manifested by a metamodel of information dynamics based on first principles of physics and computation.

The methodology applied in this study is a variant on a pseudo-qualitative approach to abstract mathematical modeling. Abstractions are based on general patterns of process and structure and their evolution. Patterns of behavior of matter and structure in organization are the central issue, not the mostly mistaken and abused methods of quantitative modeling and prediction that fall short of consistently bringing forth a better understanding of the process and properties of emergence and the evolution of organization from information dynamics. Business analytics which are dependent on classical statistical methodologies of causal/explanatory and correlation/prediction/control do not address nonlinear dynamics (which include deterministic and stochastic chaos), evolutional behavior, self-organization, and other emergent phenomena. In this study, real-time organization pattern analysis, through the use of novel nonclassical information-theoretic notions, replace these mindsets.

Purpose Statement

The purpose of this research was to contribute to a three-fold goal that addressed the incompleteness of classical organization theories to universally model organization information dynamics. This goal consists of (a) developing a robust physical theory of information based on a novel concept of information field and particle, the informaton, akin to physical field theories (Kaku, 1999; Weinberg, 2005), (b) modeling the structure of prototypical organizations as complex emergent intelligent organisms technically defined as intelligent holonic multiagent holon systems (HMASs) structured over a generalized probability and causal manifold—the causaloid model Hardy (2008), and (c) developing an information theory based on a (1) leading background independent (spacetime not assumed to be fixed in order to resolve ultra-violet divergence) unified physical theory—loop quantum gravity Rovelli (2008), and (2) a general theory of uncertainty (GTU) which generalizes quantum, fuzzy, and classical and nonclassical probabilities Zadeh (2005).

Additionally, microscopic general uncertainty Itô processes, models, and games and generalized entropy macroscopic processes with binding mesoscopic control functions will be developed to describe the evolution of holarchical systems. This triad of connected and differently scaled processes defines an extension to the concept of info-macrodynamics which utilizes variational principles—minimizing energy from uncertainty in observations and maximizing entropy to dynamically define system constraints that shape the organization of entites in that system (Lerner, 2004). Prototype adaptive holarchies as holonic multiagent systems (HMASs) were abstractly modeled. In this way, a more powerful method for constructing a programmable structure that describes the general patterns of behavior and dynamics of organization can be implemented. This metamodel will be fundamental in overcoming the inadequacy of static or classically dynamic models that purport to predict and model phenomena.

Nature of the Study

This study employed a hybrid grounded theory methodology for presenting a new variant on abstract mathematical information and organization theories. Grounded in this context means constructing abstract theories of metamodels based on prior well formed and tested theories, not on data. Classical techniques as done in quantitative analysis for prediction and modeling will not be used because they are woefully inadeqauate for gleaming the holistic patterns appearing in nature and in human-inspired constructs. Dynamic models for information flow within an organization will be proposed based on well known classical and nonclassical paradigms such as quantum mechanics, general relativity, quantum gravity, complexity, emergence, dynamic and general systems theories, network theory, organization theory, game theory, and uncertainty frameworks such as general fuzzy, intuitionistic, and other nonAristotelian logics.

These models will then be synthesized to construct a proposal for universal information dynamics in general organizations. Abstract metamodels generalized from these approaches to information dynamics and organization were constructed to emulate natural and man-made systems. Rather than measuring the accuracy of these representations, general patterns will be compared, as in higher order mathematics (Brown & Porter, 2004). The abstraction of important patterns of organization and information dynamics will be held to be more important and applicable to constructing an information-theoretic based metamodel for matter and organization than quantitative studies which are limited in their scope because of the use of apriori model assumptions. In this study no static models were assumed for human organizations such as business entities.

Dynamic abstract structures are to be constructed based on the informational framework build from the premise of a generalized information theory and the informaton model of this research study. It will be proposed that complex adaptive systems which are holonic in structure, otherwise known as HMASs, will be created, evolve, and terminate based on the general information model and dynamic aforementioned. Theory-based simulations dependent on these mathematical models may be designed in followup studies which will attempt to mimick inference machines, particularly, the structural form of brains and of a general organization information flow via a holographic dashboard-cave as proposed in this study. Patterns of behavior from such simulations may then be compared to their real world physical analogies. Additionly, it is the intent of this study to design information dashboards that will aid in the visualization of such simulation and phenomena comparisons. Such comparisons will include spacetime historicities of these differences. At each spacetime coordinate, a divergence measured the difference between a generated observable from the simulation and its real system counterpart. Mathematical holographic representations of such system observables served as a framework for these immersive visual dashboard presentations.

Research Questions and Hypotheses

The major construct of this study is the conceptualization of a new information calculus for constructing general organization. This calculus emanates from (a) an abstract information particle (the information) and an accompanying Bayesian statistical field theory based on a general framework for causality (the causaloid), a unified quantum gravity theory for spacetime (loop quantum gravity), a generalization of uncertainty, holonic organizations, and high-level mathematical representations of these; (b) the duality and entanglement properties of the event-observer model for informatons; and (c) a multiscaled dynamic process mixture for a multiagent system of autonomously acting entities, the generalized Itô process defined by an info-dynamic framework. Current models for complex systems either macroscopically describe ensemble behavior or microscopically describe inter-agent dynamics, as in physical models of subatomic particles. These statements beg the following research questions relevant to this study:

1. Can the proposed metamodel of this study, the info-holarchy, adequately categorize evolutional patterns for general organizations?
2. Can a mesoscopic bridge be built from this study's metamodel that can adequately adjoin its macro and micro subcomponents in a realistic and verifiable manner?
3. Can a high-level, general mathematical framework using topos theory, causaloids, intelligent multiagent systems, and nonclassical physical theories and logical systems be built as an information calculus for complex organization?
4. Will general models constructed from such an information calculus more aptly describe emergent and evolutionary systems?
5. Can information, viewed through the lens of this study's proposals, (a) an abstract particle, the proposed information, (b) a generalized information field theory, and (c) a process model (GUT info-macrodynamics) that incorporates micro, meso,

and macro aspects of an organization; be elevated to the stature of energy-matter as a building block for nature?

6. Can this study's metamodel be appropriately and functionally specialized to inference organisms such as neural systems and to complex information business organizations?

This study hypothesized that the proposed info-holarchy metamodel functionally described general patterns of behavior and dynamics of organisms and organizations based on a calculus of organization of physically inspired intelligent and self-aware HMASs that follow the physical rules of the very small and large worlds and of the mesolevel observation scale of anthropoids. The main hypothesis of this study can more compactly be stated as: the info-holarchy metamodel as constructed from contemporary quantum gravity, general nonAristotelian logic, and uncertainty, utilizing the abstract representations of topos theory and generalize probability from the causaloid model more powerfully patternize organization evolution in an information-based universe.

Theoretical Base

Classical statistical methodologies and physical theories no longer can adequately describe the evolutionary dynamics of living organizations nor those of general material morphogenesis that develop from holistic multiagent groups such as cosmological clusters, techno-socioeconomic groups such as businesses, or the simple evolution of automata (Kilmann, 2001; Marion, 1999; Taleb, 2007; Wheatley, 1996, 2006; Wolfe, 2009; Wolfram, 2002; Youngblood, 1997). Since the inception of quantum mechanics, general relativity, nonlinear dynamics, evolutionary and adaptative modeling, and the complexity sciences, the proverbial game of modeling the universe and life has changed (Laszlo, 2004). There are now metamodels in use for more accurately describing patterns of development of these multiagent organizations and organisms. The foundation for the info-holarchy—this study's metamodel for patternizing organization—is taken from the scientific literature of chapter 2 that outlines the common properties of information from these new emerging sciences and investigations. In particular, by expanding and generalizing the core ideas of emergent studies reviewed from the literature, this study develops and introduces (a) novel methods for describing the unification of GR and QM, in the form of an entanglement information-based loop quantum gravity, utilizing a general causality framework for physical theories—the causaloid, (b) forming a new holistic physical form of information that generates matter and organization—the informaton, and a comprehensive Baysian statistical field theory, and (c) from complexity, network, and organizational studies, a general framework for structural organization—the info-holarchy.

In this study, theoretical prowess emanates from the physical theories of QM and GR, the sciences of complexity, the phenomena of emergence in the nonlinear dynamics of chaotic, adaptive, and multiagent systems; and from cutting-edge ideas from the fringes of post-modern stochastic and nonAristotelian logics. Patterns of evolution that are postulated from these new paradigms give a sense of the more important holistic properties of the universe, oftentimes supplanting the details of ill-advised and narrow predictive modeling done through the lens of classical thinking. The domain of business management guided in large part by the premises of model-predict-and-control organization theories are ostensible examples of the limits of classical mechanics applied to ultra-complex human situations (Olsen, & Eoyang, 2001). One need not go far to find important examples of systemic meltdowns emanating from unpredictable black swans, long-tail phenomena, positive feedback downward spirals, and bifurcations (e.g., the 2000 dot-com bubble burst and repeat potential (Gaither & Chmielewski, 2006, July 16), the 2007-2009 US banking, financial, and housing collapses (Foster & Maqdoff, 2009; Krugman, 2009), limitations in controlling terrorism (Gottlieb, 2010), successful but unexplained suboptimal technology markets (Taleb, 2007), and information-overload and confusion caused by the ubiquity and nonuniform verifiability of the contextual world-wide web (Lanier, 2010)).

The info-holarchy paradigm of this study is a possible way out of the doldrum of control complacency and misdirected business mindsets. Info-holarchies manifest intelligent informational multiagent-based simulation of organization evolution—paving the way to novel methods of viewing pattern dynamics and recognition. This patternization dynamic replaces the control aspect of classical paradigms. Business concerns the natural organization and evolution of things, more than classical business analytics—having a false perception of being capable of optimizing an *n*-sum game relative to one's environment. Business strategies—the metaprocesses of organizations, must not only be adaptive, but must also recognize spatio-temporal pattern gradients (rates of change) occurring at different scales and levels within the organization. It is posited in chapters 4 and 5 that these pattern gradients and evolution of morphology of organizations are more important and accurate indicators of business health than control-centric measurements, prediction, and reaction.

Definition of Terms

Category theory: a branch of mathematics that generalizes the concepts of geometry, logic, sets, topology, and other mathematical abstractions (MacLane, 1971).

Causaloids: a novel framework for constructing mathematical physical theories that incorporate indefinite causal structures (possibly ambiguous, disjointed, or nonexistent temporal ordering) with a probabilistic reasoning calculus (Hardy, 2005).

Clifford algebra: are generalizations to the algebras of complex and quaternion numbers—octanion algebras. They are also used as matrix algebras and act on spinors used extensively to represent physical spaces in quantum field theory (QFT) and string theories (de Traubenberg, 2005). Real Clifford algebras are associative algebras generated by a unit element *1* and $d = t + s$ elements, $e_1, ..., e_d$ satisfying the relations

$$e_m e_n + e_n e_m = 2l_{mn}, \text{where } l_{mn} = \begin{cases} 0, & \text{if } m \neq n \\ 1, & \text{if } m = n = 1, ..., t \\ -1, & if m = n = t + 1, ..., d \end{cases} \qquad (2.4)$$

Here, Clifford algebras are used to generalize holographic representations for holonic objects which are to act as metaphors for information particles (see the holography term below).

Complex adaptive systems (CASs): complex dynamic systems which contain large number of subsystems with highly adaptive characteristics and connectivity including some reasoning and self-organization capabilities. They are coevolutional in behavior.

They were first named by Gell-Mann and Holland during their work at the Sante Fe Institute (Waldrop, 1992).

Connection: given an *n*-dimensional manifold, *S* (an open subset of $\mathcal{R}^n$), consider the tangent spaces $T_p(S)$ at each point $p \in S$. A connection (affine), Π, is a set of (linear) mappings (C^∞ in each coordinate), $\Pi_{p,q} : T_p(S) \to T_q(S)$ that can be expressed as: $\Pi_{p,q}((\partial_j)_p) = (\partial_j)_q - d\xi^i (\Gamma^k_{ij})_p (\partial_k)_q$, $j = 1,\ldots,n$ and $\left\{(\Gamma^k_{ij})_p; i, j, k = 1,\ldots,n\right\}$ are n^3 reals that depend on *p* and that are C^∞ (Amari, 1993, p.13). Intuitively, Π is a system that connects (relates) the tangent spaces of a smooth manifold in a smooth manner to each other. This sets the stage for defining transport mappings that carry points from one tangent space to another in a smooth manner, the so-called parallel transports.

Emergent game theory: a series of game theoretic concepts involving the emergent fields of quantum information, evolution, chaos, and general uncertainty. In the context of this study, emergent game theory will depict a synthesis and general model of emergence in game theoretic processes.

Holarchy: the organizational structure of a group of holons, which are subsystems that exhibit internal dependence on lower-level subsystems, but independence of higher level subsystems while retaining high adaptive connectivity at all levels. This is a generalization of a hierarchy (Koestler, 1967/1990).

Holons: generalization of entities that are both wholes consisting of parts (other holons) and parts of other wholes (holons). The term was first coined and described by Koestler (1967) and has been expanded by others including most notably, Wilber (1995), his collegues and critics (Edwards, 2003a, 2003b; Fisher, 1997; Meyerhoff, 2003; Smith, 2009).

Holography: the development of a 3-D (*n*-dimensional) image by superimposed 2-D (*d*-dimensional, where $d < n$) images taken at various points in a representation space. The removal of any small number of projection subimages will not appreciably distort the full image. The analogy to information fields is to represent a multiattribute holon, such as an informaton-based holarchy (info-holarchy), by hypercomplex numbers which are then compressed using the linear span of octonions (or generalized Clifford algebras).

Hypercomplex numbers: a generalization of complex or real-plane number systems, they are represented by generalizations to complex numbers such as octonions which are numbers that are represented by a span of 8 complex numbers. Clifford algebras are higher dimensional hypercomplex numbers. Hypercomplex numbers can be embedded into simpler expressions in order to compress high attribute objects. Hurwitz's theorem showed a limitation to hypercomplex numbers in the sense that higher order hypercomplex algebras can be represented by isomorphisms to octonion or lower level hypercomplex algebras, i.e., octonion algebras exhibit the richest structure for hypercomplex numbers (Hurwitz, 1898).

Qubit: an information container than generalizes the classical information Boolean bit which contains either 0 or 1, to a quantum linear and convex (combination) superposition of such classical values, and is expressed as (Schumacher, 1995):

$$|\varphi\rangle = \alpha|1\rangle + \beta|0\rangle, \;\; \alpha + \beta = 1, \;\;\; 0 \le \alpha, \beta \le 1 \qquad (2.5)$$

Information field: a mathematical field equation describing the flow of information as a physical-like field using relevant properties of symmetry, coherence, invariance, and diminishing information.

Info-macrodynamics: a mathematical framework incorporating Itô stochastic calculus to describe microstate dynamics while bridging ensembles of these microstates by information entropy to macrostates for controlled information flows (Lerner, 2003)

Informaton: the concept of a unit of information to be represented in a mathematical information field. This is a structured generalization of the fundamental computing units of bits in classical mechanics and qubits in quantum computation and communication.

Manifold (topological): an *n*-dimensional space that is Hausdorff (open sets separate points), second countable (has a countable cover, i.e., a countable number of opens sets that cover the space), and is locally Euclidean—that is, every point has a neighborhood—(an open set containing it, that is homeomorphic) and is continuously mapped to each other bidirectionally, either to an open sphere in the *n*-dimensional reals or to the *n*-dimensional reals itself (Lee, 2000). Manifolds are then generalizations of spaces that have a similiar topology to that of a Euclidean space. Differential manifolds share differentiability features as well as topological features (diffeomorphisms) with Euclidean spaces.

Planck scale: the smallest scale of energy in the universe that is consistent with any singular discrete movement of light-waves, rated at 1.22×10^{28} eV and corresponding to its mass-energy equivalent of the Planck mass at 2.17645×10^{-8} kg. This scale also corresponds to a Planck time, 5.39121×10^{-44} s, the time light takes to travel in a vacuum of Planck length, $l_P = 1.61625281 \times 10^{-35}\,\mathrm{m}$, and its corresponding Planck area, l_P^2 and volume, l_P^3. Theoretically, at these scales, gravitational forces are on par with the other forces (Baez, 1999).

Topos: a basic abstract mathematical object representing a category of sets and arrows (maps) between those sets in which (a) exponentiation of an object by another is done by the set of maps between those two objects, (b) a subject map or classifier exists that maps the identity set to the universal set, (c) there exists a terminal object, one in which for every other object in the category, some arrow exists that maps it back to the terminal object, and (4) an initial object, one in which for any other object, there exists an arrow that maps the initial object to that object. Topoi generalize the concept of sets and organization in any mathematical or computational setting (Goldblatt, 2006; MacLane & Moerdijk, 1992).

Assumptions

The informaton particle model resembles a bipartite entangled quantum system. Therefore, the subcomponents in an informaton or informaton cluster are not quantum separable. The observer and event subcomponents of an informaton are entangled via the information flow that connects them. This dual component is the basis for constructing other informatons as the microlevel apparatus for HMASs. Informatons are an abstraction of information flow, not seated in any known physical counterpart.

Nevertheless, information, as an abstraction, has been linked mathematically to thermodynamic flow and energy, two hallmark observables of physics. The standard model (SM) of particle physics consists of a hierarchy of fundamental particles – bosons (forces) and fermions (mass), the latter being further reduced into quarks and leptons as the constituent components of matter. The theoretical particle mass-originator Higgs boson is predicted to be the constructive component of all SM fundamental particles, but is presently not confirmed by experiments, although situated for indirect confirmation in near future Large Hadron Collider (LHC) experiments.

Limitations

Generalized information fields are conceptual objects that describe how information processes may flow or be contained from one general information system to another and within each. It is further based on the concept of physical fields which are mathematical smears or statistical estimates of ensembles of individual particles that are much too plentiful and complex to be described deterministically. The smearing functions in physical fields are usually statistical functionals of attributes of these particles, such as ensemble averages or moments. There is no physical experimental evidence for the detection of and existence of information fields in quantum systems. They are instrumentalist tools for grasping the movement of hypothetical information particles that are here labeled as informatons. Informatons consist of bi-partite entangled systems representing the event generator and an observer. The construction of such information particle systems makes it possible to visualize general quantum observer-dependent realities as a network. The dynamics of such bi-partite systems are then motivated by microscopic stochastic processes, mesoscopic bridge ensembles and macroscopic classical information theory.

Scope and Delimitations

While the info-holarchy metamodel can be applied to any organization or structure that represents an organism, the abstraction of the model limits the scope of the study to generalized patterns of behavior and dynamics of such entities. Detailed quantitative modeling or prediction of localization dynamics is not specifically addressed. Information organization is posited to be an emergent and evolutional process and structure. Hence, the bounds of this study are reflected by the lack of local description in the globality of patterns of organization. No quantitative studies were done - no samplings of a realized phenomenon were compared to a simulation based on the info-holarchy metamodel. Quantitative methodologies are unrealistic and impractical in this study based on the sheer volume of data that would have to be generated or collected. Patterns are king in this study.

Significance of the Study

Presently, no unified theory of information exists. There are two opposite dialectic poles. Analytic credibility comes from the Shannon era of information theory. In Shannon (1948), information is a measure of entropy, the amount of uncertainty that remains about a signal after its physical retrieval. In this definition, an a-priori probability distribution is used to describe the scattering of values that a signal may propagate. The classical Shannon entropy is given by:

$$H(X) = \sum_{i=1}^{n} p(x_i) \ln p(x_i) \tag{2.6}$$

for a finite discrete random variable *X* taking on possible values $\{x_1, \ldots x_n\}$ with probability distribution function $p(x)$. If *X* takes on an infinite number of values, then the summation is infinite. If *X* is continuous then the summation is replaced by an appropriate integral taken over the space of all permissible values of *X*, i.e., an integral measure is used (e.g., Lebesgue, for a real space).

Other measures of information regarding and comparing two random variables can be constructed from this definition, namely, mutual information and a divergence measure between the two respective pdfs of those random variables. Generalizations of Shannon entropy have been constructed based on the notion of thermodynamic entropy, including α-Rényi entropies and its predecessors and nonclassical entropies from quantum mechanics and deterministic functions. In all these Shannon entropy-defined measures of information, no measure of semantics or meaning is given or implied. In fact, no meaning of receiver interpretation is attempted. In addition, the use of entropy as a measure of uncertainty or disorder is a relativistic concept. Disorder of a system is a function of the interpretation of its contents and organization. Attempts have been made to define structural complexity as a means to describe disorder in an organization based on entropy measures. These have included thermodynamic depth and computational mechanics (Lloyd & Pagels, 1988; Crutchfield & Shalizi, 1999). However, these measures remain content-meaning deaf and do not address the ability of information to organize or build organization.

Shannon information is quantitative—a means to optimize the amount of physical bits into a closed system. Nonetheless, there exists, in concept, two other types of information: shaping and semantic information (Ward, 2010). Shaping information is the coding necessary to build or construct organization, the general DNA of organization. Semantic information is that which is necessary to give meaning to objects, the mapping or interpretation of content to things. Shaping information is the scaffolding or essence of (self) assemblage. The physics of organization through the use of network dynamics attempts to address shaping information. This study approached shaping information through the concept of holons and holistic organization. What of semantic information? Enter the concept of semiotics (i.e., the study of signs—what Peirce hypothesized as the atoms of interpretive communication and its sister concepts of semantics and pragmatics (Korzybski, 1994; Peirce, 1931)).

Semantics attempts to measure how information is interpreted and what meaning is assigned to it. The general semantics project, as developed by Korzybski (1994), expands this premise by differentiating human expressiveness of observation from the state or condition of reality of the object being described. Semiotics is the study of how signs interact with each other. Pragmatics studies how signs are interpreted by and tied to observers. Signs are to semiotics what bits are to entropy. The studies of linguistics, semiotics, semantics, and pragmatics attempt to qualify information, while Shannon-Boltzmann type entropies attempt to quantify it.

In this study, particle entities of information have been defined. These entities, *informatons*, consist of generalized-uncertainty entangled bi-partite particles that, in total, contain the physical information to propagate, the media (spacetime or matter occupying spacetime) that it travels in, and the receiving interpretative partner, the interpretant that emanates the linguistic mechanisms. These linguistic mechanisms include those of its signs, intra-sign functionals, and its maps to an observer particle. An attempt is then made to mechanize informatons by the concept of an appropriate version of an information field. Once these structures are laid out, it is posited that informatons can drive the construction of organized evolutional groups of networks—holarchies and their calculus.

The significance of this study is the presentation of an alternative theory for a more complete description of information and its connection to the genesis and sustainability of general intelligent multiagent systems as universal organizations. These constructs are manifested through the use of holarchies, a new information particle, and field concepts for information organization. Info-holarchies are propelled, in part, by the dynamics of general quantum entanglement manifested in the loop quantum gravity physical model. This implied information dynamic may be applied to intelligent adaptive organizations. The evolution and management of information in these organizations, in turn, may then be described more succinctly—monitored and manipulated from virtual reality performance dashboard-caves. The performance measurement of causative agent subgroups within an organization, along with the surveillance of their respective (game) strategies, achieves a new perspective in business analytics and intelligence applications.

Proposed dashboard performance metrics are anchored in the physical and logical systems described in this study. Hence, a generalized information management theory for organizations is proposed. I propose that the info-holarchy serve as a metapattern for specialized dynamic information models applicable to business organizations and their skeletal information intrastructure. The goal is to construct, using this class of specialized dynamic models, multidimensional performance dashboards that tie together the flow and symbiosis of information content, organization complexity, and their dynamic, real-time effect in determining the integrity of those organizations. This concept of immersive displays would represent a natural evolution in performance dashboards from web portals displaying side-by-side classical and traditional performance measures, analytics, numerics, and pivot tables to ones emanating visual, holistic, multidimensional, and holographic views of the dynamic health of organizations and their supporting and component substructures. These displays represent more effective and enterprising versions of a bird's-eye-view for organization leaders and their decision-making potential. With this more powerful description of the dynamics and evolution of organization, the lifecycle of entities can be patternized to dramatically improve their maintenance and economics.

Implications for positive social change in this study include having a dramatic constructive impact on viable societies and groups that thrive on successfully networked organizations and their respective cultures. Specifically, this study posits that an info-holarchy more adequately describes a general information-based society, the prototypical organization of a heightened technological epoch. Additionally, the hypotheses and results of this study represent a positive change to the manner in which societies can govern themselves—the autonomous creation of structure and processes. Finally, this study presents a metamodel that represents a fuller picture of information around and in one's life, within and outside of the necessitated societal groups surrounding that life. In this study, businesses are treated as specialized socio-economic groups and hence follow these patterns of information engagement, this study's major positive impact on society's business space.

Summary and Transition

In this chapter the concept of info-holarchies constructed from a generalized information field theory and bipartite information particles labeled as informatons was briefly introduced. Model limitations, assumptions, their significance in the conceptual models of information, research hypotheses of the information model, terms to be expanded on and utilized, the flow and mature of the study to be conducted on these models, an ontology to the study, and a general background to the relevant conceptualizations were all discussed and reviewed in anticipation of the development of proposed concepts.

In chapter 2 a thorough review and expansions on existing ideas of micro, meso, and macroscopic processes to be utilized as components in a model for this dynamic information framework will be presented. Previous studies into the modeling of information, mathematical network organization construction, quantum dynamics, and general uncertainty principles will be reviewed and studied in detail for the eventual usage in building a novel information field, particle, and network calculus. Further in chapter 2, the evolutional behavior of and emergence within information and organization emanating in concepts from quantum gravity, general uncertainty, complexity, networks, intelligent CAMSs, and holarchies will be exploited and generalized for info-holarchies.

In chapter 3 the research methodologies leading to the generalization of physical and logical models of information, matter, and organization manifested by the study metamodel—the info-holarchy will be discussed, along with the study's ethical considerations, theory construction as a metaphor for data collection and sampling, and the researcher's role in this relationship.

In chapter 4, these tools will be presented as the foundation for a calculus in constructing intelligent multiagent networks serving as the structural prototype for info-holarchies. This information model is posited to unify the quantitative strengths of entropy-based information and the qualitative advantages of the interpretive prowess of the linguistic studies and models of semiotics, semantics, and pragmatics. These novel concepts will be developed and exposed. These metamodels will be specialized and staged as novel models of information and organization by their respective application to higher pattern analysis and seeking of certain network organizations in nature and man-made societal artifacts. Some of these examples will include the modeling of dendrite, neuron, and axial network development in brains and their generalizations in inference machines and the development a novel holographic temporal performance dashboard for information flow in business organizational dynamics.

Finally, in chapter 5, a summary of concepts and applications discussed in this study will be presented along with computational thought experiments, proposed ideas and appeals for possible future research and development.

Chapter 2: Literature Review

Introduction

In this chapter a review of classical and contemporary developments of information in physical theories and organization will be done. Some expansions and generalizations will be established alongside the reviewed theories, to be utilized in the construction of a new metamodel for information in this study. Nonclassical physical theories include QM, GR, and so-called unified theories of everything including loop quantum gravity (LQG). Classical physical theories include Newtonian and celestial mechanics and their extensions in deterministic paradigms of interaction. Statistical and information theories are stochastic, but expound classical ideas in the sense of mostly being linear and straight-forwardly causal. Emergent paradigms such as complexity, emergence, chaos, fractility, self-organization and evolutionary sciences are holistic in nature and nonclassical in approach.

This study will utilize these nonclassical physical and information-theoretic approaches in constructing a generalized information theory and particle model to connect a physical concept of information to complex adapted multiagent systems (CAMS), holarchies and holons in organization, generalized uncertainty principles, information semiotics, nonlinear dynamics, and mathematical objects - topoi and causaloids used for constructing higher order abstractions of information systems. The two major premises of this study—the informaton particle and field-process theories of information for organization evolution and dynamics—will heavily utilize and generalize concepts borrowed from this section.

This chapter will be organized as follows: (a) Classical information theories will be reviewed, along with contemporary extensions, including and most importantly, quantum information, (b) LQG will be introduced to be used as the physical basis for information structure in current physics literature, (c) field theories will be reviewed as a basis for an theory of information fields, including quantum fields, an earlier approach to a Bayesian information field, biological fields, and the classical physics fields of electromagnetism, (d) adaptive agent systems via complex adaptive systems and holonic agents will be reviewed as the network structure for the later development of the info-holarchy, (e) a general introduction to the components and properties of complexity and complex systems will be started as a means of incorporation into an information-based physics model, (f) adaptation, evolution, and holarchies will be reviewed to inspect the very important properties of organization in dynamic entities and as a requisite part of this study's metamodel, (g) nonlinear dynamics will be touched upon to describe the processes involved in a robust system, (h) quantum game theory will be introduced as a lead in to developing more general game-theoretic structures for the info-holarchy, (i) semiotics, as developed by Pierce, is described for forming a generalized theory of perception of information, and finally, (j) the topos theory of mathematical category theory is introduced as a means of more adequately describing the general power of this study's models.

Along with these discussions will be the introduction of a robust generalization to probability and causal analysis as applied to physical phenomena, namely the causaloid structure (Hardy, 2005).

In this section, studies of prior models of contemporary information theory and its extensions to nonclassical settings, hypothesized novel physical theories, complexity studies, evolution, adaptation, emergence, causaloid and topos and category theory have been garnered from peer-reviewed science journals and the ubiquitous Cornell University Library's arXiv.org e-print archive repository for scientific articles that are fast tracked for the scientific community. The premise of this study's literature search was to surmise the most current, interesting, and novel approaches to information, physical theories, organization, and their respective abstract representations. In particular, my interest laid in the past work done to promote any development of an information-based theory of physics from first principles.

The literature reviewed was generally of abstract models of information, proposed novel physical theories, abstract field theories, general discussions on emergent sciences such as complexity, evolution, adaptive systems, holarchies, and finally, higher order mathematical representations of abstract objects. In this manner, this study is a hybrid grounded study based on a tradition of abstract mathematical research, building on generalizing patterns of prior work on abstract models and logic (Brown & Porter, 2004).

Quantum Information and Field Theories

Wheeler (1990), the highly influential theoretical physicist, first coined the phrase, "it from bit" to posit that all physical reality emanates from a computation involving information bits (logic units of any base, most notably Boolean) and their respective transport. Wheeler's thesis was based on digital physics, the category of concepts that postulates that the universe is actuated via a discrete manifestation of a spatiotemporal medium, via information. The concept of a universal hyper-computer which computes the evolution of the universe in existential real time lies at the foundation of this proposal.

Fredkin (2000) and Zuse (1970) were the first to publish general hypotheses of universal computers using reversible automaton to achieve this. Fredkin generalized the concept to cover a discretized version of the philosophy of scientific development known as digital philosophy and digital mechanics, an atomism reducing all of life to categories of finite automata. This idea has been most recently championed by Wolfram (1983,2002) utilizing his taxonomy of cellular automaton and his principle of computational equivalence (PCE) which describes a concept of computational categorization. Lloyd (2006a, 2006b) followed up on this line of thought with quantum and black-hole versions of universe-wide computers. See Appendix B for my version of a quantum-gravity universe hyper-computer and chapter 4 for a higher-order computation based on generalized fields—morphic and informaton computation, that dramatically extend the traditional information bit representation.

Despite the vagueness, oversimplification, and nonoriginality in Wolfram's statements, digital physics as had a rigorous upheaval with the recent advent in quantum information theory. Wheeler conceived a gedankenexperiment by devising a quantum version of the popular 20-questions game. In this game, a participant is allowed to ask 20 questions regarding an entity that they must then guess by the end of the twentieth question (or sooner). The answers must be Boolean: yes or no. In Wheeler's version of this game, the participant asks questions to 20 different people and each person asked a question must decide what the entity is prior to the question. The entity selected may be different from the others prior to each question. In this manner, the ensemble of 20 people emulates a quantum superposition of information on the state of an entity. Each question then imitates a measurement or observation of the experiment. The answer to each question is not known until an infinitesimal moment (perhaps one Planck time unit, $t_p = \sqrt{\frac{\hbar G}{c^5}} \approx (5.39124...)10^{-44}\text{ s}$) just prior to the question being asked because the concealed entity involved has not been decided upon by the confidant until then.

Conceptually, this quantum binary or qubit game could be the basis for finding an answer to any informational question as presented in the universe. Wheeler's version of the 20-question game is observer-participant dependent because the answer has not been decided until the last possible moment before asking the binary question. His emphasis on the physical ubiquity of qubit information motivated work into the possible discrete representations of the universe via quantum computation with generalized qubits. More basic is the result that the number of binary questions necessary to converge to an answer to an original question with random variable *X* is proportional to $I(X) = -\ln p(X)$. The entropy of X, $H(X)$ is the expectation of $I(X)$. For a quantum question game, this requires the quantum density operator of *X*, ρ. This quantum extension to entropy comes in the form of the von Neumann entropy, $S(\rho) = -Tr\rho\ln\rho$. Quantum entropy will be generalized later in this chapter and in chapter 4 utilizing the GTU and other generalizations to classical entropy.

One of the main and most perplexing tenets of quantum mechanics is the observer-participant dependency. Observations of a quanta, the generalization of a physical particle, are mathematically represented by a self-adjoint operator, A acting on the state vector, $\psi(x) \in \mathcal{H}_0$ of the quanta with spatial-temporal coordinates x and $\mathcal{H}_0$ a complex separable Hilbert space. The observables can have possible values in the spectrum $S(A)$, the eigenstates of A. States evolve based on the Schrodinger equation:

$$i\hbar\partial_t\psi(t) = H_0\psi(t) \tag{3.1}$$

where H_0 is the Hamiltonian operator that corresponds to the energy level of the quantum system. If observables are used instead of states, then observables evolve based on the Heisenberg equation:

$$\frac{d}{dt}A(t) = -\frac{i}{\hbar}\left[A(t), H_0\right] \tag{3.2}$$

Hence, a quantum system is defined by an algebra, $\mathcal{F}$ of operators defined over $\mathcal{H}_0$. This is a continuum problem although a quantization can take place for physical quantities of the system especially at the Planck scale (Madore, 1992). Digital physics states that a quantum system is manifested in a complete or a spectrum of discrete spaces instead. The quanta in a physical system move in an unknown potential $V(x)$. At best only incomplete information about $V(x)$ is possible. Enter perturbative techniques for finding local approximations via a quantization of the system. Quantization is the separation of classical and quantum parts of the system around these locales using large numbers of quanta representing a condensate state (Rozali, 2008). A local approximation to $V(x)$ in a location x_0 using quantization and perturbations is called background dependence. Long evolutions are hard to compute away from these locales and so these methods are used to approximate the quantum states locally. Consequently, an important property that a physical system should retain is that of background independence (BI). BI is satisfied if the system has gauge invariance with respect to spatial-temporal transformations (active diffeomorphisms) (Rovelli, 2008). BI systems then have invariant fields within spatial-temporal diffeomorphisms or as smooth topological mappings change the structure. Essentially one aspires to a physical system theory that takes into account global effects in generality.

In order to also satisfy general relativistic mechanics a physical theory must satisfy local Lorentz invariance. The Lorentz invariance of a field value is the invariance of a field value under the Lorentz spatio-temporal transformations:

$$\begin{bmatrix} t' \\ z' \end{bmatrix} = \frac{1}{\sqrt{1+\kappa v^2}} \begin{bmatrix} 1 & \kappa v \\ -v & 1 \end{bmatrix} \begin{bmatrix} t \\ z \end{bmatrix}, \tag{3.3}$$

where $\kappa = -\frac{1}{c^2}$, and c is the speed of light. Simple discretization of spatial-temporal structures via a lattice representation unfortunately does not posses Lorentz invariance. In this regard, the idea of a fuzzy sphere was devised to represent space at or under the Planck scale with generalized fuzzy points that resemble cells while maintaining a usual continuous approximation at super-Planck scales. Within this representation is the notion of fuzzy points on a sphere that in fact satisfy these conditions while retaining Lorentz invariance. Field theories are satisfied on such representations using definitions for a path integral and algebras of matrices replacing algebras of observable operators for quanta (Madore, 1992).

The classical container of discrete information is the bit, the abstract representation of a Boolean state variable. In quantum mechanics this abstract container is generalized for the possibility of superimposed states to be discussed next. These containers are called *qubits* and were first coined by Schumacher (1995). In the next chapter a generalization of a minimal or atomic discrete information container along the lines of a generalized uncertainty concept will be introduced. In addition, later in this chapter, it will be pointed out that a physical system described by a fuzzy probabilistic logic further generalizes a quantum system. Qubits represent idealized quantum states of an attribute or observable of a quantum system. Traditionally, the $\pm\frac{1}{2}$ spin of a particle is used as an analogy for the pure, nonsuperimposed Boolean states of a quantum particle or quanta. In a quantum physical system, a qubit can be used to measure any Boolean state attribute of a quanta. In this representation, the poles of a 2-sphere (Bloch sphere) are the two pure states of the superposed state of a qubit:

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle, \qquad (3.4)$$

where $\alpha, \beta \in \mathbb{C}, |\alpha|^2 + |\beta|^2 = 1$ and the respective squared absolute values are the probabilities (amplitudes) of each state occurring. A point on the surface of the Bloch sphere represents a general (mixed) quantum state of a single qubit with 2D spherical or 3D Cartesian coordinates representing the probabilities respectively of each pure state part of that superimposed state, i.e., the spherical coordinates (θ, φ) as below with the state representation $|\psi\rangle = \frac{\cos\theta}{2}|0\rangle + e^{i\varphi}\frac{\sin\theta}{2}|1\rangle$. Here, the probability amplitudes are determined by the azimuthal angle, φ and the phase angle, θ.

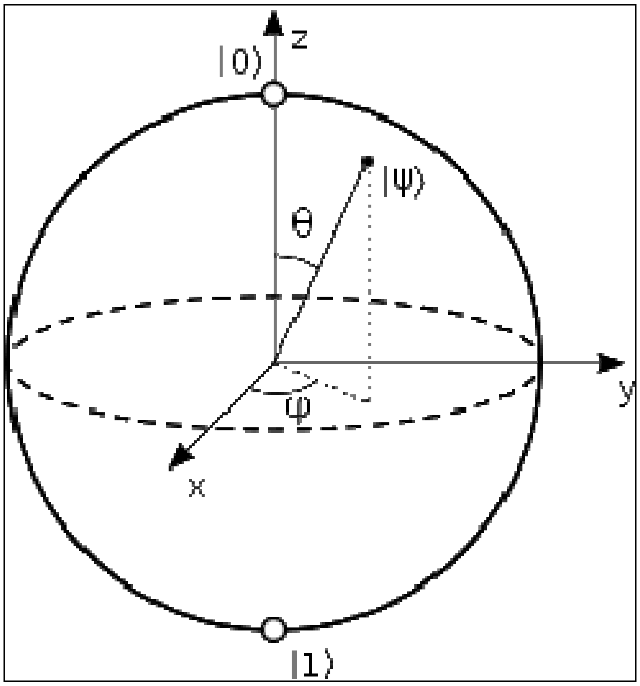

Figure 1. The Bloch sphere for qubit representation
Adapted from “The Bloch-sphere”. By Smite-Meister, 2009. Copyright 2009 by Smite-Meister. Reprinted with permission under the GNU Free documentation license – Creative Commons Attribution-ShareAlike 3.0.

Because the surface of the Bloch sphere is parameterized by the component (θ, φ), a potentially infinite amount of information is contained within a qubit. However, the classical realization of a qubit is the collapse to a bit. The Bloch sphere will be revisited when projections of states are considered in qubit computations. Now consider composite quantum systems, starting with the composition of two separate qubit systems *a* and *b*. Superposition in quantum mechanics allows for the existence of superimposed states of the tensored composite system $a \otimes b$:

$$\left|\psi\right\rangle = \alpha_{00}\left|0\right\rangle_a\left|0\right\rangle_b + \alpha_{01}\left|0\right\rangle_a\left|1\right\rangle_b + \alpha_{10}\left|1\right\rangle_a\left|0\right\rangle_b + \alpha_{11}\left|1\right\rangle_a\left|1\right\rangle_b \tag{3.5}$$

where $\sum_{\substack{i=0,1\\j=0,1}} \left|\alpha_{ij}\right|^2 = 1$, this representing a general superimposed state where each individual qubit is the same and different from each other with respective probability $\left|\alpha_{ij}\right|^2$. There are many such superimposed states. Of specific importance are those superimposed states that cannot be expressed as products of each individual qubit state, that is:

$$\begin{gathered}\alpha_{00}\left|0\right\rangle_a\left|0\right\rangle_b + \alpha_{01}\left|0\right\rangle_a\left|1\right\rangle_b + \alpha_{10}\left|1\right\rangle_a\left|0\right\rangle_b + \alpha_{11}\left|1\right\rangle_a\left|1\right\rangle_b \neq \\ \left(\alpha_0\left|0\right\rangle_a + \beta_1\left|1\right\rangle_b\right)\otimes\left(\alpha_1\left|1\right\rangle_a + \beta_0\left|0\right\rangle_b\right)\end{gathered} \tag{3.6}$$

where $\sum_{\substack{i=0,1\\j=0,1}} \left|\alpha_{ij}\right|^2 = 1,\ \left|\alpha_0\right|^2 + \left|\beta_1\right|^2 = 1,$ and $\left|\alpha_1\right|^2 + \left|\beta_0\right|^2 = 1$. These superimposed states are labeled as *entangled*. Of even more importance experimentally are the subset of entangled states named *Bell states* given by:

$$\begin{aligned}\left|\psi^+\right\rangle &= \frac{1}{\sqrt{2}}\left|0\right\rangle_a\left|1\right\rangle_b + \left|1\right\rangle_a\left|0\right\rangle_b \\ \left|\psi^-\right\rangle &= \frac{1}{\sqrt{2}}\left|0\right\rangle_a\left|1\right\rangle_b - \left|1\right\rangle_a\left|0\right\rangle_b \\ \left|\varphi^+\right\rangle &= \frac{1}{\sqrt{2}}\left|0\right\rangle_a\left|0\right\rangle_b + \left|1\right\rangle_a\left|1\right\rangle_b \\ \left|\varphi^+\right\rangle &= \frac{1}{\sqrt{2}}\left|0\right\rangle_a\left|0\right\rangle_b - \left|1\right\rangle_a\left|1\right\rangle_b\end{aligned} \tag{3.7}$$

The Bell states opened the way to show that the measurement of one qubit gives the value of the other qubit if they are entangled as such, regardless of their separation distance.

This led to the famous EPR (Einstein-Podolsky-Rosen) paradox and of Einstein's skepticism towards the completeness of quantum mechanics without further variables (Einstein, Podolsky, & Rosen, 1935). This represents the gist of the nonlocality of quantum mechanics. Bell established inequalities of measureable physical quantities that real and local-based physical theories must satisfy. Quantum mechanics violates these conditions in addition to not feasibly requiring hidden variables, hence the nonlocality of quantum causality (Bell, 1964). Clauser, Horne, Shimony, and Holt generalized Bell's inequalities to a new condition for physically plausible correlations, the CHSH inequality for a bi-partite system given by:

$$s = \sum_{x,y \in \mathbb{Z}_2} p(m_x^A \oplus m_y^B \equiv x \wedge y) \leq 3 \tag{3.8}$$

where $\mathbb{Z}_2$ is the moduli 2 group, A and B (A stands for Alice and B for Bob, two abstract physical, possibly human detectors or quantum systems used in the quantum information and entanglement literature) are separate quantum systems and their respective bit measurement results for x and y are m_x^A and m_y^B . On the other hand, separable quantum states involving local hidden variables with independent observers satisfy Bell-type inequalities (Loubenets, 2005; Loubenets, 2009).

Nonetheless, recently, it was shown that three possibilities can be had by quantum states. Quantum states (a) do not allow for a local realistic model, (b) do not possess the required EPR-type correlations, or (c) satisfy both (a) and (b) (Zukowski, 2006). Entanglement is sometimes referred to as stronger-than-classical correlations because of this seemingly nonlocal causality. Unentangled or separable states satisfy the condition $Tr(\psi)=1$, while entangled states do not. The operation $Tr(\psi)$ is the trace operator on ψ, defined as $Tr(\psi)=\sum_n \langle \lambda_n \mid \psi \mid \lambda_n \rangle$ where $\{(\lambda_n)\}$ is any orthonormal basis in the Hilbert space of states. Superpositions for bi-partite systems can be generalized to multiqubit systems. In these multiqubit states, coalitions of qubits may be entangled with other distinct coalitions, but not individually within each. Combinatorial entanglement and unentanglement in and outside of subsets of qubits ensues. In an *n*-qubit system, a simple superposition can be expressed as:

$$\left|\psi\right\rangle = \sum_{(i_1,\ldots,i_n)} \alpha_{i_1\ldots i_n} \left|i_1\right\rangle_1 \ldots \left|i_n\right\rangle_n \qquad \text{(3.9)}$$

where $\sum_{(i_1,\ldots,i_n)} \left|\alpha_{i_1\ldots i_n}\right|^2 = 1$ and the sums are a subset of all combinations $(i_1,\ldots,i_n)\in\{0,1\}^n$ entailing the distinct products of binary states of *n* bits.

Entanglement of a general *n*-qubit superposition sum,

$$\sum_{(i_{m_1},\ldots,i_{m_l})\in M} \alpha_{i_{m_1}\ldots i_{m_l}} \bigotimes_{(i_{m_1},\ldots,i_{m_l})} \left|i_{m_1}\right\rangle_1 \ldots \left|i_{m_l}\right\rangle_m \qquad \text{(3.10)}$$

entails the condition that there does not exist a separable tensor product over some subset of *n*-vector combinations $(k_1,\ldots,k_n)\in K\subseteq\{0,1\}^n$, such that,

$$\sum_{(i_{m_1},\ldots,i_{m_l})\in M} \alpha_{i_{m_1}\ldots i_{m_l}} \bigotimes_{(i_{m_1},\ldots,i_{m_l})} \left|i_{m_1}\right\rangle_1 \ldots \left|i_{m_l}\right\rangle_m = \bigotimes_{(k_1,\ldots,k_n)\in K} \left|k_1\right\rangle_1 \ldots \left|k_n\right\rangle_n \qquad \text{(3.11)}$$

for each of the superposition sum's *l*-vector combinations $(i_{m_1},\ldots,i_{m_l})\in M\subseteq\{0,1\}^n, l\le n$, over some combination of tensor product states involving subsets of *n* systems and $\sum_{(i_1,\ldots,i_n)} \left|\alpha_{i_1\ldots i_n}\right|^2 = 1$. Each factor in the tensor product under the sum is of length at most *n*. A subset of the component states in a superposition could be entangled or each one could be entangled in the case of a fully entangled superposition state. Again, as in the case of a 2-qubit system, $Tr(\psi) = 1$, for an unentangled state in an *n*-qubit system. Otherwise it is an entangled state.

An information container or system that is in an entangled state is referred to as an *e-bit*. Computation and information manipulation with an e-bit can be achieved and will be reviewed and further enhanced. In the case of an e-bit, the act of *decoherence* is detrimental to an end computational result being realized. Decoherence is the leakage of information from a qubit that has been coupled to a neighboring environment. This is expressed as the degeneration of the coherency of an entangled state. It is evolutionary since decoherence happens over a period of time. In more detail, consider the qubit-environment coupling induced by a joint unitary time evolution operator, *U*:

$$\left| j \right\rangle E \rangle \overset{U(t)}{\mapsto} \left| j \right\rangle E_j(t) \rangle,\ j = 0,1 \tag{3.12}$$

where *E* is a fixed initial state of the environment and E_j *,j=0,1* are the corresponding evolution operators for the qubit states at time *t* induced by *U*. In terms of an entanglement between a superimposed qubit, $\phi = \alpha_0 \left|0\right\rangle + \alpha_1 \left|1\right\rangle$ and its environment, *E*, the time evolution operator, *U* induces the entangled state:

$$\left(\alpha_0 \left|0\right\rangle + \alpha_1 \left|1\right\rangle\right) \otimes \left|E\right\rangle \overset{U(t)}{\mapsto} \alpha_0 \left|0\right\rangle E_0(t) \rangle + \alpha_1 \left|1\right\rangle E_1(t) \rangle \tag{3.13}$$

The corresponding reduced density matrix of a qubit-environment entanglement, $\phi + E$, is given by the trace of $\phi + E(t)$ over all the states of ϕ (Ekert, Palma, & Suominen, 2001):

$$\rho_\phi(t) = Tr_{E(t)\rho_{\phi+E(t)}} = \begin{bmatrix} |\alpha_0|^2 & \alpha_0\alpha_1^* \langle E_1(t) \mid E_0(t)\rangle \\ \alpha_1\alpha_0^* \langle E_0(t) \mid E_1(t)\rangle & |\alpha_1|^2 \end{bmatrix} \quad (3.14)$$

The off-diagonal terms approach 0 as $t \to \infty$, since the information leakage of the qubit into the environment, given by the decoherence of the qubit particle, increases with time. Subsequently, the operators, $E_1(t)$ and $E_0(t)$ approach mutual orthogonality with time.

Specifically, $\langle E_1(t) \mid E_0(t)\rangle = e^{-\Gamma(t)}$ for some increasing function in time, $\Gamma(t) \to \infty$. In this case, the (reduced) density matrix of the entangled system approaches a diagonal matrix and so, the entangled state, $\phi + E$, approaches being a pure state, consequently losing the efficiency of an entangled computational unit. In the more general case of multipartite qubit systems, it has been shown that the decay of the coherences of the registers or information containment of n qubits scales as $e^{-poly(n)\gamma(t)}$. $poly(n) \sim n$ for independent interactions with the environment, while, $poly(n) \sim n^2$ for collective interactions with the environment (Ekert, Palma, & Suominen, 2001). This implies that multiple qubits coupled with each other would reduce decoherence, i.e., the use of multiple qubits to compute results for a single classical bit are less prone to decoherence.

Quantum noise may also be introduced into the evolution of a qubit in a quantum channel, as noise is introduced in a classical bit stream. Quantum noise is manifested by the quantum uncertainty in the position of a particle and hence an unknown or unwanted change in the density matrix (operator) of a quantum system. This class of noise further degrades the coherence of qubit systems and error-correcting schemes have been developed to combat this phenomenon, including symmetrisation operators, quantum

Hamming codes, multiple qubit codes, and fault-tolerance (Macchiavello & Palma, 2001). More generally, the evolution time operator, U, acts on a set of qubits with initial state, $|\phi\rangle$ and states of the environment E, $|\psi\rangle_E$ by:

$$|\phi\rangle|\psi\rangle_E \xrightarrow{U(t)} \sum_i \left(E_i(t)|\phi\rangle\right)|\psi_i\rangle_E \tag{3.15}$$

where each E_i, is a general error operator, expressible as a tensor product of Pauli operators acting on the qubit streams, $|\phi\rangle$. Generalized noise and decoherence can then be expressed in terms of the Pauli operators σ_x, σ_y, and σ_z operating on the qubits. The following convention is used: $X \equiv \sigma_x, Z \equiv \sigma_z, Y \equiv -i\sigma_y = XZ$ and $X_u Z_v$ where u and v are n-bit binary vectors that indicate where the X and Z operators appear in a general tensor product operation of the form $\underset{i}{\otimes} A_i$ where $A_i = X, Y, Z, I$. Any error correction operation then translates an error-prone state, $X_u Z_v |\phi\rangle \rightarrow A|\phi\rangle$ where $A = Z_v, X_u$. The succession of these error correction operators is to eventually correct a general error operator, E_i, so that, $E_i|\phi\rangle \rightarrow |\phi\rangle$. Note that only error correction operators for X and Z are sufficient to correct a general quantum error (Macchiavello & Palma, 2001).

Further generalizing multipartite quantum systems are n-qudits. In an n-qudit system, n quantum logical units are present, the i^{th} unit with multiple possible states (levels) $d_i > 2$. The Hilbert space for this multipartite, multilevel system is composed as the tensor product of n Hilbert spaces each with dimension, $d_i \geq 2$, for $i = 1, 2, ..., n$, i.e., $\mathcal{H} = \bigotimes_{i=1}^{n} \mathcal{H}_{d_i}$. A typical state in an n-qudit system which is *totally separable* can be expressed in terms of density operators as:

$$\rho_{A_1 \ldots A_n} = \sum_{i=1}^{n} p_i \otimes \rho_{A_i}^{i} \tag{3.16}$$

where A_i is the i^{th} subsystem (which may consist of a single qudit), $\rho_{A_i}^{i}$ is its density operator, and $\sum_{i=1}^{n} |p_i|^2 = 1$. Separability and hence entanglement in multipartite systems can come in subset combination. Partial separability and entanglement as well, can come in subset combinations. To review, entanglement comes when a state cannot be expressed as a product of mixed or pure states. Because of subset entanglement, entanglement swapping can be actuated when partial entanglement between subsystems of qudits leads to the allowance of entanglement with other subsystems of qudits.

One result illustrates the variety of possibilities in such swapping schema. When subsets of qudits, say, m_j of them form the j^{th} subsystem, A_i, and each of these subsystems is not correlated with each other, and when a set A^M consisting of $\sum_i a_i$ qudits is formed with a_j qudits from the j^{th} subsystem are measured and subjected to a generalized Bell-type measurement, then the remaining set, $A \setminus A^M$ consisting of $\sum_j (m_j - a_j)$ qudits collapse into a maximally entangled state (Bouda & Buzek, 2002). In general, entanglement of multipartite, multilevel systems becomes exceedingly diverse and no standard entanglement in terms of simple e-bits has been defined in such cases

(Morikoshi, Santos, & Vedral, 2004). Generally, a measurement of entanglement is given by the von Neumann entropy, $S(\rho) = Tr(\rho)\log\rho$ or its generalized Renyi $\alpha-$entropies. A more practical measure of entanglement comes from the *entanglement of formation*:

$$E_f(\rho_{A_1\ldots A_n}) = \inf_{\{p_i|\psi_i\rangle\}} \sum_i p_i S(\rho^i_{A_1\ldots A_n}) \tag{3.17}$$

where the supremum is taken over all decompositions, $\{p_i|\psi_i\rangle\}$ of the multipartite system $A_1 \ldots A_n$ and $\rho^i_{A_1\ldots A_n}$ is the reduced density operator of ψ_i. $E_f(\rho_{A_1\ldots A_n})$ is the minimum average entropy over all decompositions of the multipartite system. The case for bipartite systems is given in (Jaeger, 2008, p. 51) and has been generalized here for multipartite qudit systems. The dual to entanglement of formation is the *localizable entanglement* defined as:

$$E_f(\rho_{A_1\ldots A_n}) = \sup_{\{p_i|\psi_i\rangle\}} \sum_i p_i S(\rho^i_{A_1\ldots A_n}) \tag{3.18}$$

For completeness, a final measure of entanglement will be mentioned, the *entanglement of distillation* (free entanglement):

$$E_D(\rho_{A_1\ldots A_n}) = \lim_{m\to\infty}\sup\left(\frac{k(m)}{m}\right) \tag{3.19}$$

where $k(m)$ is the number of singlet states $|\psi^-\rangle^{\otimes k}_{A_1\ldots A_n}$ that can be extracted from m copies of $\rho_{A_1\ldots A_n}$. $E_D(\rho_{A_1\ldots A_n})$ is a measure of the asymptotic separability of the system $\rho_{A_1\ldots A_n}$ (Jaeger, 2008, p. 51-52) In general, $E_D(\rho_{A_1\ldots A_n}) \le E_f(\rho_{A_1\ldots A_n})$. For pure states,

$E_D(\rho_{A_1 \dots A_n}) = E_f(\rho_{A_1 \dots A_n})$. Forming the difference defines a *bound entanglement* measure (Jaeger, 2008, p. 53):

$$B(\rho_{A_1 \dots A_n}) = E_f(\rho_{A_1 \dots A_n}) - E_D(\rho_{A_1 \dots A_n}) \geq 0 \tag{3.20}$$

See Horodecki, et al. (2007) for a more complete treatment of entanglement measures. Essentially an entropic measure of entanglement is a measure of the amount of information that is available to be entangled. However, in general, surmising if a state is entangled in a general multipartite system is difficult because of the combinatorial possibilities involving multiple dimensions and large numbers of quanta.

The problem of developing a condition to test for entanglement has been solved for limited numbers of quanta and state dimension (Rungta, Munro, Nemoto, Deuar, Milburn & Caves, 2000). Conditions also exist in which a minimum of pairwise entanglement is necessary in order for multiple entanglement or subset entanglement to be possible (Bouda & Buzek, 2002). It seems that in a multipartite qudit system, too strong of an entanglement between any two quanta prevent entanglement sharing among other quanta. In particular, by performing a Bell-type measurement on a subset of entangled subsystems, the remaining unmeasured quanta collapse into a maximally entangled state (Bouda & Buzek, 2002).

The concept of *entanglement monogamy* refers to the quantum condition that if two qudit systems, *A* and *B* are maximally entangled, then neither one can be correlated to a third, *C* by a local-operation-classical-communication (LOCC) measurement (Coffman, Kundu, & Wooters, 2000). However, under submaximal or partial entanglement, this monogamy condition can be relaxed. It has been posited that as the dimension *d* of the number of states in each subsystem increases, entanglement sharing increases (Adesso, Ericsson, & Illuminati, 2007; Dennison & Wooters, 2001). This tendency is called *entanglement promiscuity* in the limit.

Entanglement is referred to as a super-correlative theory of causality, that is, a basis for causality that is stronger than probabilistic correlations. No-signal theories are those which posit that information cannot be transferred between quanta at superluminal speeds, though correlations can be made. In a generalization to this description of information transfer limitation, the concept of Information Causality was introduced—transmission of *m* classical bits can cause an information gain of at most *m* bits (Pawlowski, et al., 2009). Communication of *m* qubits, on the other hand, can produce information gains greater than *m* qubits experimentally shown using a quantum super-dense coding protocol (Bennett & Wiesner, 1992). Information causality describes a principle that is applicable to theories of physics that are more general than QM in the sense of possessing stronger correlations, i.e., super-quantum correlations. Super-quantum correlative systems violate the information causality principle, while those of classical and quantum systems do not. One point this dissertation will question—is there some form of intelligible information transferred where only super-correlations are present? Just as statistical correlation does not imply physical causality, does super-correlation imply a version of super-causality in the universe?

Superluminal communication via e-bit relations is controversial because within the constraints of the classical communication theory of sources and receivers and the mutual management of how a signal between the two may be translated, no information is relayed. Information in a quantum e-bit situation is classically reached by collapsing a quantum e-bit to a classical bit. There are intermediate levels of communication strength that depict mixed degrees of classical and quantum causality and there are super quantum causalities as well. Stronger than quantum correlations are produced by relaxing the constraint of relavistic causality, that is, admitting superluminal classical communication. Label such theories as s-quantum or super-quantum theories and their information containers as *s*-bits (superluminal). The maximal bound on the CHSH inequality is 4 in the case of independent correlations. For quantum correlations it is $2\sqrt{2}$. It was posited that the condition of relativistic causality prevents a version of a quantum theory from reaching this upper bound.

However, in an insightful argument used by Popescu and Rohrlich, by considering a one-dimensional jamming effect from a third party, *J*, onto two spacelike separated quantum systems, *A* and *B*, making measurements on a locally past-correlated event, a quantum-like theory can be developed in which relativistic causality is preserved, while, correlative or physical motion effects may preclude causes, in a consistent manner (Popescu & Rohrlich, 1996). In this experiment *J* is spacelike separated from both A and B and produces a jamming of the measurements made by *A* and *B* in the following manner. Without jamming, the entangled event measured by *A* and *B* violates the CHSH inequality, while with jamming it is classically correlated.

Additionally, the jamming mechanism must satisfy the conditions of *unarity* and *binarity* which prevent *J* from senting superluminal signals to *A* or *B* individually or jointing in such a way that it can be informationally read, i.e., comprehended. In these classes of theories, the bound on the CHSH inequality can reach 4. These are the so-called PR-box or nonlocal information channels. In other words, quantum mechanics is not the only theory that reconciles relativistic causality with nonlocality. Moreover, stronger correlative theories exist which are consistent with general relativity and quantum mechanics. We label such theories as *nl*-correlative and the participating information containers as *nl*-bits. One can then order the strength of information communication theories as: $bit \prec e-bit \prec nl-bit \prec s-bit$ (Jaeger, 2009, p.254). Superluminal signal communication, while being contradictory in a classical sense, is possible, in a quantum mechanical and relativisitic realm, if the signal propagated is not comprehended until after the trailing light is received. Essentially, this means that super-quantum signals abound from all corners of the universe to each abstract brain, but its classical understanding lags behind its light cone. The proverb, "the answer has always been inside you", rings true, only in a super-quantum universe.

Generalized entanglement in multipartite systems based on a notion of a generalized uncertainty system which, in turn, is based on composites of different notions of uncertainty is a key component in forming a new model of super-information and communication. Quantum and fuzzy systems generalize probabilistic ones. Would systems based on general uncertainty and super-quantum theories generalize quantum and fuzzy systems? What of the notion of macroscopic development of larger systems from smaller ones that follow such rules? The bigger question may be "how do systems coalesce into organizations and can generalized entanglement as proposed above serve as a calculus for forming categories of macrosystems such as CAMSs and HMASs?" This possibility will be discussed and developed. One must first look at theories of formation of microsystems and macroorganization.

Quantum gravity as seen through the lens of loop quantum gravity (LQG) spinfoam models, a version of a topological quantum theory which combines in a consistent and physically plausible way the structural laws of general relativity and quantum mechanics, will be reviewed as the preferred approach to unifying QM and GR. This will be followed by a review of the concept of information fields as a manner in which signal processing can be combined with physical field theory to compute and view ensemble information flow. Next, returning to Wheeler's call for a discrete computational space, the notion of a fuzzy sphere discretization of quantum mechanics and qudit computation on it will be discussed as a physical information theory usurping a computational one.

Loop Quantum Gravity and the Spinfoam Formalism

In the LQG rendition of a unified theory (among many others, Witten (1998) superstring/M-Theory, canonical quantization, Halvorson & Mueger (2006) algebraic quantum field theory (AQFT), Penrose (1967) twistor theory, and Lisi (2007) geometric Lie group E_8, being other major unified conceptual frameworks, albeit background dependent), the prescient discrete spacetime model is the spinfoam network (SFN). Loop quantum cosmology (LQC) is an analogous formalism for the cosmological studies leading to a replacement of the Big Bang Theory with LQ phenomena that posits that a quantum bounce was followed by super-inflation (Ashtekar & Sloan, 2010). In this brief overview of LQG spinfoam networks, the expository reviews of Rovelli (2008) and

Thiemann (2002) of LQG, and the spinfoam network formalism of Pérez (2003) will be utilized. Penrose (1971) had developed the spin network formalism for representing states of quanta and their fields as an abstract directed graph with nodes representing states of a quanta and edges representing fields between quanta. LaFave (1993) first developed the formalism for spinfoams as path histories for spin networks analogous to Feynman path histories and diagrams for particle path histories.

These ideas were followed up for the more general case of quantum gravity spinfoams as histories of spin networks by a group of physicists; most notably Ashtekar (1986) and Rovelli (2008). In this configuration a space cell volume is represented by a node, i_n, in an abstract graph, Γ, while adjacent surface boundaries between cells are represented by edges, j_l of that graph. SFNs are abstractions for the physical connections in the universe and are labeled by the triplet, $s = (\Gamma, i_n, j_l)$ (Rovelli, 2008, p. 19). A quantum state $|\psi\rangle$ that comprises *N* cells or grains in space is otherwise represented by a graph, Γ with *N* nodes. The graph Γ is a member of an equivalence class of embedded subgraphs in a 3-manifold space.

Specifically, these graphs are embedded spin-networks (ESNs). These states represent the polymeric excitations of the gravitational field. Each ESN equivalence class, $\hat{s}$ is defined under smooth deformations (diffeomorphism invariance), ξ and is called an *s*-knot. Two members of the same equivalence ESN class are gauge equivalent because the diffeomorphism gauge values are shared. ESNs define the abstract reference global structure for space. Quanta observables are then localized into these networks and their quantum states are defined by an *s*-knot. The relevant observables in LQG using ESNs are geometric measures—cell boundary surfaces and volumes.

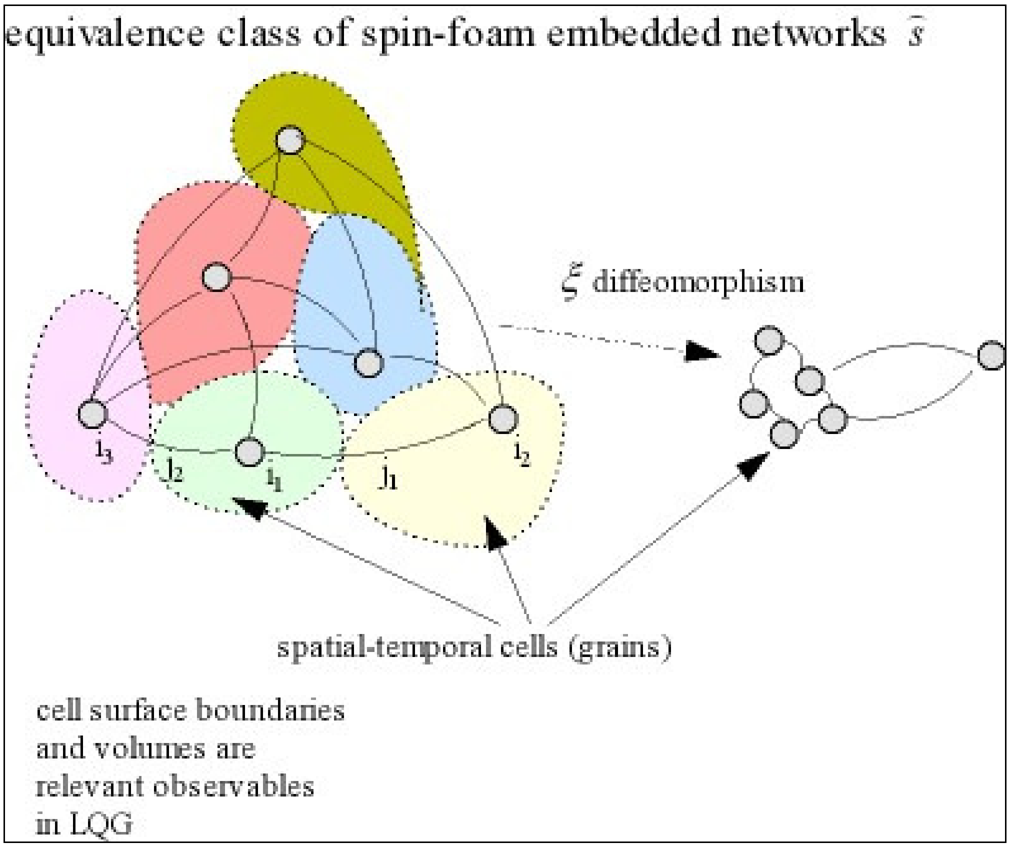

Figure 2. Spinfoam embedded networks

In the transition from one spatial-temporal coordinate (x,t) to another (x',t'), the transition probability amplitudes (propagator) given by:

$$W((x,t),(x',t')) = \langle x | e^{-\frac{i}{\hbar}H_0(t-t')} | x' \rangle = \langle x,t \mid x',t' \rangle \tag{3.21}$$

where H_0 is the Hamiltonian, $|x,t\rangle$ is the eigenstate of the Heisenberg position operator x(t) and we define $|x\rangle = |x,0\rangle$, completely describes the quantum dynamics of that path.

See Appendix A for a review of Hamiltonians, propagators, and transition probability amplitudes with respect to Feynman path histories. In general, for any observable states measured at the two points, $|\psi(t')\rangle$ and $|\psi(t)\rangle$ respectively, the propagator is given as:

$$W(\psi',\psi) = \langle \psi | e^{-\frac{i}{\hbar}H_0(t-t')} | \psi' \rangle \tag{3.22}$$

The pair (ψ',ψ) maps to a tensor product state $\psi = |\psi'\rangle \otimes \langle \psi|$ in the appropriate tensor product of the input and the dual of the output Hilbert spaces respectfully. The propagator can now define a generalized state labeled as $|0\rangle$ (the covariant vacuum state), through the calculation $W(\psi',\psi) = \langle 0 | (|\psi'\rangle \otimes \langle \psi|) = \langle 0 | \psi \rangle$ (Rovelli, 2008, p.22-23).

Rovelli extends this definition to field values for Quantum Field Theory (QFT) by replacing the coordinate states (ψ',ψ) with the pair (Σ,φ), where Σ is a 3-D surface bounding a region in space and φ is the field configuration on that surface,. The pair (Σ,φ) resides in a space $\mathcal{G}$ that is the product of 3-D surfaces and all possible field configurations on those surfaces. Formally, one defines:

$$W(\Sigma,\varphi) = \sum_{\substack{\eta \text{ has value} \\ \text{of } \varphi \text{ and is a} \\ \text{field configuration} \\ \text{on } \Sigma}} W(\Sigma,\eta) \tag{3.23}$$

using the Feynman sum over all histories (in this case, of all possible field configurations η over Σ with the value of φ) from one point to another.

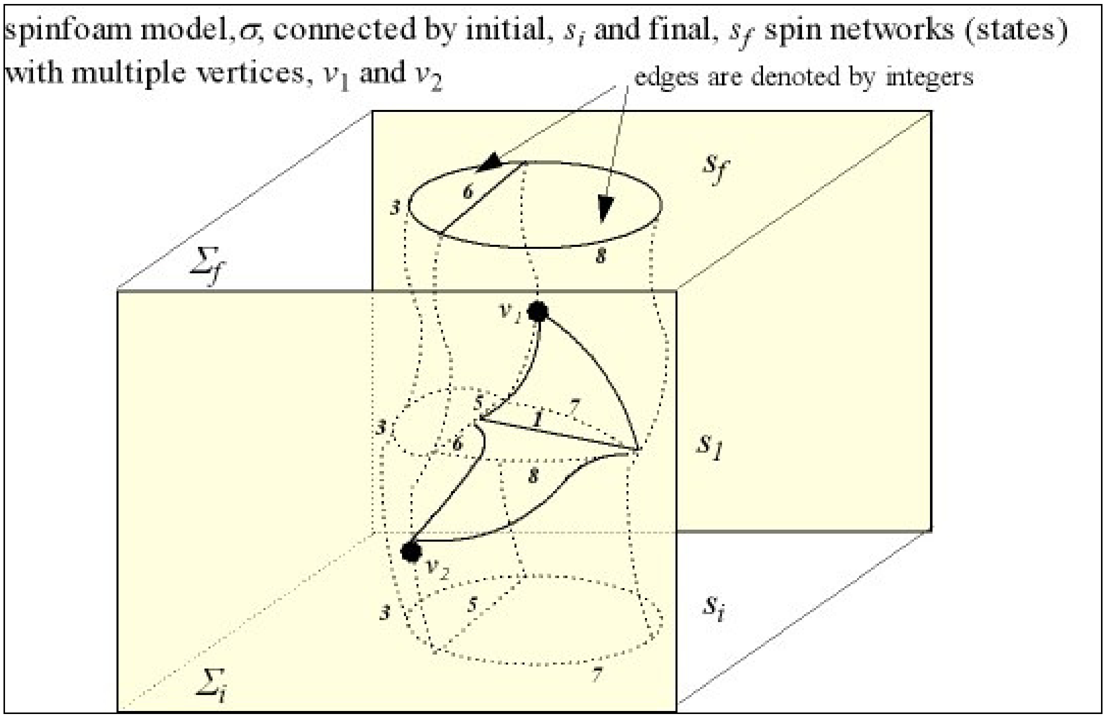

Figure 3. Spinfoam worldsheet
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In a background independent physical system, because of diffeomorphism invariance, $W(\Sigma, \varphi)$ does not dependent on the geometric surfaces, Σ, only on the field values on those surfaces (cell boundaries), φ. Since one can take as one example of a field configuration that of a gravitational field, then utilizing relativistic dependence of spacetime on that field, the propagator will remain dependent only on the combined spatial-temporal separation of the points, not the underlying geometric structure. In LQG therefore, measurements are in the form of propagators that combine the difference effects of dynamic field values (such as particle forces) and spatio-temporal coordinates. These are separated measurements in a background-dependent QFT. In spin-foam geometry, an ESN replaces the spatial-temporal coordinates, (x,t). Furthermore, in a quantum experiment, one can rig measurements so that the starting ESN, *s*, is the state to measure and the ending ESN, s', is the observed state measurement. In this way, the propagator, $W(s,s')$, gives the correlational probability of observing s', given that the actual state was *s*. These transition amplitudes are defined as:

$$W(s,s') = \langle s \mid P \mid s' \rangle \tag{3.24}$$

where s and s' are s-knot states and the operator *P*, is the projector onto the space of solutions of $H\psi = 0$ (kernel), and *H* is the Hamiltonian of the quantum spin-foam system. More explicitly, this propagator may be expressed as a sum taken over the histories of the spin networks connecting s to s' of amplitudes:

$$W(s,s') = \sum_{\sigma} A(\sigma) \tag{3.25}$$

where $\sigma = (s, s_m, ..., s_1, s')$ is a sequence of spin networks that is called a spinfoam.

Additionally, each amplitude can be expressed as a product of step amplitudes, $A(\sigma) = \prod_{\gamma} A_{\gamma}(\sigma)$, each given by a matrix with elements:

$$A_{\gamma} = \left\langle s_{m+1} \mid e^{-\int d^3x H(x) dt} \mid s_m \right\rangle_{\kappa} \tag{3.26}$$

where κ is the Hilbert space of gravity and matter. Spinfoams represent a generalization of Feynman diagrams for discrete quantum gravity in the following sense. Spinfoams are embedded 4-D graphs in spacetime which sweep along the time dimension creating worldsheets according to the Hamiltonian operation *H*. The surface faces, denoted by *f*, of a spinfoam are the world surfaces and denote the links in the graph. The edges, denoted by *e*, are the worldlines of the nodes of the graph. In addition to the embedded graph representation of a spinfoam, Γ, there are irreducible representations, j_f associated with the faces *f*, and intertwiners, i_e associated with edges *e*. A spinfoam is then formally expressed as a 2-complex, $\sigma = (\Gamma, j_f, i_e)$.

Associated with the graph Γ will be a set of weights determined by the graph and denoted by $w(\Gamma)$. Each node (vertex) will have an amplitude that is determined by the edges and faces adjacent to it and hence to their representations and intertwiners respectively, $A_\sigma(j_f, i_e)$. This amplitude can be extended and expressed as a separable product. Using the weights of the graph, the propagator of a spin network, *s*, which is the boundary ∂ of a possible set of spinfoams, σ, that is, those spinfoams such that $s = \partial\sigma$, can then be written as:

$$W(s) = \sum_{\partial\sigma = s} w(\Gamma(\sigma)) \prod_f A_f(j_f, i_e) \prod_e A_e(j_e, i_e) \prod_\gamma A_\gamma(j_f, i_e) \qquad \underline{(3.27)}$$

The propagator, in the case of a spinfoam that is connected by two spin networks, s and s' can be expressed as:

$$W(s, s') = \sum_{\partial\sigma = s \cup s'} w(\Gamma(\sigma)) \prod_f A_f(j_f, i_e) \prod_e A_e(j_e, i_e) \prod_\gamma A_\gamma(j_f, i_e) \qquad \underline{(3.28)}$$

the transition amplitude between two quantum states of a gravitational field (Rovelli, 2008, p. 329). This is the general form of the computation of a spinfoam model propagator.

Discretization of the spinfoam model is the means toward its realistic computation. To this effect, triangulation of a spinfoam is constructed. In 4-D spacetime, the faces are 4-simplices, denoted by Δ_4. The duals of such faces are what are used in the formalism and are denoted by Δ_4^*. When Δ_4 is a 4-simplex, Δ_4^* is a point vertex. Additionally, if Δ_4 is a (hyper) tetrahedron then Δ_4^* is an edge joining 4 faces, if Δ_4 is a triangle then Δ_4^* is a face, if Δ_4 is a segment then Δ_4^* is a 3D region and if Δ_4 is a point then Δ_4^* is a 4-D region. Vertices of a spinfoam are embedded in each 4-simplex of a triangulation. The collection of faces, edges, and vertices of a 4-simplex, of Δ_4^*, along with their respective relations at the boundaries is called the 2-skeleton of Δ_4^*. This is a 2-complex. Note that 4-simplices of the triangulation can be generalized to n-simplices of n-polyhedra in a tile covering of the spinfoam model. Then one can consider the *n*-skeleton of the dual of the tile covering Λ_n^* consisting of lower dimensional polyhedra, edges, vertices, and their respective duals and their boundary relations and adjacencies.

To greatly simplify the development of an expression for the calculation of the sum-of-paths, an intermediate theory of models called BF-Theory will be used to setup the terms. Denote by *e*, an edge of Δ_4^* and g_e the holonomy of ω (a connection in the spacetime manifold), along *e*. Generally, a holonomy is a relative measure of how much geometrical information is preserved from the curvature of a connection during its parallel transports, i.e., going from one point of the connection to another along a parallel path on the surface of a manifold. Connections of a manifold are roughly the smooth mappings from one tangent space to another in that manifold (see definitions for connection, holonomy, and tangent spaces to a manifold). Holonomies are closely related to the curvature of forms of the manifold by the Singer-Ambrose Theorem. Hence, the group of holonomies of a connection of a manifold gives information about the geometric curvature of the surface. This, in turn, will give an indication of the effect of the curvature of the surfaces of the triangulations to preserving the geometry of the underlying spacetime manifold. Define the connection, g_e, along the edge *e*, using the Lie Group interpretation, $SU(2)$, as:

$$g_e = \mathcal{P}\exp[\int_e \omega^i \tau_i] \in SU(2), \tag{3.29}$$

using Einsteinian notation. Additionally, let l_f^i be the line integral of e^i along a segment *f* in the triangulation Δ. The pair $(l_f^i, g_e) \in SU(2) \times \mathcal{R}^3$ is then chosen as the variable of discretization of a spinfoam. The Hamiltonian action functional can then be written as:

$$S[l_f, g_e] = \sum_f l_f^i tr[g_f \tau_i] \tag{3.30}$$

where we write the product of group elements, $g_f = g_{e_1}^f \dots g_{e_n}^f$ over *n* edges.

Following Rovelli, one writes the path integral as:

$$Z = \sum_{j_1 \cdots j_n} \prod_f \dim(j_f) \int dg_e \prod_f tr[R^{j_f}(g_{e_1}^f \dots g_{e_n}^f)] \tag{3.31}$$

where R^{j_f} is the 2-form curvature of the 1-form ω^{j_f}. Regressing momentarily to the 3-D space manifold, the integral in the sum above can be expressed in the form:

$$v^{\alpha\beta\gamma} v_{\alpha'\beta'\gamma'} \tag{3.32}$$

where $v^{\alpha\beta\gamma}$ is the normalized intertwiner operator for the relationships between the triplet spin state (j_1, j_2, j_3). At each vertex there are four contracting tensors resulting in a function of six spins of the six faces that bound the vertex point. Expressing this in a compact visual symbol:

$$\{6j\}_{v^{\alpha\beta\gamma}} \equiv \begin{pmatrix} j_1 & j_2 & j_3 \\ j_4 & j_5 & j_6 \end{pmatrix} \equiv \sum_{\substack{h \neq k \neq l \\ h,k,l \in \{1,\ldots,6\}}} v^{\alpha_h \alpha_k \alpha_l} \tag{3.33}$$

The trivalent vertex 3-tensor $v^{\alpha\beta\gamma}$ can be further visualized as a tetrahedron using the intertwiner 6-j Wigner symbols it contains at each edge joining into the vertex:

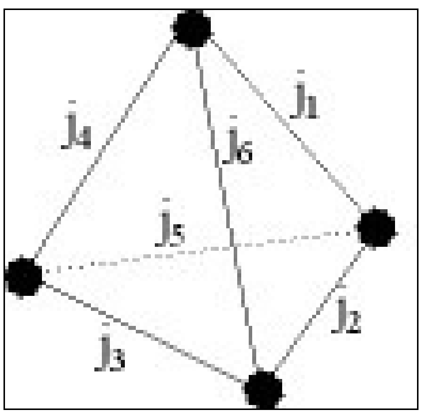
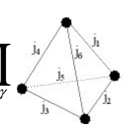


Figure 4. Spinfoam intertwiner operator in Wigner 6j symbology
Adapted from "Quantum Gravity" by C. Rovelli, 2008, p. 334. Copyright 2008 by Cambridge University Press. Reprinted with permission.

The partition function for a 3-D space spinfoam triangulated model can be written as:

$$Z_{PR} = \sum_{j_1 \cdots j_n} \prod_f \dim(j_f) \prod_v \{6j\}_v = \sum_{j_1 \cdots j_n} \prod_f \dim(j_f) \prod_{v^{\alpha\beta\gamma}}$$



(3.34)

This is the Ponzano-Reggae spinfoam model. This model can be further discretized and simplified by taking the physical length of each segment to be in integer units of a normalized Planck length, $l_P = 1$:

$$Z_{PR} = \sum_{j_1 \cdots j_N} \prod_v e^{iS_v(j_n)} \tag{3.35}$$

Returning to the 4-D spacetime manifold case, the intertwiner sums can be generalized to the 15-j Wigner symbol (hyper-polyhedron):

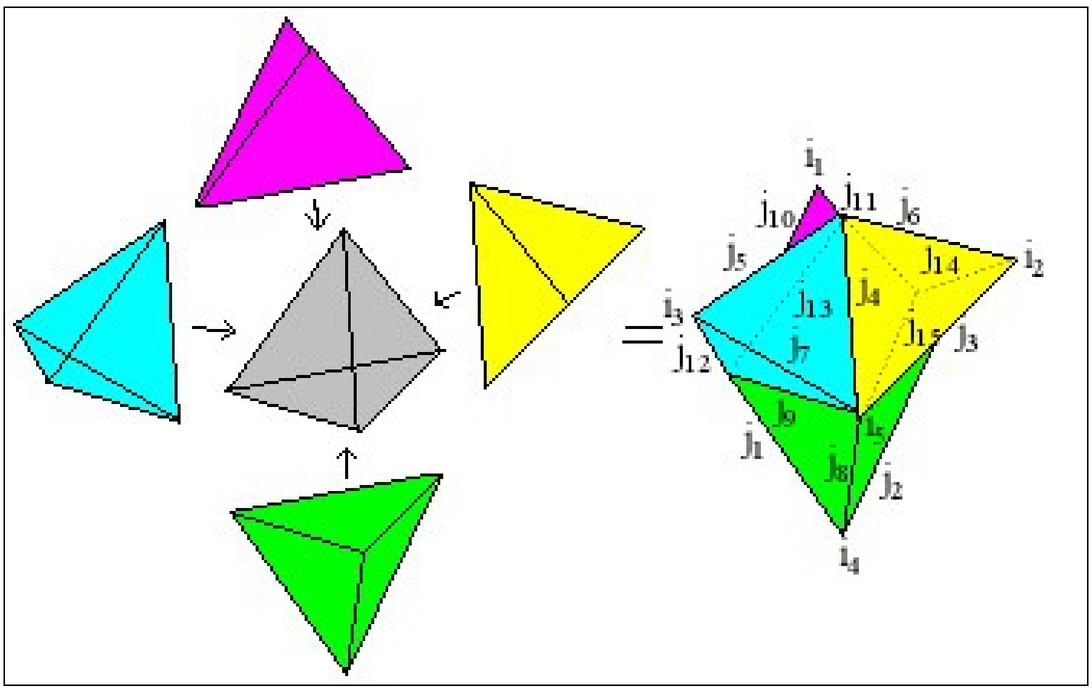
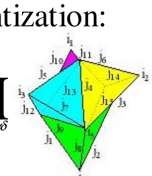


Figure 5. Deconstructed 4D hyper-polyhedron intertwiner in 15-j Wigner symbology

to obtain the partition sum for the TOCY (Turaev, Ooguri, Crane, and Yetter) model of the BF-Theory of a 4D spacetime manifold triangulation quantization:

$$Z_{TOCY} = \sum_{j_f, i_e} \prod_f \dim(j_f) \prod_v \{15j\}_v = \sum_{j_f, i_e} \prod_f \dim(j_f) \prod_{v^{\alpha\beta\gamma\delta}} \tag{3.36}$$



In this partition sum, the set of 2-complexes that are summed over is the 2-skeleton of a the dual of a 4-D triangulation, Δ^*. Additionally, the representations, j_n, are the unitary irreducibles of the group $SO(4)$ and the vertex amplitude is given by $A_v = \{15j\}$. Crane and Yetter (1997) give a variation on this model that bypasses the condition of infrared divergence, that is, of the divergence of the sum (integral) in the sum of paths expression due to high energies or phenomena happening at large distances and Barrett (1998) gives a formalism for the graph invariant for relativistic spin networks over 4-simplices.

The above partition function involves an infinite number of degrees of freedom. In order to capture the situation for a true 4-D spacetime spinfoam model, a sum over 2-complexes must be made in the partition function. We review the general approach taken by the method of group field theory. To this end define a field $\phi(g)$ on the group $SO(4)$ as invariant under the Hamiltonian action H when,

$$\phi(g) = \phi(gh), \quad \forall h \in H \tag{3.37}$$

if and only if its representations, (j_f, i_e) are irreducible. Define the Hamiltonian action functional under these fields as:

$$S(\phi) = \frac{1}{2}\int \phi^2 + \frac{\lambda}{5!}\int \phi^5 \tag{3.38}$$

and define projection operators, P_G and P_H as:

$$P_G[\phi(g_1, g_2, g_3, g_4)] = \int_{SO(4)} dg\phi(g_1 g, g_2 g, g_3 g, g_4 g) \tag{3.39}$$

$$P_H[\phi(g_1, g_2, g_3, g_4)] = \int_{H^4} dh_1 ... dh_4 \phi(g_1 h_1, ..., g_4 h_4) \tag{3.40}$$

Next, define the new Hamiltonian action based on these projection operators:

$$S_C(\phi) = \frac{1}{2}\int (P_G[\phi])^2 + \frac{\lambda}{5!}\int (P_G P_H[\phi])^5 \qquad (3.41)$$

Now define the expectation value of the transition amplitudes of the spin network, *s*, based on the action operator, $S_C(\phi)$, as:

$$W(s) = \int \mathcal{D}\phi f_s(\phi) e^{-S(\phi)} \qquad (3.42)$$

This will then be a well-defined amplitude function of 3-D space models, i.e., Euclidean quantum gravity (QG). What is needed for a relativistic version is a Lorentzian model. This can be accomplished by replacing the group $SO(3)$ by the Lorentz group $SO(3,1)$ and then considering the two cases (1) fixing the subgroup, $H = SO(2,1)$ or (2) $H = SO(3)$ of $SO(3,1)$ and finally, redefining the Hamiltonian action, $S_C(\phi)$, as:

$$S_H(\phi) = \frac{1}{2}\int (P_G[\phi])^2 + \frac{\lambda}{5!}\int (P_G P_H[\phi])^5 \qquad (3.43)$$

One of the more interesting points about this model is that the field, $\phi(g)$ used in (3.43) is essentially arbitrary, i.e., aside from being in the group $SO(4)$, it may have arbitrary behavior. In particular, in this study, the case for a generalized information field, $I(g)$, that depends on the group element g_e associated with an edge *e* of Δ^*, will be constructed into the 4-D spinfoam model and extended into an arbitrary infinite dimensional spinfoam model. Spin networks, in this framework, will be associated with a richer structure that represents information field values with respect to observers and transmitters (event generators) of information, i.e., the proposed informaton particle. We next consider the role information can play in LQG.

Information and Entanglement in Loop Quantum Gravity

After the review of LGQ and spinfoam networks, one may become curious about how information manifests itself in such a setting. Specifically, since the mechanisms of LQG and spinfoam networks carry the states of quanta through a relativistic constraint in the geometry, quantum information will be investigated in such settings. The approach of Terno and Livine will be used for this exposition (Terno, 2006; Terno & Livine, 2006a; Terno & Livine, 2006b). For a spin network and hence a spinfoam, the goal will be to develop a definition for the partial trace operator acting on a bipartite subsystem so that standard manipulation of entangled elements can be performed. This will give an indication of the potential for accommodating entangled information in spinfoams and spin networks. Start with a spin network, Γ with vertices *v* and oriented edges *e*. For a spin network state, an *SU(2)* representation, labeled V^{j_e}, is assigned to each edge *e*, and a *SU(2)*-invariant linear map that was named an intertwiner, labeled,

$\mathcal{I}_v : \underset{e \text{ ingoing}}{\otimes} V^{j_e} \rightarrow \underset{e \text{ outgoing}}{\otimes} V^{j_e}$, is assigned to each vertex *v*.

This is the typical structure of a spin network. Let $\mathcal{H}_v^0 \equiv \{\mathcal{I}_v \text{ for each state}\}$ be the Hilbert space of intertwiners (one per state) for a given vertex, *v*. Next, denote a spin network state of Γ as, $\left|\Gamma, \vec{j}, \vec{\imath}\right\rangle$ where the vectors are the V^{j_e} and $\mathcal{I}_v$ components of the edges and vertices respectively. $\left|\Gamma, \vec{j}, \vec{\imath}\right\rangle$ will in turn, define a function of the holonomies, *g*, along the edges of Γ, given by $\Gamma_{\left|\Gamma, \vec{j}, \vec{\imath}\right\rangle}(g)$. Now fix Γ and its components, V^{j_e} and $\mathcal{I}_v$. Form the tensor product space of intertwiners for all vertices as: $\mathcal{H}^0 \equiv \underset{v}{\otimes} \mathcal{H}_v^0$. Form a basis state in $\mathcal{H}^0$ and label it as $|\vec{\imath}\rangle = |\imath_1,...,\imath_v\rangle$. The component $\imath_k$ represents the intertwiners at *v*. Form the corresponding function of the holonomies along the edges by:

$$\Gamma_{\vec{\imath}}(g) \equiv \langle g | \imath_1,...,\imath_v \rangle \equiv tr\left[\bigotimes_{v=1}^{V}\left\{\bigotimes_{e_v=1}^{S_v} D^{j_{e_v}}(g_{e_v})\right\}\cdot \mathcal{I}^{\imath_v}\right] \tag{3.44}$$

Here $D^{j_{e_v}}(g_{e_v})$ is the group of elements of the holonomies of g_e along the edges outgoing from the same vertex, *v*. $\Gamma_{\vec{\imath}}(g)$ is then the contraction of these matrix representations, $\bigotimes_{e_v=1}^{S_v} D^{j_{e_v}}(g_{e_v})$ with the intertwiners, $\bigotimes_{v=1}^{V} \mathcal{I}^{\imath_v}$. Let E_v denote the total number of adjacent edges to *v*. Let T_v be the total number of ingoing edges to *v* and S_v the total number of outgoing edges to *v*. Then $E_v = T_v + S_v$. E_v as a function of *v* is gauge invariant and preserves its value under the *SU(2)* gauge group of *v*. In this way, the functions E_v are the analogy of the wave functions for quantum geometry and are called gauge invariant cylindrical functions. In the rest of the discussion, it will be assumed that the intertwiner functions are normalized, i.e., $\left\|\mathcal{I}^{\imath}\right\| = 1$ and as such $\langle \imath_1 | \imath_2 \rangle = \delta_{\imath_1 \imath_2}$. In addition, the factors, $\sqrt{d_j} = \sqrt{2j+1}$ are absorbed in the representation matrices. A pure state in the spin network is then expressed as: $|\psi\rangle = \sum_{\imath} r_{\imath} |\vec{\imath}\rangle$, where $\sum_{\vec{\imath}} |r_{\vec{\imath}}|^2 = 1$.

One now considers a bounded connected region, *B* in a closed connected spin network with graph, Γ. Vertices and intertwiners of the sets, $\mathrm{int}(B), ext(B)$, and $\partial \mathrm{B}$ will now be defined along with their respective Hilbert spaces in order to define a course-graining procedure defining generic patch surfaces on the spin network. These will, in turn, be defined in terms of partial tracing, an operation that defines quantum information and entanglement between two quantum subsystems in general (see (3.14)). Course-graining in a spin network is the procedure in which a larger region in the spin network is patched from smaller ones and the dynamics are redefined to the larger region from the smaller patch areas. Define the following sets:

$$\begin{aligned}
&\mathrm{int}(B) = \{\text{vertices } v \in \Gamma \cap B\} \\
&ext(B) = \{\text{vertices } v \in \Gamma \setminus \mathrm{int}(B)\} \\
&\partial B = \left\{ \begin{aligned} &\text{edges } e \in \Gamma \,|\, \text{if } v_e^1 \text{ and } v_e^2 \text{ are ends of } e,\ v_e^1 \in \mathrm{int}(B), \\ &v_e^2 \in ext(B) \text{ or } v_e^2 \in \mathrm{int}(B), v_e^1 \in ext(B) \end{aligned} \right\}
\end{aligned} \tag{3.45}$$

Define the state of the region B as the tensor product of all the intertwiners of the vertices, $v \in B$. Then, the Hilbert space of intertwiners of B is: $\mathcal{H}_B = \mathcal{H}_{\mathrm{int}(B)} \equiv \bigotimes_{v \in B} \mathcal{H}_v$. The corresponding Hilbert space of $ext(B)$ is: $\mathcal{H}_{ext(B)} \equiv \bigotimes_{v \notin B} \mathcal{H}_v$. Lastly, the Hilbert space of the boundary, ∂B ,$\mathcal{H}_{\partial B}$, is defined as the space of intertwiners between the representations j_e that are attached to the edges that cross ∂B . This turns out to be the space of states of B when one course-grains B to a single vertex.

Consider an arbitrary subset of edges of B, E_B, and the group of holonomies along those edges, $\{g_e, e \in E_B\}$. In general, the parallel-transport dependent boundary state is given by:

$$\int_{SU(2)} dg \ tr\left[\left\{ \bigotimes_{e \in \partial B} D^{j_{e_v}}(g)^{\varepsilon_e} / \sqrt{d_j} \right\} \otimes \left\{ \bigotimes_{e \in Int(B)} D^{j_{e_v}}(g_e) \right\} \otimes \left\{ \bigotimes_{v \in \mathrm{int}(B)} \mathcal{I}^v \right\}\right] \in \mathcal{H}_{\partial B} \tag{3.46}$$

where $\varepsilon_e = \begin{cases} 1 & \text{if } s(e) \in B, \text{ (outgoing edge of } B) \\ -1 & \text{otherwise} \end{cases}$. The trace operator is taken over all the *SU(2)* representations, V_{j_e} around each vertex $v \in B$. The integration of the tensor product integrand in (3.46) over *SU(2)* implies that it is *SU(2)* invariant, which, in turn implies that it is a bonafide normalized intertwiner operator.

Now consider two subsystems, *A* and *B* and their tensor product Hilbert space of states, $\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$. A basis in $\mathcal{H}$ can be formed by the direct products $|mn\rangle = |m\rangle_A \otimes |m\rangle_B$ with wave functions, $\psi_m(x) = \langle x|m\rangle$ for *A* and $\psi_n(y) = \langle y|n\rangle$ for *B*. In this setting, a general state in $\mathcal{H}$ can be expressed as $|\psi\rangle = c_{mn}|m\rangle|n\rangle$. We will form the elements of a general operator *O* on $\mathcal{H}$ by using the reduced density operator that is standard in the development of entangled information and was touched upon earlier in this chapter. First, let $\langle mn|O|m'n'\rangle = o_{mm'}\delta_{nn'}$ be the (matrix) elements of *O*. The partial trace operation between *A* and *B* can then be defined as:

$$\langle\psi|O|\psi\rangle = o_{mm'}c_{mn}c_{m'n} = tr(o\rho_\psi^A) \tag{3.47}$$

Note that the reduced density operator given by $\rho_\psi^A \equiv tr_B(\rho_\psi)$ is constructed by tracing out B, and $\rho_{mm'}^A = \rho_{(mn,m'n)}$. The operator *O* can be expressed as:

$$O\big((x,y),(x',y')\big) = o(x,x')\delta(y-y') \tag{3.48}$$

The reduced density operator, $\rho_\psi^A(x,x')$ w.r.t. the coordinates (x,x') may then be written as:

$$\rho_\psi^A(x,x') = \int \psi(x,y)\overline{\psi}(x',y')\delta(y-y')dydy' = c_{mn}c_{m'n}\psi_m(x)\overline{\psi}_m{}'(x') \tag{3.49}$$

and the partial trace operator is written as $\langle\psi|O|\psi\rangle = \int o(x,x')\rho_{\psi}^{A}(x,x')dxdx'$. Note that we have used the condition that $\{\phi_n\}$ is an orthonornal basis. Then $\langle\psi|O|\psi\rangle$ is the partial trace on the space $\mathcal{H}^O$, the abstract Hilbert space of operator O states. Terno considers the application of this definition to a class of local operators acting on the vertices of spinfoam, Γ which is generalizable to other classes of operators. This is a formal definition of the partial trace operation for entangled subsystems acted upon by a general operator on a spinfoam.

We consider the case of Schwarzchild nonrotating black holes and a spinfoam representation with qubits on its surface patches. A case for rotating black hole computation is given at the end of Appendix B. The case for relativistic quantum information independent of the quantum gravity framework was reviewed and investigated in Adami (2004) and Bais and Farmer (2007). However, LQG information theory, as discussed here, provides a subsuming conception to that of relativistic quantum information. Black-hole information theory, as discussed in Appendix B, is done so in the context of quantum gravity and hence serves as a basis for general information in moving information containers and organisms.

We are interested in computing the entropy of such spinfoams and hence of the information dynamics on such devices. Measures of entanglement on the space of intertwiners of $2n$ qubits, in a spin-$\frac{1}{2}$ system, $\mathcal{H} = Int(\mathbb{C}^2)^{\otimes 2n}$ in the case of a black hole horizon with area, $A = \alpha_{1/2} 2n$ have been shown to be equivalent. We will use the entanglement of formation as a representive measure of entanglement on this discrete computational space of $2n$ qubits. Spin-s systems which represent the patches on the surfaces of spinfoams are considered, starting with the spin-$\frac{1}{2}$ qubit systems. These are akin to quantum computer registers residing on spinfoams. In effect, they are LQG spinfoam computers. We are interested in the general information flow in such devices as a model for information flow in bi-partite systems such as our informaton model in this paper. The informaton model is a system of bipartite-entangled event-observer pairs and as such represent information particles that live by entanglement. Let $\rho = \sum_\alpha w_\alpha \left|\Psi_\alpha\right\rangle\left\langle\Psi_\alpha\right|$ be a general state expressed as a convex combination of pure states. The entanglement of formation is then expressed as:

$$E_F(\rho) = \inf_{\Psi_\alpha} \sum_\alpha w_\alpha S(\rho_\alpha) \tag{3.50}$$

where $S(\rho_\alpha)$ is a suitable entropy, such as the von Neumann entropy of the reduced density operator, ρ_α. Now divide the set of qubits into two subgroups (subsystems), A and B of size $2k < n$ and $2n-2k$ respectively. The respective Hilbert spaces of states are denoted by $\mathcal{H}_A \equiv (\mathbb{C}^2)^{\otimes 2k}$ and $\mathcal{H}_B \equiv (\mathbb{C}^2)^{\otimes 2n-2k}$. The intertwiner space is then expressed as:

$$\mathcal{H}^O = V^O \otimes \sigma_{(2n,0)} = \oplus V^0_{(j)} \otimes (\sigma_{(2k,j)} \otimes \sigma_{(2n-2k,j)}), \tag{3.51}$$

where $V^0_{(j)}$ is the singlet state in the product space, $V^j \otimes V^j$. Let

$N = c_{(2n,0)} = \sum_{j=0}^{k} c_{(2k,j)} c_{(2n-2k,j)}$. Denote the basis states of A and B by

$\left| j,m,a_j \right\rangle$ and $\left| j,m,b_j \right\rangle$ respectively, where $0 \le j \le k(n-k)$, $-j \le m \le j$ and the degeneracy labels, a_j and b_j account for the subspaces, V^j. The intertwiners are then expressed as:

$$\left|\mathcal{I}^{(a_j,b_j)}\right\rangle \equiv \tfrac{1}{\sqrt{2j+1}} \sum_{m=-j}^{j} (-1)^{j-m} \left| j,-m,a_j \right\rangle \otimes \left| j,-m,b_j \right\rangle \tag{3.52}$$

One can then express the entanglement of formation of the state ρ by:

$$E_F(\rho) = \tfrac{1}{N} \sum_{j=0}^{k} c_{(2k,j)} c_{(2n-2k,j)} \log(2j+1) \tag{3.53}$$

Consider the limit as $N \to \infty$ while equally splitting the subsystems, i.e., $k = \frac{n}{2}$. Then

$$\lim_{N\to\infty} E_F(\rho : k = \frac{n}{2}) \sim \frac{1}{2}\log n = \log\frac{m}{2} \tag{3.54}$$

for spin networks with general spin, s. Terno shows that under general conditions for all bipartite partitions of a horizon spin network with large enough numbers of edges and general spin, $s \geq \frac{1}{2}$, the quantum mutual information between the black hole horizon, A, and its component parts, B, equates to three times the inter-component entanglement, i.e.,

$$I_\rho(A:B) = S(\rho_A) + S(\rho_B) - S(\rho) \simeq 3E_F(\rho \,|\, A:B) \tag{3.55}$$

Furthermore, if $\frac{k}{n} \equiv C$ is fixed, as $n \to \infty$, using a logarithmic correction:

$$\lim_{\substack{n\to\infty \\ \frac{k}{n}\equiv C}} I_\rho(A:B) = \tfrac{3}{2}\log n \tag{3.56}$$

This implies that in a black hole horizon with independent, uncorrelated qubits, the entropy scales linearly with the number of qubits in the horizon, $2n$. This correction is due to the condition that qubits begin to correlate because of their invariance under

SU(2). This is the qubit black hole model of information. Terno points out that when the qubit black hole model starts with one outside segregate pair of qubits, then the fraction of unentangled states in the model follows as $s \sim \frac{1}{4} + \frac{3}{8n}$. This has possible implications for the hypothesis of information loss in black holes and the evaporation model. The consequences of this are that information loss is possible because of the lessening of entanglement in such limited segregated start up qubit black hole models.

It should be noted that these results are generalizable to the qudit case where the dimension of the state space for each spin particle on a surface patch is *d*. In this case, a singlet state is expressed as:

$$\left|\varsigma\right\rangle = \frac{1}{\sqrt{d!}} \sum_{i_1 \ldots i_d} \varepsilon_{i_1 \ldots i_d} \left|i_1 \ldots i_d\right\rangle \tag{3.57}$$

where $\varepsilon_{i_1 \ldots i_d}$ is completely antisymmetric and $\varepsilon_{0,1,\ldots,d-1} = 1$. The qudits then become *SU(d)* invariant. The infinite-dimensional case for the state space (and continuous variables, *CV* state space) becomes interesting because it has been posited that in such a setting, entanglement sharing becomes readily available to all infinite-state ∞-qudits (Dennison & Wooters, 2001). This means that under a condition of marginal pairwise entanglement (i.e., the entanglement of formation measure of the system which is the maximum of the minimum entropies between pairs of qudits in the system is sufficiently small), all qudits can share entanglement throughout the system.

Pérez (2010) develops a scheme in which information from a qubit system, *S* interacts with an environment, *E* which is considered generally as an *n*-level quantum system, i.e., a qudit. In this method quantum information decoheres into the environment.

Partial information from *S* is then measured using a fraction, $f = \frac{n_F}{n}$ of levels from *E* that define a subsystem *F*. Mutual information, $I(S:F)$ is then a partial measurement of the information from *S*. Decoherence is normally the killer of entanglement. In this general case of a qubit and its environment, mutual information can be recovered as an indicator of the original quantum information from *S*. In the case of informatons in chapter 4, the mutual pairs of entities within each informaton can recover mutual information from other informaton subsystems that are considered part of their respective environment. This is considered a form of *quantum Darwinism* because observers get information about quantum systems through their imprint on its environment. Zwolak, Quan, & Zurek (2009) first coined the term quantum Darwinism to determine the *haziness*, *h* of a qubit system, *S*, which by definition is the mutual information (initial mixed entropy) gathered about *S* from a mixed environment *E* involving *S*. The storage capacity of a fragment, *F* of the environment, *E* which contains the decohered qubit information of *S* is reduced by a factor of $1-h$, where *h* is the haziness of *S*. Haziness then determines the capacity to store information about a qubit in an environment or part thereof containing it. We next consider a model for information fields using generalized Bayesian signal processing in the form of an information field theory constructed in the tradition of QFT. It will be extended to apply to LQG spinfoams, lattice models, and a general uncertainty framework to be applied to construct a generalized information field theory for this study's intended information metamodel.

Information Field Theory

Combining the mechanisms of physical field theories that utilize Feynman's sum-of-paths integral definition and the partition function using a Hamiltonian action (operator) with classical definitions of a signal process via linear response theory in a Bayesian probabilistic setup, an attempt was made to construct an information field theory (Enßlin, Frommert, & Kitaura, 2008). Faraday originally defined the concept of physical fields to accommodate the spatial dynamics of electromagnetism (Faraday, 1839). A field is constructed on a space by assigning a vector value at each "workable" point of that space where workable means computable. These vectors represent arrays of values relating to observables of an entity. The underlying space may be Einstienian and quantum in nature and hence a creditable physical field theory must take into account the constraints of such frameworks. In this work it was assumed that a signal contained within the full physical state of a system in spacetime is a representation of an observer's filtration, i.e., is purely the limited part of the system that is of interest to the subjective observer. Additionally, only one data observation is taken and so this technique is not an ensemble statistical decision problem as in the derivation of an estimator based on a repeated iid sampling. Let s depict the signal of interest and d the data sample collected by the observer. The signal may be interpreted as a function acting on the state, $s = s(\psi)$. The linear response model is used by the authors where the data is modeled as a response R to the signal s, plus a noise term n,

$$d = R(s) + n(s) \tag{3.58}$$

The response is then defined canonically as that part of the observed data which correlates with the signal. One expresses this as:

$$R(s) = \langle d \rangle_{d|s} \equiv \int_d d \ [p(d \mid s)] \ d\mu_d \tag{3.59}$$

with the noise simply being the remainder, $n(s) = d - R(s) = d - \langle d \rangle_{d|s}$. By this definition, the noise component is linearly uncorrelated to *s*, given the data *d*, i.e.

$$\langle n(s), s \rangle_{d|s} = \left[\langle d \rangle_{d|s} - R(s) \right] s^{\dagger} = 0 s^{\dagger} = 0 \tag{3.60}$$

The response function methodology is not diffeomorphic-invariant, in fact, it is not even coordinate invariant since for any transformation, *T* of the data *d*, if $d' = T(d)$, then

$$\begin{aligned} R'(s) &= \langle d' \rangle_{d|s} \\ &= \langle T(d) \rangle_{d|s} \\ &= \int T(d) p(d \mid s) \mu_d \\ &\neq T\left[\int d \ p(d \mid s) \mu_d \right] \text{ (in general)} \\ &= T\left[\langle d \rangle_{d|s} \right] \\ &= T\left[R(s) \right] \end{aligned} \tag{3.61}$$

Hence, linear response functions are not unique in defining a reconstruction of signals. Criteria must then be used in order to differentiate the performance of such transformations of data to construct spaces of response functionals. What is desirable is the maximization of the response function, $R_T(s)$ corresponding to the data transformation *T* on *d*.

This mapping should also recover (or be as nearly invariant to *s*) as much of the signal as possible, i.e., $T(d) \cong \langle s \rangle_{s|d}$. In this sense, the search for an optimal estimator of the signal based on the observed data and prior knowledge takes on the form of the min-max problem:

$$\begin{aligned} &\min_{T \in T^*} \left| T(d) - \langle s \rangle_{s|d} \right| \\ &\max_{T \in T^*} R_T(s) \end{aligned} \tag{3.62}$$

where T^* is the dual space of transformations from the data manifold to the reals. One can combine this problem into a single minimization (or maximization), provided $R_T(s)$ is not degenerate (or $T(d) = \langle s \rangle_{s|d}$):

$$\min_{T \in T^*} \frac{\left| T(d) - \langle s \rangle_{s|d} \right|}{R_T(s)} \tag{3.63}$$

Since there is no guarantee in general that this will not happen, approximations to this optimization problem are done using variational methods. One requirement would then be that $R_T(s)$ is positive definite *w.r.t.* s, i.e., $\frac{\partial R_T(s)}{\partial s} \geq 0$ so that the response does not decrease with an increasing signal. To minimize $\left| T(d) - \langle s \rangle_{s|d} \right|$ in L^2, one defines the quadratic loss (uncertainty):

$$\sigma_T^2 = \left\langle [s - T(d)][s - T(d)]^\dagger \right\rangle_{d|s} \tag{3.64}$$

and the expected value of its square, the trace of σ_T^2:

$$\begin{aligned} tr(\sigma_T^2) &= \int \left\langle | s_x - T_x(d) | \right\rangle_{s|d}^2 dx \\ &= E_{s|d} \left[s_x - T_x(d) \right]^2 \end{aligned} \tag{3.65}$$

where x is the variable of integration *w.r.t.* a random variable $X \underset{dist'n}{\sim} p(s \mid d)$.

Here s_x and $T_x(d)$ both depend on this random variable through the posterior, $p(s \mid d)$. One would then minimize the trace, $tr(\sigma_T^2)$ over the space of plausible data transformations, *T*. The authors suggest limiting their search space of plausible signal functions on the information state field, ψ to analytic, linear, and smooth operators. In addition, they suggest choosing a signal such that optimal knowledge of the information field can be extracted, in particular when the posterior and prior, $p(d \mid s)$ and $p(s)$ could reliably give a good approximation to the nature of the information field ψ. This was made unclear and could be further crystallized through the use of optimal statistical estimators such as complete and minimally sufficient statistical estimators. These, however, require repeated quantum experiments. Nonetheless, in general, a signal operator could be a filter applied to an input stream in producing an output or in a quantum mechanical setting, a measurement operator of the quantum state producing an observation.

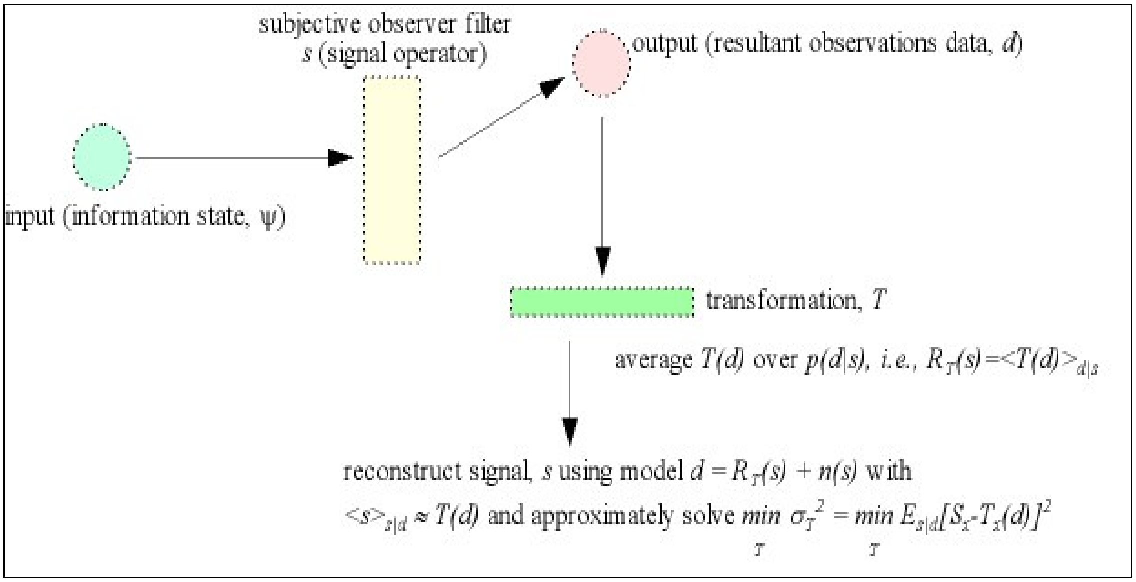


Figure 6. Quantum information signal processor

Considering the whole information state variable, ψ , the density function of *d* can be expressed in the tradition of Feynman sum of path integrals (histories) as the sum of paths integral over all realizations of ψ :

$$p(d) = \int_{\psi} p(d \mid \psi)\, p(\psi) d\psi \tag{3.66}$$

See Appendix A for a review of Feynman path histories based on Hamiltonian operators and field theory as an alternative to computing quantum states using wave equation derivations. When concentrating on a particular signal functional, *s*, of the information state, ψ , and by utilizing Bayes Theorem, the posterior probability of that signal given the data is expressed as:

$$p(s \mid d) = \frac{p(d \mid s)\, p(s)}{p(d)} = \frac{e^{-H_d(s)}}{Z_d}, \tag{3.67}$$

where $p(d \mid s)$ is the probability of achieving the observed data, *d*, given a signal *s*, $p(s)$ is the prior probability of the signal s, $p(d)$ is the unconditional probability of achieving the signal s, $H_d(s)$ is the Hamiltonian operator of the signal *s*, and Z_d is the partition sum for the data state *d*. This posterior can also be constructed by a Taylor-Fréchet expansion around a suitable field variable *t*. The Hamiltonian, dependent on the data *d*, is expressed as:

$$H_d(s) \equiv -\log\left[p(d,s)\right] = -\log\left[p(d \mid s)p(s)\right] \tag{3.68}$$

while the partition (also dependent on the data) is written as:

$$Z_d = \int_s e^{-H_d(s)} ds = \int_s p(d \mid s)p(s)\,ds = p(d) \tag{3.69}$$

As a convenient tool to compute moments of *s*, the authors introduce into the definition of *Z*, the moment generating function, *J* (utilizing the Fréchet partial differentiation operations, $\Delta_F^n = \dfrac{\delta_F^n}{\delta J(x_1)...\delta J(x_n)}$ acting on functions on Banach spaces and write:

$$Z_d(J) = \int_s e^{-H_d(s)+J^\dagger(s)} ds \tag{3.70}$$

Then the (connected) correlation functions, $E_n^d(s)$, may be expressed as:

$$E_n^d(s) = \left\langle s(x_1),...,s(x_n)\right\rangle_d = \Delta_F^n[\log Z_d(J)]\,|_{J=0} \tag{3.71}$$

Finally, the Hamiltonian can be Taylor- Fréchet expanded as:

$$H(s) = \frac{s^\dagger D^{-1} s}{2} - J^\dagger s + H_0 + \sum_{n=3}^{\infty} \frac{1}{n!} \Lambda^n_{x_1...x_n}[s(x_1)...s(x_n)] \tag{3.72}$$

where $H_0 = \log\left[\int_s ds \int_d e^{-H_d^{'}(s)} \mu_d\right]$ is the normalized Hamiltonian and $H_d^{'}(s)$ is the unnormalized Hamiltonian. Additionally, μ_d is simply the measure of integration w.r.t. the data variable d. D^{-1} is defined as the quadratic coefficient of the information propagator, $D(x.y)$ that in turn propagates an information field from the signal, $s(y)$, that is at y, to another location x. The operators Λ^n, act as anharmonic tensors, creating interactions between the modes of the free, harmonic theory (Enßlin, et al., 2008). The authors give as their example a Gaussian data model in a free theory (free of interactions). Next, they form the interaction version under the Gaussian model for which the above Hamiltonian is split between the free and interacting Hamiltonian parts as:

$$\begin{aligned} H(s) &= \frac{s^\dagger D^{-1} s}{2} - J^\dagger s + H_0^G + \sum_{n=3}^{\infty} \frac{1}{n!} \Lambda^n_{x_1 \ldots x_n} [s(x_1)\ldots s(x_n)] \\ &= H_G(s) + H_{\text{int}}(s) \end{aligned} \tag{3.73}$$

where $H_0^G(s)$ is the nonnormalized Gaussian Hamiltonian, $H_G(s)$ the free Hamiltonian part and $H_{\text{int}}(s)$ the interaction Hamiltonian. Using a shifted field, $\phi = s - t$ for computational convenience, the Hamiltonian can be expressed as:

$$\begin{aligned} H(\phi) &= \frac{\phi^\dagger D^{-1} \phi}{2} - J^\dagger \phi + H_0^{G'} + \sum_{n=0}^{\infty} \frac{1}{n!} \Lambda^{'(n)}_{x_1 \ldots x_n} [\phi(x_1)\ldots \phi(x_n)] \\ &= H_G(\phi) + H_{\text{int}}(\phi) \end{aligned} \tag{3.74}$$

where the shifted version terms are:

$$H_0^{G'} = H_0^G - J^\dagger t + \frac{t^\dagger D^{-1} t}{2},\ J^{'} = J - D^{-1} t,\ \text{and}\ \Lambda^{'(n)}_{x_1 \ldots x_n} = \sum_{n=0}^{\infty} \frac{1}{n!} \Lambda^{(m+n)}_{x_1 \ldots x_{m+n}} [t_1 \ldots t_n] \tag{3.75}$$

The partition sum can then be expressed as:

$$Z(J) = e^{\left[-H_{\text{int}}\left(\frac{\delta}{\delta J}\right)\right]} Z_G(J) \tag{3.76}$$

where $Z_G(J) = \int e^{-H_G(s)+J^\dagger s} ds$ is the partition function of the free (noninteracting) case. Using the Feynman diagrammatic expansions, the logarithms of $Z(J)$ and hence any connected moments are calculated. Feynman diagrams are constructed from the symbols of lines, vertices with line attachments and without line attachments according to the Feynman rules:

1. Open-ended lines represent external coordinates. In the case of information fields, the moments $m^n(x) = \frac{\delta^n[\log Z(J)]}{\delta J(x_1)\ldots\delta J(x_n)}|_{J=0}$ are coordinate dependent and hence must be expressed with open-ended lines.
2. The propagators, D are represented by a line connecting the coordinates defining D.
3. Vertices with one line attachment represent the expression $j_{x'} + J_{x'} - \Lambda^{(1)}_{x'}$
4. Vertices with n attachment represent the expression $\Lambda^{(n)}_{x'_1 \ldots x'_n}$
5. All internal coordinates are integrated over (external coordinates are not)
6. Each diagram is divided by its symmetry factor which is the number of permutations of vertex attachments leaving the topology invariant.

This model of an information field presumes a design that involves a linear response function, single observation experiments, and a separate signal operator as a outside process. In this proposal, to be expanded in the next chapter, these models will be replaced by generalizations and extensions to include multiple experiments, generalized response functions, a signal that will be represented as a physical particle that is holistically included in a quantum system, not as a separate observer, and a generalized uncertainty framework that will include quantum probabilities as a special case. Furthermore, the Hamiltonian operators that are derived from these extensions as generators of an information field will be applied in the more general spinfoam formalism.

With respect to the partition sums, a measure of negative entropy is given by:

$$\begin{aligned} I(d) &= -\langle H(s)\rangle_d - \log Z_d \\ &= -\langle H(s)\rangle_d - \log Z(d,0) \\ &= \frac{\partial F_\beta(d,J)}{\partial\beta}\Big|_{\beta=1,J=0} \end{aligned} \tag{3.77}$$

where $Z_\beta(d,J) = \int e^{-\beta[H(s)-J^\dagger s]}ds$ and $F_\beta(d,J) = -\dfrac{\log Z_\beta(d,J)}{\beta}$. F_β is the Helmholtz free energy of the system written as a function of the inverse temperature, β. In terms of the generated signal, *s*, F_β may be expressed as:

$$F_\beta = -\frac{1}{\beta}\log Z_\beta^G(J) - \frac{1}{\beta}\log\left\langle e^{-\beta H_{\text{int}}(s)}\right\rangle_{s|J+j,G} \tag{3.78}$$

where the last average is taken over the Gaussian *pdf* defined as $p_{J,\beta}^G(s) \propto e^{-\beta[H_G(s)-J^\dagger s]}$.

These quantities may be calculated using the Feynman diagrams rules as defined above. With this definition of entropy (Boltzmann-Shannon), one may calculate $I(d)$ using some simplifications, such as in the free theory case (no interactions) and the underlying Gaussian distribution for the signal:

$$I(d) = -\frac{tr\left[1+\log(2\pi D)\right]}{2} \tag{3.79}$$

and the information prior to the data observations as:

$$I(0) = -\frac{tr\left[1+\log(2\pi S)\right]}{2} \tag{3.80}$$

obtaining the information gain from the data as:

$$\begin{aligned}\Delta I(d) &= I(d) - I(0) \\ &= \frac{tr\left[\log(SD^{-1})\right]}{2} \\ &= \frac{tr\left[\log(1+SR^{\dagger}N^{-1}R)\right]}{2} \\ &= \frac{tr\left[\log(1+R^{\dagger}QR)\right]}{2}\end{aligned} \tag{3.81}$$

where D is the propagator, R is the response, N the noise, and S, the signal operators respectively. Additionally, $Q \equiv RSR^{\dagger}N^{-1}$, the signal response-to-noise ratio, In this manner, the information gained from the data, $I(d)$ is dependent on Q and hence on the fidelity of the signal. The fidelity between two quantum information states (pure or mixed), commonly called the transition probability between ρ and φ is defined as:

$$F(\rho,\varphi) = tr(\rho^{1/2}\varphi\,\rho^{1/2}) \tag{3.82}$$

and is a measure of the similarity between ρ and φ (Petz, 2008, p.83). F is not a metric but can be turned into one using the Bures distance:

$$D(\rho,\varphi) = \sqrt{2-2\sqrt{F(\rho,\varphi)}} \tag{3.83}$$

It has been shown that an upper bound for fidelity is the so-called super-fidelity defined as (Miszczak, Puchała, Horodecki, Uhlmann, & Życzkowski, 2009):

$$U(\rho,\varphi) = tr(\rho\varphi) + \sqrt{1 - tr(\rho^2)}\sqrt{1 - tr(\varphi^2)} \tag{3.84}$$

with a general lower bound (subfidelity) of:

$$L(\rho,\varphi) = tr(\rho\varphi) + \sqrt{2}\sqrt{[tr(\rho\varphi)]^2 - tr[(\rho\varphi)^2]} \tag{3.85}$$

In signal processing, the reconstruction signal taken from the response functional given the data is measured and compared to the original signal. Their similarity is the fidelity of the signal reconstruction. This may also be measured by the quantity Q and reframed in the context of F, U, and L above in the following sense. Let $\rho = s(\psi)$ and $\varphi = R_T(s) = \langle T(d) \rangle_{s|d}$. Then $F(\rho,\varphi) = tr[S^{1/2}\mathrm{R}\ \mathrm{S}^{1/2}]$. In the case of higher order interacting theory, the information gain will have a remainder term, $O(\gamma^2)$ added above in the order of a parameter, γ that is proportional to the interaction perturbations. General definitions of entropy, such as quantum and general uncertainty versions of the family of Renyi entropy, Bergmann entropy, von Neumann quantum entropy, and the α and β families of entropies will be designed into this definition for information field theory in the next chapter. The corresponding expressions for fidelity and quality of information measurement will follow as well.

An information field theory (IFT) can be similarly constructed in the LQG case by the use of the Hamiltonian action from (3.43). In (3.73), the hamiltonian, H_0^G is substituted with the LQG hamiltonian action, $S_H(\phi)$.

Holonic and Complex Adaptive Systems

Although the spinfoam model, information field theory, and digital renditions of a universe computer (hypercomputation or Turing) address nonlocal and global formulations of entities as reviewed before, the issue of the formation of complex systems from microlaws and the bridging to macrolaws is largely missing or unconnected. Can one construct evolutionary rules for complex adaptive systems development from general rules for spacetime and information fields? Towards this end, the next section will review complex adaptive systems (CASs) and their most currently practical models, multiagent ensembles. To a lesser extent, holonic systems have become attractive as a means of describing the organizational behavior of natural complex systems. Holarchies will be combined with multiagent complex adaptive systems (MCASs) in the final portion of the review on CASs. This will set the stage for the proposals to be made in the next chapter that attempt to pull together information field models as a calculus for constructing general computational holonic multiagent complex adaptive systems as models for realistic ensembles and organization.

In the vernacular of complex systems theory, a system consisting of multiple agents or entities is complex if it exhibits collective behavior that cannot be explained by the microlevel rules of its components. This behavior is at times self-referential, emergent, evolutional, self-organized or adaptive. Therein lay ambiguities to the technical definitions of complex systems. Does one characteristic causally link to another or are all of these properties linked to a separate mechanism? In an attempt to unify the understanding of properties of complex systems a proposed standard was presented grouping these properties utilizing information theoretic concepts, specifically entropy (Prokopenko, Boschetti, & Ryan, 2009). These authors used classical definitions of entropy to define categories of complexity. Here quantum and more general extensions of entropy will be used in that context. Complex or at least a measure of complexity may be viewed as the amount of information needed to describe a system. Shannon entropy was proposed to measure this concept for signals in a noisy channel defining his famous entropy quantity (Shannon, 1948). Previous to Shannon's work on entropies for probability distributions, von Neumann (1955) expressed a quantum version of entropy for a quantum density operator (matrix), ρ :

$$S(\rho) = -Tr\left(\rho \ln \rho\right) \tag{3.86}$$

An alternative definition of entropy comes from Rényi and its quantum version is given by (Hayashi, 2006, p. 40):

$$S_{\alpha}(\rho) \equiv \ln\left[Tr\rho^{1-\alpha}\right],\ \alpha \neq 1 \tag{3.87}$$

One of the most general definitions for classical entropies comes from Salicru (Salicru, 1993) who defined the $(h,\phi)-$entropy as:

$$H_{\phi}^{h}(X) = h\left(\int_{\chi} \phi\left(f_{\theta}\left(x\right)\right) d\mu(x)\right) \tag{3.88}$$

where X is a random variable distributed by the *pdf* f_θ (parameterized by θ), μ is an appropriate measure of integration, χ is the space of possible values of X, $\phi:(0,\infty)\rightarrow\mathbb{R}$, $h:\mathbb{R}\rightarrow\mathbb{R}$, h is differentiable and either:

1. ϕ is concave and $h^{'} \geq 0$ or

2. ϕ is convex and $h^{'} \leq 0$

The discrete case can be handled by replacing the integral with a summation over the probability space. Additive entropies can be classified under this very general form. Both the classical Shannon and Rényi entropies are special cases of this entropy family. These general entropy measures are examples of functionals defined on the space of probability distributions of the underlying random variables. Tsallis studied and developed functionals in this manner using the principle of maximum entropy. Havrda and Charv´at before him, published the original version as a theoretical entropy (Havrda & Charv´at, 1967). Tsallis generalized Shannon entropy for nonadditive entropies using a parameter *q*. The quantum version of this is (Hu & Ye, 2006):

$$S(\rho) = (1-q)^{-1}\left[Tr\rho^{q} - 1\right],\ q > 0,\ q \neq 1 \tag{3.89}$$

the *quantum Tsallis entropy of degree q*. The most general version of this form of Tsallis-like entropy in quantum systems is called the *quantum unified* $(q,r)-$entropy and is given as:

$$E_q^r(\rho) = \begin{cases} S_q^r(\rho), \text{ if } q \neq 1, r \neq 0 \\ S_q(\rho), \text{ if } q \neq 1, r = 0 \\ S^q(\rho), \text{ if } q \neq 1, r = 1 \\ {}_qS(\rho), \text{ if } q \neq 1, r = 1/q \\ S(\rho), \quad \text{if } q = 1, r > 0 \end{cases} \tag{3.90}$$

Here

$${}_qS(\rho) = (1-q)^{-1}\left(\left[Tr\rho^{1/q}\right]^{q} - 1\right),\ q > 0,\ q \neq 1 \tag{3.91}$$

is the *quantum entropy of type q*,

$$S_q(\rho) = (1-q)^{-1} \ln\left[Tr \rho^q\right],\ q > 0,\ q \neq 1 \tag{3.92}$$

is the *quantum Renyi entropy of degree q*, and $S(\rho)$ is the before mentioned von Neumann entropy. Note that $\lim_{q \to 1} E_q^r(\rho) = S(\rho),$ for $r > 0.$ A class of more general functionals on classical probability distributions is the so-called *q-expectation* given by:

$$\langle u_m \rangle_q = \int u_m(x) p(x)^q dx,\ m = 1, \ldots, M \tag{3.93}$$

where the *normalized q-expectation* is given by:

$$\langle\langle u_m \rangle\rangle_q = \frac{\int u_m(x) p(x)^q dx}{\int p(x)^q dx} = \int u_m(x) k^{(q)}(x) dx,\ m = 1, \ldots, M \tag{3.94}$$

and the set of functions, u_m, are akin to energy state levels with the so-called escort probability distributions, $k^{(q)}(x) = \frac{p(x)^q}{\int p(x)^q dx},\ m = 1, \ldots M$ acting as the moment functions. Statistical estimation based on the Tsallis entropies is known as Tsallis statistics and is of importance because it arises and is useful in the case of dimensional reduction in quantum gravity black-hole thermodynamics and as a generalization to Boltzmann-Gibbs statistics in the case of nonadditive entropies used in the nonextensive statistical mechanics of quantum gravity or large range effects (Tsallis, 1988; Cantcheff, & Nogales, 2005). Extensive systems are those in which total system energy is proportional to system size. Criticisms of Tsallis statistics as a means of thermodynamic construction of systems mainly point to certain nonphysical conclusions reached by them. It continues to be a controversial theoretical construct with some successful implementations.

This dissertation will develop further generalizations to these entropies and their properties in regards to LQG-spinfoam and Zadeh (2005) general uncertainty (GU) information models. Since complex systems consist of multiple entities, their interactions are as important as each individual action and as such one would like to utilize the concepts of joint and conditional entropies and mutual information of two or more quantum random variables. The joint entropy of two quantum states ρ and σ respectively of two subsystems of a quantum system is defined as:

$$S(\rho,\sigma) = S(\rho\otimes\sigma) = -Tr\left[\left(\rho\otimes\sigma\right)\ln\left(\rho\otimes\sigma\right)\right] \tag{3.95}$$

where $\rho\otimes\sigma$ is the joint state. The conditional entropy of ρ given σ written also in terms of joint and singular entropies, is defined as:

$$S\left(\rho\,|\,\sigma\right) = S\left(\rho,\sigma\right) - S(\sigma) \tag{3.96}$$

The mutual information about the quantum state ρ of one subsystem that remains after knowing the quantum state σ, of a second subsystem written in terms of conditional, singular, and joint entropies, is given by:

$$\begin{aligned} I\left(\rho;\sigma\right) &= S(\rho) + S(\sigma) - S(\rho,\sigma) \\ &= S(\sigma) - S(\sigma\,|\,\rho) \end{aligned} \tag{3.97}$$

Mutual information is especially useful in signal processing since it presents a concept of shared information between a source quantum state, ρ and a receiver quantum state σ, and a means to maximize a signal between the two. The quantum relative entropy for two density operators, ρ and σ is given by:

$$D_{KL}(\rho\,||\,\sigma) \equiv Tr\left[\rho\left(\ln\rho - \ln\sigma\right)\right] \tag{3.98}$$

This is called the quantum *KL*-divergence or *relative entropy* of ρ and σ and is a pseudo-metric. It can be turned into a metric by considering the symmetrized divergence:

$$D_{KL}^{s}(\rho \,||\, \sigma) = D(\rho \,||\, \sigma) + D(\sigma \,||\, \rho) \tag{3.99}$$

Divergence distances such as the *KL*-divergence and its more general *f*-divergence family, are relative entropy measures because they measure the minimum amount of information needed to be transmitted to one entity when using a language or code from another entity. Here the quantum version of the *f*-divergence between two density operators, ρ and σ on $\mathcal{H}$ is defined by:

$$D_f(\rho \,||\, \sigma) = Tr\left[f\left(\frac{d\rho}{d\sigma} \right) \right] \tag{3.100}$$

where *f* is a convex function such that $f(\mathbb{I}) = 0$. The classical versions were developed in (Ali & Silvey, 1966; Csiszár, 1967). In this regard, an observer would be represented by the use of a code with density operator ρ and a system component would be represented by the use of a code with density operator σ. A divergence is therefore a measure of the relative complexity of ρ relative to σ. Divergences will be used in the definition of the info-macrodynamics of the informaton model to be developed in chapter 4. Utilizing the $(h,\phi)-\text{entropy}$, quantum divergences can be further generalized as:

$$D_\phi^h(\rho \,||\, \sigma) = Tr\left[H_\phi^h\left(\frac{\rho+\sigma}{2} \right) - \frac{H_\phi^h(\rho) + H_\phi^h(\sigma)}{2} \right] \tag{3.101}$$

where H_ϕ^h is defined as in (3.88). We label these the family of quantum (h,ϕ)-divergences.

The quantum version of Fisher information will now be reviewed in connection with showing the Cramér-Rao inequality for quantum systems and hence a lower bound on uncertainty in quantum estimation problems. Consider a random variable $X:(\Omega,\mathcal{F},P)\rightarrow(G,\mathcal{G},P_X)$ defined with the quantum probability rules where M is a POVM , i.e., $P_X(G,\theta)=Tr\left[\rho(\theta)M(G)\right],\ \forall G\in\mathcal{G}$. To this end Holevo and Helmstrom defined the quantum Fisher information of a quantum system with density operator ρ as (Holevo, 2001):

$$i(\theta,M)=E\left[l_{/\theta}^2\right]=\int\limits_{G_+}p(x;\theta)^{-1}\left[Tr\left(\rho_{/\theta}m(x)\right)\right]^2\mu(dx)\qquad(3.102)$$

where l is the score function of the quantum system parameter θ, $G_+=\left\{x\in G:p(x;\theta)>0\right\}$, G is the range space of the quantum random variables and the partial differential operation is abbreviated as:

$$\left[\rho_{/\theta}\right]_{ij}=\frac{\partial}{\partial\theta}\left[\rho(\theta)\right]_{ij}\qquad(3.103)$$

An upper bound for $i(\theta,M)$ was given by (Braunstein & Caves, 1994):

$$I(\theta)=E\left[\rho_{//\theta}^2\right]=Tr\left[\rho(\theta)\rho_{//\theta}^2\right]\qquad(3.104)$$

The Cramér-Rao inequality then takes the form:

$$Tr\rho(\theta)M^2\geq\frac{1}{i(\theta,M)}\qquad(3.105)$$

The quantum Fisher information, $i(\theta,M)$ is interpreted as the amount of relative information or relative entropy that grows with small perturbations in the quantum system parameter, θ. This can be depicted as (Shalizi, 2009):

$$D\left(\theta \,\|\, \theta+\varepsilon\right) \approx \varepsilon^T i(\theta, M)\varepsilon + O\left(\|\varepsilon\|^3\right) \tag{3.106}$$

The quantum Cramér-Rao inequality then gives a lower bound on the growth rate of uncertainty in the relative information between a perturbation of a model and itself in a quantum system, as transpires in computational-statistical estimation algorithms.

Prior to embarking on quantum communication theory, we mention a concept that runs antecedent to that of entropy, that of extropy. Extropy has been popularly defined as negentropy or the information content as disorder in a system decreases. For molecular systems this is similar to the concept of potential for life expectation. More specifically, extropy has been defined for systems as the entropy of a Markov chain that describes the states of the system. These definitions are motivated by the setup for a digital probabilistic approach to physics (Stonier, 1990, 1992, 1997). We extend this to quantum systems. In a quantum mechanical system, Q, a Markov chain describing the state of a system is a pair $\left(E, \rho\right)$ where E is a quantum channel map $E : \mathcal{B} \otimes \mathcal{B} \to \mathcal{B}$ and ρ the quantum state of the system, such that E is a completely positive trace-preserving map and $\mathcal{B}$ the C^*-algebra of bounded operators that contain the observables operators on the quantum system Q, and satisfies the following:

$$Tr\rho\left(B_1 \otimes B_2\right) = Tr\rho E\left[\left(B_1, B_2\right)\right],\ \forall B_1, B_2 \in \mathcal{B} \tag{3.107}$$

(Accardi & Frigerio, 1983). The extropy of Q , $I_e(Q)$ is then defined as the entropy of the quantum Markov chain $\left(E, \rho\right)$ that describes Q:

$$I_e(Q) = S\left[\left(E, \rho\right)\right] = -Tr\rho E \ln\left(\rho E\right) \tag{3.108}$$

with respect to $\mathcal{B} \otimes \mathcal{B}$. A generalization of quantum Markov chains to quantum Markov fields and entanglement on Cayley graphs has been shown (Accardi & Volterra, 2009). A similar extropy definition may be based on such general structures. Further generalizations to GU constraints from the GTU can be made based on the uncertainty constraint variables of a GTU. We now turn to a discussion on abstract communication channels.

The channel capacity is an upper bound for the rate of communication that is possible over a quantum noisy channel and is given by maximizing the quantum mutual information over all distributions on the source quantum system. When information is transmitted classically, as in electronic signal transmission through a wire, radio waves, or light waves, the medium is quantum mechanical. In the case the transmitting of a classical bit stream through a quantum transmission channel requires two additional processes – the encoding of the classical information into a quantum state and the decoding of the message by a quantum measurement on output (Hayashi, 2006, p.94). This situation is referred to as a *c-q* (classical-quantum) channel. To this end define the quantum channel as a map, W from an alphabet, χ to the space of quantum states, $S(\mathcal{H}_O)$ on the output quantum systems in the Hilbert space $\mathcal{H}_O$. Let $\mathcal{H}_I$ be the Hilbert space of the input quantum systems. Define the map, $\Gamma : S(\mathcal{H}_I) \to S(\mathcal{H}_O)$ as the state transmission channel from the input to the output quantum systems. Define $W(x) = \Gamma(\rho(x)),\ x \in \chi$. Next define the transmission information:

$$I(p,W) \equiv \sum_{x \in \chi} p(x) D(W(x) \,||\, W(p)) = H(W(p)) - \sum_{x \in \chi} p(x) H(W(x)) \qquad (3.109)$$

where the average state $W(p)$ for the *c-q* channel W is:

$$W(p) \equiv \sum_{x \in \chi} p(x) W(x) \tag{3.110}$$

Define a *code* as a triplet $\Phi^{(N)} = (N, \varphi, Y)$ where N is a natural number that is the size of an encoder, $\varphi : \{1,...,N\} \to \chi$ is a map corresponding to the encoder, and Y is a decoder which is a set of positive Hermitian operators $Y = \{Y_i\}_{i=1,...,N}$ such that $\sum_{i=1}^{N} Y_i \le I$. Then by a theorem of Holevo, Schumacher and Westmoreland, the *c-q* channel capacity $C_c(W)$ satisfies (Holevo, 2001; Schumacher & Westmoreland, 2001):

$$C_c(W) = C_c^{\dagger}(W) = \sup_{p \in \mathcal{P}(\chi)} I(p, W) = \min_{\sigma \in S(\mathcal{H}_0)} \sup_{x \in \chi} D(W_x \,\|\, \sigma) \tag{3.111}$$

where $\mathcal{P}(\chi)$ is the space of all distributions on the space of alphabets χ and the *c-q* channel capacity, $C_c(W)$ is defined as:

$$C_c(W) = \sup_{\{\Phi^{(n)}\}} \left[\lim_{n \to \infty} \frac{1}{n} \log \left| \Phi^{(n)} \right| : \lim_{n \to \infty} \varepsilon[\Phi^{(n)}] = 0 \right] \tag{3.112}$$

with the dual capacity defined is:

$$C_c(W) = \sup_{\{\Phi^{(n)}\}} \left[\lim_{n \to \infty} \frac{1}{n} \log \left| \Phi^{(n)} \right| : \lim_{n \to \infty} \varepsilon[\Phi^{(n)}] < 1 \right] \tag{3.113}$$

Note that $C_c(W)$ is an additive operator on the space of quantum channels while the transmission information operators $I(p, W)$ are subadditive.

For quantum transmission in a quantum channel with entanglement, a general result by Devetak gives:

$$Q(\mathcal{N}) = \lim_{l\to\infty}\frac{1}{l}\max_{\rho\in\mathcal{H}_\rho^{\otimes l}} I_c(\rho,\mathcal{N}^{\otimes l}) \tag{3.114}$$

where $Q(\mathcal{N})$ is the full quantum channel capacity of a quantum channel $\mathcal{N}$, and $I_c(\rho,\mathcal{N}^{\otimes l}) = S(\mathcal{N}^{\otimes l}(\rho)) - S(tr[U_{\mathcal{N}^{\otimes l}}(\rho)])$ is the *coherent information* with $U_{\mathcal{N}^{\otimes l}}$ being the isometric extension of the composite quantum channel $\mathcal{N}^{\otimes l}$ of l quantum copies of a quantum system (Devetak, 2008). The mapping $\mathcal{N}: B(\mathcal{H}_\rho) \to B(\mathcal{H}_\sigma)$, maps bounded linear operators on the Hilbert spaces of the input and output quantum systems ρ and σ respectively.

Returning to the discussion on proposals to define a common structural framework for complex systems, Prokopenko, et al. define the *equivocation* of a receiver quantum state, ρ about a source quantum state, σ as the conditional entropy, $I(\rho\,|\,\sigma)$. Additionally, the entropy, $I(\rho)$ is an indication of the *diversity* of the quantum state, ρ. The mutual information, $I(\rho;\sigma))$ can then be expressed as the difference between the quantum receiver's diversity and the equivocation of the quantum receiver about the quantum source. The *c-q* channel capacity, $C_c(W)$ of the quantum channel defined by the quantum receiver state σ and source quantum state, ρ represents the maximization of $I(\rho;\sigma))$ over all possible distributions of the classical signal source.

In the glossary of network theory, *assortiveness* between two nodes, x_1 and x_2 is a measure of reciprocity in the sense that highly connected nodes connect with other highly connected nodes or in the opposite side of this spectrum with other low connected nodes. Equivocation of ρ about σ is then equated with nonassortiveness between the two in a network node setting. Hence, in a connected network graph, $I(\rho \mid \sigma)$ is a measure of connections that are of dissimilar degrees or, in a sense, surprise connections between entities. Prokopenko, et. al use an argument made by Solé and Valverde that $I(\rho \mid \sigma)$ can be interpreted as a noise factor that affects the overall assortiveness of the (quantum) network in the sense that it adds to a measure of (quantum) network heterogeneity without contributing to (quantum) network information (Solé & Valverde, 2004). In essence, the gist of the argument is that in order to maximize channel capacity (*c-q* channels), $I(\rho_1,,,,\rho_n)$ for an *n*-node quantum network, the diversity in the form of joint node entropies, $S(\rho_1,,,\rho_n)$, should be maximized while the set of joint node equivocations (joint node conditional entropies):

$$S(\rho_i \mid (\rho_{j_1},,,\rho_{j_{n-1}}), j_k \neq i, k = 1,,n-1) \tag{3.115}$$

should be minimized. Their respective arguments were in the case of classical entropies and systems. Here, more general quantum versions have been extended. In the next chapter, quantum entanglement will be considered and general uncertainty operators will further extend these concepts. The vague concept of complexity may now be viewed as the maximization of channel capacity.

Complexity

Prokopenko, et al. outline the differences in various analytic definitions of complexity including those of Solomonoff-Kolmogorov-Chaitin (*SKC* algorithmic complexity) and statistical complexity involving information theoretic concepts mainly entropic measures. Algorithmic complexity is defined as the shortest program needed as computed by a universal Turing machine to reconstruct a system, a deterministic approach (Chaitin, 1987). Formally, if *s* is a system described by a bit stream and *d(s)* is a description of that system as an algorithm to be executed in a universal Turing machine, then the *SKC* algorithmic complexity is given by $K_L(s) = \min_{d \in D_s} d(s)$ where D_L is the space of all plausible description programs (algorithms) using a language *L* executed in a universal Turing machine applicable to the system *s*. $K_L(s)$ is dependent on the language *L*, but an upper bound can be established with respect to the length of the system *s*, that is, $\exists$ a constant *c*, $\ni K_L(s) \le |s| + c$. The description operators *d* may also measure the time to compute the system *s*, the time complexity. A quantum version of K_L was established. In this definition, quantum Turing machines (QTM) replace universal Turing machines, qubit streams replace bit streams, and quantum programs replace classical programs or algorithms.

Definition (Quantum Algorithmic Complexity). Let M be a QTM, $\rho \in \mathcal{T}_1^+(\mathcal{H}_{\{0,1\}^*})$, a finite length qubit string. Define the finite-accuracy quantum complexity written as $QC_M^\delta(\rho)$, $l(\sigma)$ the length of a quantum program defined on the Hilbert space of quantum states $\sigma \in \mathcal{T}_1^+(\mathcal{H}_{\{0,1\}^*})$ and $M(\sigma)$ the output of the quantum program σ, then:

$$QC_M^\delta(\rho) \equiv \min\{l(\sigma) : \|\rho - M(\sigma)\|_{tr} \le \delta\} \qquad (3.116)$$

where $\|\cdot\|_{tr} = \frac{1}{2}tr(\cdot) = \frac{1}{2}\sum_i |\lambda_i|$, and λ_i are the eigenvalues of the operand, i.e., is the norm metric defined by the trace distance (Benatti, Kruger, Muller, Siegmund-Schiltze, & Szkola, 2006).

Statistical complexity attempts to define the minimal length program that stochastically describes a system. There are various flavors of a stochastic approach to describing a measure of length or time computation of a system. Entropic measures are usually used in these approaches. One approach is to describe the complexity of monotonically and successively increasing subsystems of a system through the use of its entropy (Shannon or others, such as the Renyí entropy) and examining the limit of such entropies towards the entropy of the entire system (Grassberger, 1989). They are mostly interpreted as measures of the average memory of the configuration of a system (Feldman & Crutchfield, 1998). Computational approaches to statistical complexity include the *statistical logical depth* which is the average time required for a universal Turing machine to compute or describe a system, assuming an Occam's razor assumption of the simplest possible computation in such estimation (Crutchfield & Young, 1989). An example of a modified statistical complexity is a measure proposed by L`opez-Ruiz, Mancini, and Calbet that takes into account disequilibrium, later modified by Feldman and Crutchfield to distinguish structure within a category of disorder (preventing over-universality), and presented here in the quantum version (proposed in the next chapter):

$$C_{LMC}(s) = S(\rho)D_{KL}(\rho \,||\, \rho_{1/N}) \tag{3.117}$$

where D_{KL} is the quantum *KL*-divergence (relative entropy), $\rho_{1/N}$ is the state density representation of a uniformly mixed state of *N* states, and *s* is a system of *N iid* quantum states (L`opez-Ruiz, Mancini, & Calbet, 1995; L`opez-Ruiz, 2002; Feldman & Crutchfield, 1998).

Recall that the quantum *KL*-divergence is given by: $D_{KL}(\rho \| \sigma) = tr[\rho(\log \rho - \log \sigma)]$. An approach based on computational ε-machines was given by Shalizi (2001) and Crutchfield and Young (1989). In this method, one considers a data stream with infinite past and future halves (bi-infinite), $\overline{s} = (.....s_{t-1},, s_t, s_{t+1},.....)$ taking into account a prior probability for future values, $p(s_{future})$ and a conditional probability for future values given past values, $p(s_{future} \mid s_{past})$ .Let $s_{past}(t)$ denote the past half infinite subsequence $(.....s_{t-1},, s_t)$ and $s_{future}(t)$ denote the future half infinite subsequence $(s_{t+1},.....)$. For a given time step *t*, an equivalence class $\mathcal{S}_t$ and relation $\underset{c}{\approx}$ are then formed based on the past history subsequence states ending at *t*: for a step time *u*, where $u \neq t$,

$$s_{past}(t) \underset{c}{\approx} s_{past}(u) \iff p(s_{future}(t) \mid s_{past}(t)) = p(s_{future}(u) \mid s_{past}(u)), \ \forall s_{future} \qquad (3.118)$$

$(\mathcal{S}_t, \underset{c}{\approx})$ form an equivalence class of past history subsequences and are referred to as *causal states*. For fixed future horizon *t*, one may consider finite length members of $(\mathcal{S}_t, \underset{c}{\approx})$ of differing length l and denote a class partition, $(\mathcal{S}_t^L, \underset{c}{\approx})$ as those members of $(\mathcal{S}_t, \underset{c}{\approx})$ with length $l \leq L$. Denote by $\mathcal{S}$ the set of all causal state classes $(\mathcal{S}_t, \underset{c}{\approx})$. These notions can be expressed for general stochastic processes as well. Now for each causal state $(\mathcal{S}_i, \underset{c}{\approx})$, attach the conditional probability of the futures based on the past histories of that causal state, i.e., $p(s_{future} \mid s_{past})$ where $s_{past} \in (\mathcal{S}_i, \underset{c}{\approx})$. These distributions are unique to the causal state and are referred to as their *morphs*. Next, examine the transitions from one causal state to another. A new causal state can be created from an old one by simply observing the next random variable s_{t+1} and building a new past history equivalence.

Definition. (Causal Transitions). The transition probability of proceeding from the causal state $\mathcal{S}_i$ to the causal state $\mathcal{S}_j$ while emitting or predicting the value s is given by:

$$T_{ij}^{(s)} \equiv p(\mathcal{S}' = \mathcal{S}_j, \overset{\to 1}{\mathcal{S}} = s \mid \mathcal{S} = \mathcal{S}_i) \tag{3.119}$$

where $\mathcal{S}'$ is defined as the successor causal state to the current causal state $\mathcal{S}$ and $\overset{\to 1}{\mathcal{S}}$ is the one-step future causal state (Shalizi, 2001). From this, an ε-machine is defined:

Definition (ε-machine). The pair $M_\varepsilon(s) = (\underset{c}{\approx}, T(s))$, where $T(s) = \{T_{ij}^{(s)} : s \in \mathcal{A}\}$ and $\mathcal{A}$ is the set of all possible values taken on by the stochastic process $s = (s_i)_{i \in I}$ is called an ε-machine for the stochastic (general) process s.

Let μ be the underlying measure for the quantum stochastic processes involved. It was shown that (1) ε-machine are Markovian, (2) causal states are minimal sufficient statistics for predicting futures of stochastic processes, (3) the statistical complexity, $C_\mu = S(\mathcal{S}) = -\sum_{\mathcal{S}_i \in \mathcal{S}} p(\mathcal{S}_i) \log p(\mathcal{S}_i)$ as defined by the entropy of the probability distribution of the stochastic process is equal to the statistical complexity of the stochastic process,

$O(s)$ and (4) causal states are maximally accurate predictors of a minimal statistical complexity of the underlying stochastic process, (5) through the use of ε-machines, its transition probabilities construct an invariant probability, $p(\mathcal{S})$ on the space of causal sets, $\mathcal{S}$, and (6)

$$I(\underline{\rho}, \overline{\rho}) \leq C_{\mu} = -\sum_{\mathcal{S}_i \in \mathcal{S}} p(\mathcal{S}_i) \log p(\mathcal{S}_i) \tag{3.120}$$

The quantum version of an interesting measure of the rate of growth or decline of entropies of quantum processes is given by:

$$s_{\mu} = \lim_{L \to \infty} s_{\mu}(L) = \lim_{L \to \infty} [S(L) - S(L-1)] = \lim_{L \to \infty} \frac{S(L)}{L} \tag{3.121}$$

where $S(L)$ is the block quantum entropy of length L subsequences within a data stream measured in qubits and is given by: $S(L) = -\sum_{\rho_L \in \chi^L} tr \rho_L \log \rho_L$, where χ^L is the space of length L quantum processes. s_{μ} is called the quantum entropy rate for a quantum process sequence s. An interpretation of s_{μ} is that it estimates the information-carrying capacity in $S(L)$ that is not random, but is actually correlated locally and hence measures local predictability going from one sequence length L to another. The quantities, $\left| s_{\mu}(L) - s_{\mu} \right|$ then measure a version of the irreducible amount of randomness left after correlating in the L length sequence. Now define the excess entropy as the sum of these quantities over all lengths L:

$$E(\rho) = \sum_{L=1}^{\infty} \left| S(\rho(L)) - s_{\mu}(\rho) \right| \tag{3.122}$$

$E(\rho)$ may then represent the amount of irreducible randomness left after accounting for all correlations. The excess entropy is a measure that operates on bi-infinite sequences as above.

In this paper, the quantum version of the excess entropy is proposed. The relationship between the mutual information between the bi-infinite past and future subsequences, the excess entropy and the quantum entropies is:

$$E(\rho) = I(\underline{\rho}, \overline{\rho}) = S(\overline{\rho}) - S(\overline{\rho} \mid \underline{\rho}) \tag{3.123}$$

This is in direct analogy to the classical version of excess entropy (Crutchfield & Feldman, 2003; Crutchfield & Wiesner, 2007). In terms of quantum positive operator valued measures (POVMs) on the future quantum processes given the past and the future prior, labeled as $M_{\omega_{future}|\omega_{past}}$ and $M_{\omega_{future}}$ respectively, and qubit stream lengths, *L*, the predictive information measure is given by:

$$I_{pred}(\underline{L}, \overline{L}) = \left\langle \log \frac{tr(\rho M_{\omega_{future}|\omega_{past}})}{tr(\rho M_{\omega_{future}})} \right\rangle \tag{3.124}$$

where the average is taken over the joint quantum distribution of the past and future values, $tr(\rho M_{\omega_{future}|\omega_{past}})$, $\underline{L}$ is the length of the observed past state values and $\overline{L}$ is the length of the observed future state values. Note that $I_{pred}(\underline{L}, \overline{L}) = I(\underline{\rho}, \overline{\rho}) \geq 0$ and that $I(\underline{\rho}, \overline{\rho})$ is sublinear.

A further generalization of causal states involving spacetime considerations, that is, using causal light cones instead of general processes was given by Shalizi, Shalizi & Haslinger (2004). In this setup one considers fields, $x(\vec{z}, t)$, defined on spacetime coordinate space, an $(n+1)$-D manifold, Λ (here a manifold is used to bring in diffeo-geometric considerations later), with coordinates, $(\vec{z},t)\in\Lambda$. Define the *past light cone* of $x(\vec{z},t)$, $L^{-}[x(\vec{z},t)]$, as the totality of all points $x(\vec{z}_1,u)$ that could, within light-speed propagation, c, influence $x(\vec{z},t)$. More specifically,

$$L^{-}[x(\vec{z},t)]=\left\{(\vec{z}_1,u):u<t,D_\Lambda(\vec{z}_1,\vec{z})\le c(t-u\right\} \tag{3.125}$$

where $D_\Lambda()$ is a divergence defined on Λ. One defines the *future light cone,* $L^{+}[x(\vec{z},t)]$ in an analogous manner:

$$L^{+}[x(\vec{z},t)]=\left\{(\vec{z}_1,u):u>t,D_\Lambda(\vec{z}_1,\vec{z})\le c(u-t\right\} \tag{3.126}$$

$L^{+}[x(\vec{z},t)]$ represents those points of Λ that could be influenced within light-speed propagation, c, by $x(\vec{z},t)$. Define $p(L^{+}\mid L^{-})$ to be the conditional distribution of future light cone configurations given the configuration in the past. Now define an equivalence class of past light cones:

$$\varepsilon(L^{-})\equiv\left[L^{-}\right]=\left\{l:p(L^{+}\mid l)=p(L^{+}\mid L^{-})\right\} \tag{3.127}$$

Consider the mutual information: $I(\varepsilon(L^{-});L^{-})$. Finally, define the statistical complexity of the field configuration as:

$$C_s=I(\varepsilon(L^{-});L^{-}) \tag{3.128}$$

It was shown that the statistic ε is the unique minimal sufficient statistic for estimating the minimal amount of information needed to predict the dynamics of a system given the past of that system (Shalizi, Shalizi & Haslinger, 2004). Note that C_s takes values in the order spectrum of systems.

However, for systems that are (i) completely disordered, i.e., independent processes or (ii) completely ordered, i.e., are deterministically constant, $C_s = 0$. In general, $C_s \to \infty$ for a system as the amount of information required to describe its behavior increases. In this study, a general quantum-relativistic model via spinfoams will be applied to the above generated causal state mechanism and subsequent ε-machine.

Self-organization

Self-organization is the next trademark of complex systems. What does it mean for a system to self-organize or to exhibit self-organization from an information-theoretic complexity point of view? Self-organization in open thermodynamic systems equates to the controlled, efficient release of energy in building internal structure (Kaufmann, 2000). This definition links living with nonliving self-organizing systems. The consensus among complex system theorists and the proposal that this study presents is that self-organization is the mixture of (i) the absence or relative low levels of external influences in building system complexity— the main premise of autonomous systems, (ii) an increase in order—a measure of increase in structural complexity, (iii) system robustness and adaptability—resilience, and (iv) interaction—connectivity and relational linkage (Correia, 2006). Prokopenko, et al. use predictive information, I_{pred} and statistical complexity to set conditions for increasing order by first defining information and complexity incremental differences respectively:

$$\Delta I(t_2, t_1, T_1, T_2) = I_{pred}([t_2 - T, t_2],[t_2, t_2 + T']) - I_{pred}([t_1 - T, t_1],[t_1 - T, t_1 + T']) \qquad (3.129)$$

$$\Delta C_{\mu}^{sys}(t_2, t_1) = C_{\mu}^{sys}(t_2) - C_{\mu}^{sys}(t_1) \qquad (3.130)$$

where $t_2 > t_1$ and $T, T^{'} > 0$ are two times and time interval lengths respectively and $C_{\mu}^{sys}(t)$ is the statistical complexity of the system at time *t*. Then one has the conditions,

$$\Delta C_{\mu}^{sys}(t_2, t_1) > 0 \tag{3.131}$$

$$\Delta I\left(t_2, t_1, T_1, T_2\right) > 0 \tag{3.132}$$

for increasing order. Now let $I^{ext}(t_2, t_1, T_2, T_1)$ depict the amount of exogenous information that can influence (change the complexity of) the system at those two times and within those two time increments. Then define an autonomous system as one that satisfies the condition:

$$I^{ext}(t_2, t_1, T_2, T_1) < \Delta I(t_2, t_1, T_1, T_2) \tag{3.133}$$

for all quadruplets (t_2, t_1, T_1, T_2). This may be too strong of a condition for autonomy. One may then relax it by prefacing that for autonomous systems condition (3.133) eventually be satisfied for all future time pairs (t_2, t_1) past some finite time, t^*. Now let $C_{\mu}^{ext}(t_2, t_1)$ depict the amount of statistical complexity introduced exogenously between the times t_2 and t_1. Then, for autonomous systems:

$$C_{\mu}^{ext}(t_2, t_1) < \Delta C_{\mu}^{sys}(t_2, t_1) \tag{3.134}$$

This inequality may be interpreted as the condition that the internal complexity of the system at time t_2 increased by more than what could have been added to the system exogenously before that and after time t_1—so called *spontaneous information dynamics* produced by self-organization.

Robustness

Robustness is a condition of re-stabilization under perturbative actions, that is, an invariance to system perturbations (Adami, 2005, p. 1). This invariance to system perturbations, either exogenous or indigenous, is within the framework of the original system's general behavior and morphology. Adami considered biological systems in his definition of robustness, concentrating on the fitness, survivability, and reproducibility of systems as the key entities of robustness under the perturbative actions of genetic operations. For general systems, one is inclined to consider any information structure or substructure of the system for robustness with perturbations arriving out of the interaction with other systems or of the universe outside of the system. Prokopenko, et al. attempted to quantify this condition by proposing that robustness implies that a system weaves in and out of stages in which information transfer within channels increases, that is, when $\Delta I^{sys}() > 0$ where dominant patterns are exploited and assortative noise is relatively low and those stages where $\Delta I^{sys}() < 0$, i.e., when alternative patterns are explored and assortative noise is relatively high (Prokopenko, et. al, 2009). In this respect, robustness is described by the dynamical nature of the excess entropy $E(\rho)$ in a general quantum-relativistic system. This aspect will be investigated in the next chapter as pertains to spinfoam quantum-gravity models.

A general stability condition is much stronger than robustness because it implies the existence of a limiting state probability distribution as $t \to \infty$. In this case, a proposal for stabilization was made by Briscoe and DeWilde in the situation involving a system of

Markovian multiagents, each with state probabilities of p_X^t and the degree of instability defined as:

$$d_{ins} = H(p^{\infty}) = -\sum_X p_x^{\infty} \log p_x^{\infty} \tag{3.135}$$

where $p_x^{\infty} = \lim_{t\to\infty} p_X^t$ is the limit distribution of the occupation probability sequence, $p_x^t = \sum_Y p(X \mid Y) p_Y^{t-1}$, X is the space of values for agent states and Y is the space of values for the system state, as a whole (Briscoe & De Wilde, 2009). The multiagent system is considered to be *self-stabilizing* if p_x^{∞} exists and is nonuniform, i.e., $\exists$states, x_1 and x_2 $\ni p_{x_1}^{\infty} \neq p_{x_2}^{\infty}$. This model takes into account the environmental pressure of the system on each agent, in essence, the effect of the whole system on each component. The quantity d_{ins} is simply the entropy of the limiting agent state probability. If one considers more general systems, such as stochastic quantum systems, then the quantum entropy growth rate, s_{μ}, that was described before, would define an analogous degree of instability. If a quantum system consists of multiagent quantum subsystems, then the definitions of the transition probabilities could account for the effects from the total quantum system state, redefining the entropies involved and hence the mutual information transfers and excess entropies. Robustness would define a sort of cyclic or wobbly (semi-) stability of a system where the fitness of that system would be stable via the quantum entropy growth rate, however, in route towards a limit, the general structure may change via the intermediate changing of the excess entropy measure of information.

Interaction

Interaction within a system was defined by Correia as the minimization of local conflicts producing global optimal self-organization, yielding evolutionary stability (Correia, 2006). Minimization of local conflicts is reflected by a measure of nonassortativeness, that is, by minimizing the noise in the information channels between agents of the system. Again, this equates to the minimization of the joint equivocations between agents in the system written as the conditional entropies:

$$S\left(\rho_i \mid \rho_{j_1} \ldots \rho_{j_{n-1}}\right),\ j_k \neq i,\ k = 1,\ldots,n-1 \tag{3.136}$$

Simultaneous to this minimization is the maximization of the diversity of the system represented by $S(\rho_1,,,\rho_n)$, the joint entropies. This results in an attempt, once again, at maximizing the excess entropy and the subsequently defined system predictive information $I(\underline{\rho},\overline{\rho})$.

Emergence, Self-asssembly, and Evolution

Emergence in systems is the capability of those systems to exhibit novel behavior at various scales of observation or resolution, such as spatio-temporal, measurement precision or structure type and size, which may not be directly explained from a hierarchical causality emanating from those scales or levels. In the extreme, *strong emergence* means that the whole is greater than the sum of its parts. Hence, strong emergence is the proposal that emergence cannot be explained by any examination of lower levels from the level where emergence is manifested. Weak emergence is then the phenomenon of studying microbehaviors and understanding their collective manifestation as a starting point to the the higher level emergence. In organizations (organisms), this behavior may arise from a coalition of networked tasks. Macroscopic emergence is usually not defined directly by microscopic activity and vice-versa. Normal human observation takes place at mesoscopic levels, that is, at levels compatible with biological filtering and spectra, both below cosmological scales and above quantum fluctuations.

Anthropomorphically, emergence at mesoscopic scales equates with sensorial manifestations such as consciousness, stimuli from human senses, and social interaction. This is generalizable to any scale or level at which observers or SASs live. Quantum mechanically, this means that a POVM operator is applied to the state space that exist and is appropriately defined for that level. The properties of complexity and organization of a system are considered to be subjective by leading systems and complexity theorists (Crutchfield, 1994). Hence emergence, which will be proposed to emanate from complexity and organization, is subjective as well, according to this doctrine. Notwithstanding, this, there are those that argue for the objective nature of emergence-emergence happens without an observer (Corning, 2002). Moreover, the concepts of strong and weak emergence are not the issue. The whole is not only more than the sum of its parts—it is distinctly different from it. Emergence may be understood better by the interactions and structure of components, but also by the interactions with the system's environment at the scale of emergence. Why should emergence be relegated to a limited group of scales? In other words, an emergent at any particular scale may be defined by a plausibly infinite number of scales at work – infinite interactions at all scales propagating to form the "life" of a particular emergent, at a particular scale.

Emergence is essentially a special case of synergism – the ability for two or more components to collaborate to form or display a new type of behavior or ability not possible by individual means within their scale. Nevertheless, these manifestations are contributed to from the emergent properties of quantum phenomena (microscopic effects), as upward feedback, and cosmological perturbations (mega-macroscopic effects), as downward feedback. The rules or laws for an integration of these levels is what is missing. Quantum gravity (GR) is an attempt to model both predominant theories of physical presence. However, no mesoscopic bridge has been formulated that joins these rules together at the boundaries of scales. In chapter 4 a version of an analytic attempt to do this, utilizing information-entropic rules will be given.

That withstanding, what is the form of these feedback mechanisms? The course-grained answer to this is information transmission. To this end, one may ask, How may one use information-theoretic notions to investigate emergence? The second question may be, since emergence is relative to the observer, can a mesoscopic view be developed for emergence? This second question begs a follow-up question, what is the mesoscopic mechanism that will simultaneously make ostensible macro and microemergence at the mesoscopic level?

To answer this, Prokopenko et al. used the idea of two levels or categories of emergence from Crutchfield (1994). Crutchfield proposed two levels of expressions of emergence: pattern formation and intrinsic emergence. Additionally, Crutchfield defines how emergence could be recognized as a conceived and ostensible novel behavior. Pattern formation or emergence of patterns is the process of viewing a system externally by an observer of a pronounced developing novel pattern of behavior or structure. Intrinsic emergence is the detection of an emergence of system wide behavior by internal agents through coordinated behavior of that system from the acknowledgement of a system-wide or global measurement functional and by utilizing local connection mechanisms to actuate reactions to that system global measurement. Pattern formation is highly observer dependent. The mechanics of the contributing microscopic components do not causally link to the novel emergence as a new categorization of the system. How are macropatterns gleaned upon by the observer in their respective scale environment? The observer possesses a filter spectrum applied to signals emanating from the observed system. If information about the system's structure is contained in the noise component of the filter model, then no such structure will be discovered.

Emergence arises at different scales as discussed above. However, because pattern formation may be observer dependent or intrinsically defined by whole system effects on local causality, the level in which to observe emergence becomes important. Practically speaking, emergent patterns may be masked by observer bias. Hence, it may be advantageous to define more objective measures that endeavor to predict emergent behavior. This was the approach taken by Shalizi (2001) and promoted by Prokopenko, et al. Using a computational mechanics approach as in defining ε-machines, Shalizi defined the efficiency of prediction of a system. Here we extend that to define a quantum-stochastic version:

$$e_{pred} = \frac{E_\mu}{C_\mu} \tag{3.137}$$

where E_μ is the excess entropy of a quantum system and C_μ is its stochastic complexity. Prokopenko, et al. described this as a ratio of a measure of how much of the system can be predicted divided by a measure of how difficult it can be to predict. System predictability is then increased in three separate scenarios: (i) E_μ is larger (more predictive power), (ii) C_μ is smaller (easier to predict), or (iii) E_μ is smaller by an appreciably lesser amount than C_μ is larger. Levels of an organization such as a CMAS or what we will investigate later, a HMAS, can be seen as a scale at which functional synchronization and homogeneity takes place. This is a caricature characterization of levels of multiagent systems, but it will suffice for this purpose. The level at which e_{pred} is optimal with respect to all other levels of organization is the level at which to attempt to build a predictive model for emergence. More precisely, if L is the number of levels of an organization, O, E_μ^Q and C_μ^Q are the efficiency and statistical complexity at level l respectively, and the predictive efficiency at level l is given by:

$$e_{pred}^Q = \frac{E_\mu^Q}{C_\mu^Q} \tag{3.138}$$

then the optimal level, l^* in which to model predicting emergence in O satisfies: $\max_{l=1,\ldots L} e_{pred}^Q = e_{pred}^{O_{l^*}}$. A problem with this approach is that the levels of a system may also be observer or model dependent and so, the system structural model may also mask inter-level causality or hidden scale levels. Emergence may happen between and within defined levels of organization. Emergence is then a relative phenomenon. All emergent behavior may not be visible by this cadre of observer filters. Patterns of emergence are then gestalt experiments.

An approach to patterning which leads to a definition of emergence using psychological filters will be considered (Goertzel, 1994). We begin with a formalization and modification of Goertzel's nonmathematical definition of patterns. A quantitative measure of structure of a component and system must be attempted in order to define a measure of emergence. Measures for structure of a component will be given by the set of all patterns of that entity. A pattern, in this respect, is simply a representation of the entity whose complexity measure is at most, that of the component itself.

Definition. (Pattern) A *pattern*, $f_{PAT}(x)$ of an entity, x is a process (map) that results in x, that is, that computes x entirely, such that if C is a complexity measure then

$$C(f_{PAT}(x)) < C(x) \tag{3.139}$$

Goertzel's original definition took a computational complexity view. In it a pattern, $f_{PAT}(x)$, of the entity x, is a self-delimiting program which computes x on a universal Turing Machine (UTM), U from an input sequence, $(...000z000...)$ such that if l is a length metric on sequences of symbols, then

$$l\left[f_{PAT}(x)\right] + l\left[z\right] \leq l\left[x\right] \tag{3.140}$$

In this paper, a generalization to program length, the complexity measure, is utilized in our definition of patterns. The statistical complexity, C_μ is used. A further generalization would involve a quantum process in which a pattern would be an operator, F_C acting on a quantum state space, $\mathcal{H}$ such that $C_\mu\left(F_C(\rho)\right) \leq C_\mu\left(\rho\right)$ for a density operator ρ. In a discrete setting for this quantum statistical complexity, the pattern computes on a Bernoulli-Turing Machine (BTM) and guesses each consecutive qubit. Recall that a BTM is a UTM with a random register that accommodates simulating a Bernoulli time trial (random coin flips), B_t and hence a random process with respect to a computational process. The qubit's superposition state can then be computed with this random simulator. We will discuss a slightly more general version of this in the form of qubit computation on an LQG-computer simulated on a fuzzy sphere in chapter 4 and mention GR-inspired computers on black holes in Appendix B.

Definition. The *relative complexity* of a pattern, $f_{PAT}(x)$ of *x* relative to *x* is given by:

$$I(f_{PAT}(x) \mid x) = D(f_{PAT}(x), x) \qquad \underline{(3.141)}$$

where *D* is a divergence measure in the space of entities of a system.

Definition. The *intensity* of a pattern, $f_{PAT}(x)$ in *x* is given by:

$$It(f_{PAT}(x) \mid x) = 1 - \left| \frac{f_{PAT}(x)}{x} \right| \qquad \underline{(3.142)}$$

Definition. The *structure of an entity* x, denoted by $St(x)$ is the set of all patterns of *x*,

$$St(x) = \{ f_{PAT}(x) \} \qquad \underline{(3.143)}$$

Definition. The *structural complexity* of *x*, acts on the space $St(x)$ and is defined as:

$$C_S(St(x)) = \frac{\sum_{j=1}^{m}\left(\left|\bigcup_{k\in P_j} p_k^j\right| - \sum_{n\in P_j}\left|\bigcap_{k=1}^{n} p_k^j\right|\right)}{m!} \qquad \underline{(3.144)}$$

where m is the number of patterns of x, i.e., $m = Card(St(x))$, p_k^j is a distinct pattern in an ordering P_j of patterns in $St(x)$ and P_j is the jth ordering of $\{1, 2, ..., m\}$ in the number of distinct orderings of sets in $St(x)$.

This definition of structural complexity is a mathematical formulation of casuality (Goerztel, 1997, p.19-20). Structural complexity of x preserves additivity because each pattern contributes nonredundantly within $St(x)$ and represents an "average" composite pattern complexity. An associated definition of emergence between two entities follows.

Definition. The *emergence* of two entities, x and y, denoted by $Em(x, y)$ is the set:

$$Em(x, y) = St(xy) - St(x) - St(y) \qquad \underline{(3.145)}$$

where $St(xy)$ is the structure for the juxtaposition of x and y (Goertzel, 1997, p. 20).

Goertzel described this definition of emergence as a *gestalt* of x and y in the following sense: $Em(x, y)$ is the *gestalt* of x and y consisting of all patterns in the whole of x and y minus the patterns of x and y individually. Interestingly a gestalt may be established between a contained observer and a subsystem component. By this definition of an emergence set for two components of a system, if $Em(x, y) = \varnothing$ then the juxtaposition or merging of x and y produce no new holistic patterns separate from those found individually in x and y. Emergence as a set operator then involves a new kind of set logic for the usual disjunction (union) and conjunction (intersection) connectives that Boolean logic is not sufficient to build with. These inherent structures of x and y and the pattern definitions dictate that set operator. This emergent-based logic is an example of a para-consistent and multivalued logic that is shared with quantum logics.

Other definitions of emergent systems have involved computational criteria. One such definition labels a *computationally, thermodynamically,* or *relative-to-a-model emergent* phenomena as one in which the optimal means of prediction is through simulation. The condition used is $u(n) \geq s(n)$ where $u(n)$ is the amount of computation (information complexity) needed to calculate predictions for a system of size n when one has perfect knowledge of some part of the system and $s(n)$ is the amount of computation needed to simulate the system and arrive at a prediction of the emergent phenomena with totally unknown structural information. A phase change in the phase transition diagram of the system happens when these values coincide. An *emergence ratio*, $\xi = \frac{u_{opt}(n)}{s(n)}$ is given where $u_{opt}(n)$ is the optimal (minimum) amount of computation needed to predict future states of the emergent phenomena of the system, i.e., $u_{opt}(n) \leq u(n) \; \forall u(n)$. The quantity, $\zeta = -\frac{\log \xi}{\log n}$ is referred to as the *emergence coefficient*. Finally, a measure of *emergent understanding* is given by the *relative understanding*, $\lambda = \frac{u_{opt}(n) - u(n)}{s(n)}$. For an emergent system, $\lambda \leq 0$ and the best understanding is achieved when $\lambda = 0$ (Darley, 1994). The set of possible $u(n)$ are given by the set of possible understandings of the system. These measures give a way of categorizing emergent systems through given "knowledge scenarios" of the structure.

Other categorizations of emergent systems for simpler structural families have been given, most notably by Wolfram using the mechanics of cellular automata. Wolfram (1983) classified emergent cellular automata into the types: (a) homogeneous, (b) periodic, (c) chaotic, and (d) complex. This classification is subsumed by the more general taxonomy of attractors as precursors to emergence: (a) fixed point, (b) simple limit cycles, (c) quasi-periodic cycles, and (d) strange attractors. However, this categorization of emergence is defined on a continuum.

Natural specializations of cellular automata are classes of mass agent system behavior such as insect (locust, bee, and ant), bird, animal, or human crowd group optimization. In these scenarios, groups of biological species exhibit emergent behavior based on a finite set of local rules and a sometimes limited amount of self-awareness. These individual behaviors can be represented by a set of local rules as in the more general setup for cellular automata. Three such universal rules of engagement for agents in biological group optimization which include ant optimization have been posited by Reynolds (1987) in his famous Boid agent model. They are (a) threshold attraction or avoidance to neighbors (no bumping into neighboring agernts), (b) alignment with or movement of direction with neighbor subgroups (go in the average direction of the local agent neighborhood, and (c) attraction to or movement to a local averaged position (go to a local neighborhood average position. In addition, as in the case for ant colonies, a sense of self-awareness is also present as in the feedback mechanism of pheromone detection from other ant agents.

These rules have been modified to take into account the scale of the mass of agents, that is, the threshold of group awareness of neighboring agents as influenced by the density of the agent system (Parrish & Viscido, 2005). The role of information transmission between biological agent systems can be encapsulated by this set of algorithms and feedback (both positive and negative) mechanisms. Emergent behavior is manifested through events such as cascading and group patternization. Novel levels of organization are created. In this way, new forms of information are also created because the pattern created is interpretable only through a novel vocabulary that is necessary on the new levels.

A generalization of these multiagent systems collectively displaying emergent behavior and computational prowess is particle swarm optimization (PSO). Adaptability has also been introduced into a PSO model (Zhan, Zhang, Li, & Chung, 2009). Since adaptive particle swarm optimization (APSO) exhibits the most general model of swam intelligence, we review this model in terms of information transfer. In a classical version of APSO as depicted in (Zhan, et al., 2009), with each particle in a swarm, are associated velocity and position vectors respectively, $\overline{v}_i = \left[v_i^1, \ldots, v_i^D \right]$ and $\overline{x}_i = \left[x_i^1, \ldots, x_i^D \right]$ for *D*-dimensional spaces. We correct here for a unit discrepancy and time step absence in the ESE model of Zhan, et al. Let *N* be the number of particles. An evolutional state estimation (ESE) model is introduced:

$$\begin{aligned} v_i^k(t_{j+1}) &= \omega v_i^k(t_j) + c_1 U_1^k({}_b x_i^k(t_j) - x_i^k(t_j)) + c_2 U_2^k({}_n x_i^k(t_j) - x_i^k(t_j)) \\ x_i^k(t_{j+1}) &= x_i^k(t_j) + v_i^k(t_{j+1}) \Delta t_{j+1} \end{aligned} \qquad \underline{(3.146)}$$

where ω is an inertia weight, $c_i, i = 1,2$ are acceleration coefficients, $\Delta t_{j+1} = t_{j+1} - t_j$ is the *(j+1)*th time interval, $U_i^k \sim U([0,1]),\ i = 1,2,\ k = 1,...,D$ uniformly distributed random numbers, ${}_b x_i^k(t_j)$ is the position with the best fitness found up to the current evolution and ${}_n x_i^k(t_j)$ is the best position in the neighborhood for the i^{th} particle's k^{th} coordinate at time step t_j. Best in this sense may be a global or local optimum. Genetic and fuzzy version of the inertia weight ω have been proposed in order to tune the stability and search capabilities of this model. However, an adaptive version of ω will be reviewed. To this end, define the distance measures between particles that are instrumental in displaying the three rules of Reynolds. Define the mean distance of all particles from the l^{th} particle at the time step t_j by:

$$d_l(t_j) = \frac{1}{N-1} \sum_{\substack{j=1 \\ j \neq l}}^{N} \sqrt{\sum_{k=1}^{D} \left(x_l^k(t_j) - x_j^k(t_j) \right)^2} \tag{3.147}$$

Denote by $d_g(t_j)$ as the globally best particle *w.r.t.* the measure (3.147). Let $d_{\max}(t_j) = \max_{i=1,...,N} d_i(t_j)$ and $d_{\min}(t_j) = \min_{i=1,...,N} d_i(t_j)$. Now define the evolutionary factor, f by:

$$f(t_j) = \frac{d_g(t_j) - d_{\min}(t_j)}{d_{\max}(t_j) - d_{\min}(t_j)} \tag{3.148}$$

In this definition, $f(t) \in [0,1]$. Finally, define an adaptive sigmoidal version of ω as:

$$\omega_{f(t)} = \frac{1}{1 + \alpha e^{-\beta f(t)}} \tag{3.149}$$

Zhan, et al. experimentally set $\alpha = 1.5$ and $\beta = 2.6$. In this setup, $\omega_{f(t)} \in [0,4,0.9]$ and is initially set to the maximum, $\omega = 0.9$ in the evolutional process. The accelerator coefficients, c_i, $i = 1,2$ represent the "self-cognition" and "social influence" respectively of the particles. Self-cognition means the propensity of a particle to go to its own historically best position, therefore aiding in finding local niches and global diversity of the swarm. Social influence means the ability of a swarm to move towards the current globally best region and acceleration to convergence to a best solution of the task of the swarm. In the model of Zhan, et al. the acceleration coefficients are utilized in order to enter into convergence, exploitation, exploration or jumping out (of local optima) regimes before applying the inertia weights, ω_f to an elitist particle-the particle with the globally best particle. In summary, in an APSO model, information is transmitted via local and global particle rules, including self-cognition and social influence and the tuning from the acceleration coefficients which may be viewed as exogenous influences, e.g, from an external observer to the swarm organization.

Here we introduce further adaptation by varying the acceleration coefficients based on observer and evolutional states. Then we have the acceleration coefficients given by $c_i\,(f(t))$, $i = 1,2$ which are dependent on the evolution factor, *f*. Moreover, we introduce a quantum version of the APSO model (3.146):

$$
\begin{aligned}
U_j(t_{j+1})\left|\rho_i(t_j)\right\rangle = \omega_{f(t_j)}\left|\rho_i(t_j)\right\rangle + \\
c_1^{f(t_j)}U_1^k\left(D\left(\rho_b(t_j)\,\|\,\rho_i(t_j)\right)\right) + \\
c_2^{f(t)}U_2^k\left(D\left(\rho_i^n(t_j)\,\|\,\rho_i(t_j)\right)\right)
\end{aligned} \tag{3.150}
$$

$$
U_n(t_{n+1}) = \bigotimes_{k=0}^{n} U_k(t_{k+1})
$$

$$
\left|\rho_i(t_{j+1})\right\rangle = U_j(t_{j+1})\left|\rho_i(t_0)\right\rangle
$$

Here, at time step t_j, U_j is an evolutional operator defined by the first expression, D is a diveregence measure between quantum states, $\rho_i(t_j)$ is the quantum state for the i^{th} particle, $\rho_b(t_j)$ is the quantum state for the best fitness particle, and $\rho_i^n(t_j)$ is the quantum state of the particle with the best fitness in the neighborhood of the i^{th} particle. This quantum setup deserves closer scrutiny because of the implied nonlocality of entanglement of particles. A neighborhood in this sense is redefined to mean an entanglement neighborhood. Proximate particles are more likely to produce entanglement than distance ones during the early steps of evolution. However, as time passes, these entanglement neighborhoods expand. The quantum evolutional operators, U_j may be further generalized to include GTU-operators as defined by Zadeh (2005) GTU constraints. This sets up general uncertainty operators which include fuzzy logic operators utilizing Lukasewicz operations. These will be discussed further in our treatment of GTU operators for the info-macrodynamic model in chapter 4. A generalization to this setup is the set of evoloutional operators defined in the spirit of true physical evolutional processes in biology. This is developed in chapter 4 as well.

In addition to the capability of verifying and categorizing emergent systems, the descriptive power of metapatterns of emergent systems is all important. Several useful metacategorizations of emergent systems, notably, poietic systems have been proposed in various fields. Poiesis is the process of manufacturing and producing components, their relations, and their organization within a system. Course limit delineation has been posited by Dempster as follows. Autopoietic systems generate their organizational structures in a strictly autonomous manner (Dempster, 2000). They may nonetheless receive input from outside the boundaries of their system, but these inputs do not directly affect their internal organizational apparatus. Sympoietic systems, in contrast, are open to outside influences in forming these organizational possibilities. These are essentially subjective descriptions because the mechanics of changing boundaries and patterns of organization of a system from outside influences can be blurry.

Now consider systems exhibiting self-organized criticality (SOC). These systems organize their patterns of behavior based on autonomous microstates of behavior and an ensuing macrodynamics based on power laws of distribution, fractality, and $\frac{1}{f}$ (pink) noise (Bak, Tang, & Wiesenfield, 1987). Self-organization of critical phase transitions, transpire in a phase space region existing between stable and chaotic regions while in long transient periods. Random external perturbations may cause avalanche phenomena and so they are both independent in organization, but dependent on external perturbative energy.

Power laws are probabilistic rules governing the occurrence of entities described by bidirectional exponential distributions, i.e., $P(w) = \frac{C_{\pm}}{|w|^{1+\mu_{\pm}}}$, $w \to \pm\infty$ (Sornette, 2004, p.116). Power laws exhibit both self-similarity and scale invariance. Pink, fractal or $\frac{1}{f}$ noise is exhibited by the behavior of its power spectral density following the proportional relation $S(f) \propto \frac{1}{f^{\alpha}}$ where $0 < \alpha < 2$, but usually $\alpha \approx 1$. These are directly related to fractional Brownian processes (Mandelbrot & Van Ness, 1968). SOC systems have internal thresholds that govern when the system "breaks out" into a critical phase transition or avalanche. Bak has posited that SOC is a natural maximization of complexity (Bak, 1999). Both poietic system types build internal components based on feedback mechanisms and as such are self-organized. Quantum versions of criticality and $\frac{1}{f}$ noise have been proposed in Pitkänen (2001).

Autopoietic systems are characterized by closed boundaries and limited lifecycles in evolution. Sympoietic systems develop open or more fuzzified boundaries and are more capable of sustained lifecycles. The mechanism of receiving input information from outside of a boundary is also differentiated, but somewhat vaguely described in the literature. Reproduction is also slightly different. In autopoietic systems, the prodigy are different systems than the parent, while in sympoietic systems, the prodigy may be the same or different systems from the parent.

Dempster pointed out a final delineation between these poiesis types with regards to self-organization. Autopoietic systems depend on a stable or predictable input from outside their boundaries while sympoietic systems depend on uncertainty and a continuous process of adaptation from outside their boundaries. Additionally, sympoietic systems are combinations of two types of self-organization, creative and transmitted, while autopoietic systems perform creative self-organization. Creative self-organization refers to a reorganization based on nonequilibirum conditions in dissipative systems, such as physical chemical systems (Prigogine, 1984). They depend on counteracting influences between components and when that tension is released, the emergence may also be. Transmitted self-organization refers to reorganization based on information-dependence from within and outside. Sympoietic systems confront uncertainty in their surrounding environment at their boundaries in fully adaptable manners, while autopoietic systems, while maintaining the ability to adapt, have a smaller space of possibilities in this regard because of the limited information exchange at the boundary.

A very broad, but related measure of complexity within a hierarchy is that of *decomposability*. Systems are *decomposable* if they can be divided into identifiable parts. They are *nearly decomposable* (ND) if their parts are not completely independent (Simon, 1969). Because there is a continuum of system independence, ND may also be measured on a spectrum. Alternate definitions of ND include the follow: a complex system is ND if it can be decomposed into subsystems that are delineated by different frequencies of connectivity. ND may be structural (physical connectivity) or functional (processes).

More specifically, *modularity* is the notion that inter-system level interactions are small in number or sparse in comparison to intra-system level interactions. Subsystems defined as such are the *modules* of a complex system. Modules are also characterized by their lower, but not eliminated dependence on outside systems. In this regard, modules may be a way of defining a level of a holarchy. Modularity depends on the quantification of independence. For example, subsystems may be structurally modular through the observation that they have stronger intra-module dependencies than inter-module ones while simultaneously being functionally nonmodular. Stability is oftentimes used as a benchmark property of dynamic systems. Stability means the relative or eventual nonchanging aspect of the state of the system. Dependency can be defined with respect to stability. In an Ising model for a network of particles, the update rule for a state of a particle is usually in the form of the average sum of the contributing or connecting particles' states. For example, if $\left|\psi_i(t)\right\rangle$ depicts the state of the i^{th} particle of a quantum state at time t, then $P(\left|\psi_i(t)\right\rangle = \rho) = \dfrac{\sum_{j \in N_i} \left|\psi_j(t)\right\rangle}{|N_i|}$, where N_{G_A} is the set of indices of all the inter-system connecting particles to the i^{th} particle. The stability at time t of a subsystem may then be defined as the probability that no subsystem particle changes state at time t.

In a *free-energy measure* of an Ising model, independence of probabilities for each particle's state change can be assumed. Hence, the subsystem, A has stability at t given by:

$$S_t(A) = \prod_{j \in N_A} P(\left|\psi_j(t)\right\rangle = \left|\psi_j(t-1)\right\rangle) \tag{3.151}$$

where N_A is the set of indices of particles belonging to A. Conditional stability can be calculated as well. For example, let $S_t(G_A \mid G_B)$ denote the stability at time t of the group G_A of "characteristic" A, given the state at time t of group G_B of characteristic B. Then

$$S_t(G_A \mid G_B) = \prod_{j \in N_{G_A}} P\left(\left| \psi_j(t) \right\rangle = \left| \psi_j(t-1) \right\rangle \mid \left| \psi_k(t) \right\rangle = \rho_k : k \in N_{G_B} \right) \quad (3.152)$$

where N_{G_A} and N_{G_B} are the set of indices of the particles that belong to the groups G_A and G_B respectively. Let MS_{G_A} be the number of maximally stable configurations of the group (subsystem) G_A. Let M_G be the total number of distinct configurations in that same group. Then G_A is said to be *decomposable* if $MS_{G_A} < M_{G_A}$. If $MS_{G_A} = 1, G_A$ is said to be *separable*. If $1 < MS_{G_A} < M_{G_A}$ then the group, G_A exhibits *modular interdependency* (Watson, 2003). The concept of modular interdependency then provides a gradation of modularity, that is, a relative measure of the degree of intra-dependency versus inter-dependency in modules of hierarchies that can be utilized in holarchies, as well. Watson (2002) developed the notion of *compositional evolvability* to describe the capability of a system that has strong inter-modularity to evolve, even with the heuristic that systems that are not decomposable are not evolvable because of lack of independence between modules. In compositionally evolvable systems, modules can adapt to each other so that intra-module evolution occurs based on recombining fitness benefits from other modules. Again, this sets a precedent to develop a spectrum for measuring evolvability of a system with respect to decomposability, inter-modularity and intra-modularity. This is a finer grain metric for evolvable complex systems.

Remodularization is the dual concept of (1) component type reduction (functional and structural) as higher order systems become built from lower order ones, and (2) as higher-level systems arise, lower-level components become more differentiated and so, more intermediate component types arise as well (McShea & Anderson, 2005). As higher-level systems arise, the lower-level components that aid in building such systems may become less in diversity because the higher-level component gains the functionality that those lower-level components have specialized in. Hence these lower-level component specialists become less vital in the functioning of the higher-level component. There are socio-cultural and cell-biological examples of such phenomena. This matter is less prevalent with functional remodularization. In cell biology, intermediate types to organ cells are skin and tissue cells. The second concept of remodularization then states that as higher-level components become more functional and larger in size, more diversity in cell types is produced within that higher-level component, in addition to the manifestation of more intermediate components, i.e., those between the higher-level component and the contributing lower-level ones. Both these changes in remodularization take place within the lower-level to higher-level regime, normally one level.

Systems which possess poiesis may be capable of *self-assembly*. Self assembly is the ability to built novel subsystems within a systems environment from multiple parts. More specifically, Definition. *Self assembly* is the process in which spontaneous formation of organized structures from many discrete components, in a possibly disordered system, through a stochastic process, is reversible, and can be controlled by a proper design of those components, the environment, and the driving force (Pelesko, 2007, p. 5). When local interactions dominant this process, sympoiesis reigns. Otherwise the system is mostly autopoietic.

There are at least three separate categories of self-assembly, (a) static – assemblies that result in structures that are in local or global equilibirium, (b) dynamic – assemblies that result in structures that have stable nonequilibiriums, and (c) programmable – assemblies that carry information about the destination structure or function of that assembly. Pelesko points to four characteristics of self-assemblying systems, followed by both organic and inorganic structures (a) structured particles (components) that take part in the assembled structure, (b) binding forces that act upon those particles in the process, (c) an appropriate or proper environment that nurtures the dynamics, and (d) a consistent driving force partaking in that dynamic.

Pelesko stops short of pronouncing that the minimization of energy is a consistent dynamic of self-assembling systems because some natural systems are dissipative systems. Nonetheless, energy dynamics play a dominant role in natural systems of self-assembly, witness (a) the minimization of surface tension in bubble formation, (b) helical formation and free energy of rod configurations in nucleic acids (DNA and RNA) and protein structures, (c) chemical kinetic energy configurations in polymer formation, and (d) the energy systems of waterbug models. These structures are mimicked by engineering models depicting different tile patterns such as Wang tiles and the use of universal graph grammars (Pelesko, pp. 211-251). In particular, the study will utilize information transfer across tiling systems in order to direct self-assembly. This is a novel approach to tiling and it's self-assembling prowess. As an introduction, a definition of some recent general tiling approaches will be given based on the original tiling system idea.

Definitions. A *Wang tile*, *t* is a unit square defined by the quadruple $(\sigma_n, \sigma_e, \sigma_s, \sigma_w)$,each subscript denoting the directional side, north, east, south, and west respectively of the square and the type of *glue* on that side. Each glue is taken from an alphabet, Σ of glue types. A *null glue* type, denoted by *null* will be included in Σ. Define the *glue complexity* by the cardinality $g = |\Sigma| - 1$. The functions, *north(t), east(t), south(t),* and *west(t)* will denote the glue types on those particular sides of tile *t*. A *tile system* is an ordered triple (T, G, τ) where T is a *tileset* (distinct tiles), $G : \Sigma^2 \to \{0,1,...,\tau\}$ is the *glue function*, and $\tau > 0$ is the temperature. G is associative and $G(null, x) = 0 \;\; \forall x \in \Sigma$. In the standard model, $G(x, y) = 0,$ if $x \neq y$ and $G(x, x) \in \{1, 2,..., \tau\}$. The *tile complexity* of the system is given by the cardinality $|T|$. A *configuration* is a partial function $C : \mathrm{Z}^2 \to T \bigcup \{empty\}$ where $\{empty\}$ is a special tile that has the *null* glue on each of its four edges. The *shape* of a configuration C is the set of positions (i, j) that do not map to the empty tile (Demaine, et al., 2007).

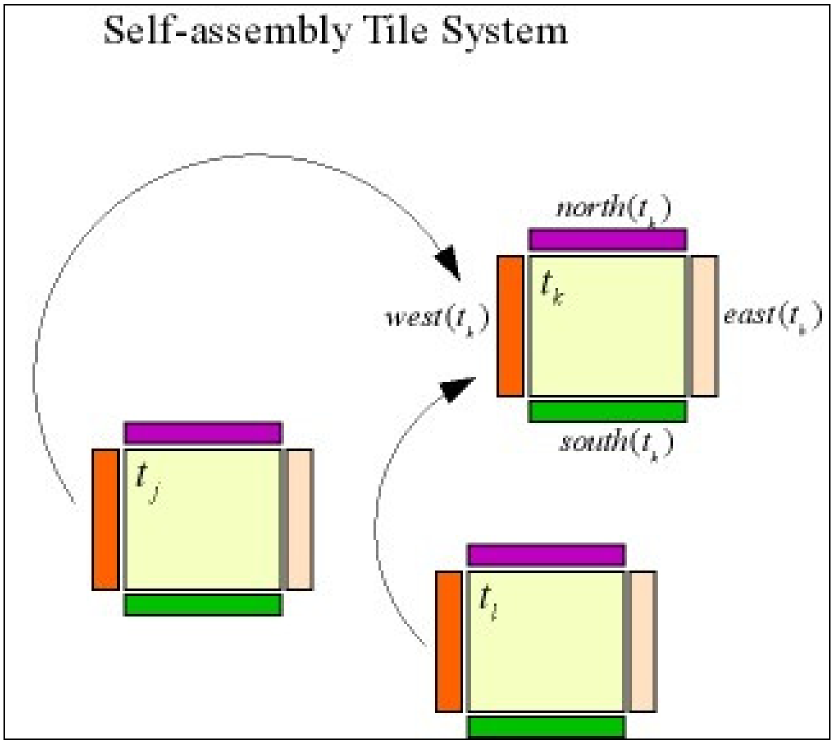


Figure 7. Self assembling tiles

Definition. A tile $t \in T$ is *attachable* at (i, j) to C if:

(1) $C(i, j) = \text{empty}$, and

(2) $G(\sigma_n, \text{south}(C(i, j+1))) + G(\sigma_e, \text{west}(C(i+1, j))) +$
$G(\sigma_s, \text{north}(C(i, j-1))) + G(\sigma_w, \text{east}(C(i-1, j))) \quad \geq \tau,$ for some $\tau > 0$

If t is attachable to C at (i, j) then the process or act of attaching is a means of producing a new configuration from C such that the empty tile at (i, j) is replaced by t (Aggarwal, et al., 2004).

Definitions. An *adjacency graph* denoted by G_C of a configuration C is defined by: (1) vertices (i, j) such that $C(i, j) \neq \mathit{empty}$ and (2) an edge exist between the vertices (x_1, y_1) and (x_2, y_2) if and only if $|x_1 - x_2| + |y_1 - y_2| = 1$. A *supertile* is a maximal connected subset $G^{'} \subseteq G_C$ such that, for every connected subset *H*, if $G^{'} \subseteq H \subseteq G_C$, then $H = G^{'}$. If *S* is a supertile, denote by $|S|$ (size of *S*) the number of nonempty positions (tiles) in the supertile. If every two adjacent tiles in a supertile share a positive strength glue type on abutting edges, the supertile is *fully connected* (Demaine, et al., 2007). A supertile *S*, whose graph is a subgraph of another supertile, $S^{'}$ is a *subsupertile* of $S^{'}$. A *cut* of a supertile is a cut of the adjacency graph of the supertile. For each edge, e_i in a cut, define the edge strength s_i of e_i to be the glue strength of the glues of the abutting edges of the adjacent tiles corresponding to e_i. The *cut strength* of a cut, *c* is then the sum of glue strengths $\sum_{i:e_i} s_i$ for all edges e_i of *c*.

Definitions. An assembly is *produced* by growing a supertile, starting with a tile *s* at $(0,0)$. Another tile, $t \in T$ which is attachable at (i, j) can be added to *s*, increasing the size of the beginning supertile. This process can be continued until no further tiles can be added and the resultant supertile is said to be *terminally produced*. Now consider an arbitrary shape, *S*. If a tile system, Γ uniquely terminally produces a supertile with shape *S*, then Γ is said to *uniquely produce* shape *S*. The *tile complexity* of *S* is given by the minimum tile set size necessary to uniquely assemble *S* under a given assembly model (Aggarwal, et al., 2005).

Counting times have also been developed for tiling systems. With each tile $t \in T$ associate a nonnegative probability $P(t)$ such that $\sum_{t \in T} P(t) = 1$. The distribution $P(t)$ models the concentration of tile *t* in the tiling system. Self-assembly corresponds to a continuous Markov process in which the states map to derived supertiles and the initial state x_0 maps to the seed. The probability $P(t)$ is the transition probability of going from a supertile without tile *t* to a supertile with tile *t*. If the tile system uniquely produces a terminal supertile, S_T, then the process of self-assembly is a Markov chain and the time to reach S_T can be treated as a random variable. Let T be such random variable. Then the expected value, $E(T)$ is called the *running time* of the self-assembling process (Cheng, et al., 2004). In thisdissertation, emergence is embedded in self-assembly so that the glue function, *G*, strength thresholds, τ, and other parameters of alternative tiling models to be reviewed below have corresponding quantum general uncertainty stochastic processes. For example, a stochastic process, ψ_t satisfying a quantum-general uncertainty Ito stochastic equation is substituted for the deterministic glue function, *G*. These generalized processes will be developed in more detail later in this section.

Some generalized models of tile self-assembly have been given by Aggarwal, et al. In the *multiple temperature model*, the lone temperature τ is replaced by a sequence of temperatures, $\{\tau_1, ..., \tau_k\}$. This *k*-phase model assembles in the following manner: in phase 1 use temperature τ_1 until no more tiles can be added, then iteratively in phase *i* use temperature τ_i until no tiles can be added or removed. If no more additions or removals can be done, regardless of the choice of tiles, then the tile system terminally produced the shape assembled. If under any circumstance of tile phase iterations, the process always ends with a unique terminally produced supertile *R*, then the tile system is said to *uniquely assemble* the shape *R* under the *k*-temperature model. In the *flexible glue model*, the restriction $G(x, y) = 0$ for $x \neq y$ is removed. There may be gradations of glue value. In the *unique shape model*, a tile system uniquely produces a shape S if the only terminal supertiles produced by the tile system are of shape S. In the q-Tile model, tiles are allowed to combine into larger supertiles of size at most q before being added to the growing seed supertile (Aggarwal, et al., 2005). The authors then show a general procedure to build arbitrary k x N rectangles.

More general models of assembly are given such that intermediate assemblies can be stored for later use. These *staged self-assembly* models utilize bins for storage of unused supertiles and stages for measuring the time required by an operator to perform the experiment. Here, unused supertiles can be "washed away" from the experiment or recombined with other unused supertiles into bins for later use. Bin and stage complexities are then calculated as added computational limits in the tile assembly experiment (Demaine, et al., 2007). Mathematical tile systems generalize the idea of regularized self-assembly of parts into more complex systems. The issues of information content and self-assembly potential in different CASs will be addressed in the author's dissertation. In addition, general mathematical models of molecular, DNA, and viral self-assembly will be addressed in review and in the context of information exchangeability in the dissertation (Twarock, 2004). These self-assembly systems will all serve as natural models of generalized organism reproduction and self-assembly in view of dynamic complexity.

One of my proposals is a general definition of information and exchange functors in a topoi of CASs that would more objectively delineated poiesis and other delineations of complex systems, such as modularity. The spectra of poiesis, modularity, and decomposability define the true possibilities of real systems, not the extremes of autopoiesis, sympoiesis, strict decomposability and strict modular independence. Hence, these gradations of systems should be treated as measures of inter-system information exchangeability. More particularly, exchangeable information may be composed into a part that strictly affects the building of components and a part that strictly affects building the relationships between components, the organizational pattern. These information subspaces may overlap and the overlap may be treated as a DNA-like subsystem of information. Additionally, these information pipes will be subdivided according to whether they are (a) DNA-like, as discussed, (b) epigenetic (nonDNA), (3) behavioral, or (d) symbolic. These evolutionary subdivisions have been posited as a new 4-dimensional approach to heredity, superseding the normal gene theory (Jablonka & Lamb, 2005, pp. 1-4).

Different levels of a system will be categorized based on their respective similarities in (a) phenotype, i.e., functional type, (b) local topology, (c) relationship or clique space type, i.e., what neighbor-component types do they connect highly to, and (d) information reception type, i.e., what information pipe types do they connect to. These information types and exchangeability criteria may be viewed as functors and natural transformations between topoi of CASs or more general systems. Despite the diversity of information, observation-dependency persists. An information measure should then be relative to the observer. Hence, each information measure should have a corresponding comeasure with respect to the observer topoi, in the form of either another component or a separate observer who is now part of the info-holarchy as posited by the timeless thermal-time hypothesis and Bohmian interpretations of QM.

Nonlinearity, Chaos, and Fractal Information

After the general discussion on emergence and information, a particular type of emergent behavior stands out in scientific investigations of nonlinear phenomena. One intriguing categorization of nonlinear behavior is that of singularities and multibranching, commonly referred to as deterministic chaos. Stochastic chaos will be discussed later in this section. In this sense, deterministic chaos captures the dynamics of a process that are separate from randomness, but that cannot be adequately predicted by a model. Note that linear systems cannot be chaotic. Henri Poincaré is credited with the discovery of chaotic processes when he investigated the three-body problem in which the dynamics of three separated bodies affect each other in complex fashions. Nonperiodic orbits in the phase space of a three-body system dynamic were observed in which neither a monotonic increase in magnitude nor convergence to a fixed point was observed. This in-between behavior in orbits of a nonlinear dynamic system was the first characterization of chaotic processes. Its generalization, the *n*-body problem, produces chaotic behavior through gravitional attraction.

Moreover, deterministic chaotic systems display sensitivity to initial conditions that lead to disparate differences in long term behavior given small perturbations in these initial conditions. However, this is not a necessary and sufficient condition for chaotic systems. Two conditions subsume the sensitivity condition for deterministic chaos. They are (a) topological mixing, and (b) its set of periodic orbits are dense. If only sensitivity to initial conditions and topological mixing are present then a system is called *weakly chaotic*. We will define what these conditions mean. Intimately involved in chaotic behavior are the structural properties of fractals and general self-similarity. In this discussion we pay closer attention to the informational aspects of these dynamics in transmission and object communication within a general system.

Consider a general stochastic process, S_t, $t \in T$ for some time index set, T. S_t is assumed to model the state evolution of some subsystem S in the universe. More generally, one can consider a quantum or Zadeh GTU process as will be discussed later. Consider general iterated maps as a model for describing deterministic chaos, $S_{t_{n+1}} = F(S_{t_n})$. The most utilized and popular example of such an iterated map is the celebrated logistic map given by:

$$x_{n+1} = rx_n\left(1 - x_n\right) \tag{3.153}$$

with $r > 0$ characterizing its behavior. Feigenbaum began his investigation of chaotic behavior using the map:

$$f(x) = 1 - \mu|x|^r \tag{3.154}$$

Let ε_0 be the amount of uncertainty or perturbation at the initial condition, S_{t_0}. The evolution of this perturbation is then given by $\varepsilon_k = M(S_{t_0}, k)\varepsilon_0$, where

$M(S_{t_0}, k) = \prod_{i=0}^{k-1} J(S_{t_i})$ and $J(S_{t_i})$ is the i^{th} Jacobian of F along the k steps of the trajectory.

By taking the singular value decomposition (SVD) of $M, M = U\Sigma V^T$ where the diagonal entries of the middle diagonal matrix, Σ are given by $\sigma_i(S_{t_0}, k)$ in decreasing order, the

Lyapunov exponents of *F* are expressed as (McSharry, 2005):

$$\Lambda_i = \lim_{k\to\infty} \frac{1}{k} \log_2 \sigma_i(S_{t_0}, k) \tag{3.155}$$

When $\Lambda_1 > 0$, the system is said to be *chaotic*. When $\Lambda_1 < 0$, the system has a *stable fixed point*. In a *2-dimensional limit cycle*, $\Lambda_1 = 0$ and $\Lambda_2 < 0$. In this case Λ_2 is the major indicator of the *rate of attraction/repulsion*, while the region of phase space in which trajectories lead into or out of the limit cycle is the *basin of attraction/repulsion*. In a *torodial limit cycle*, $\Lambda_1 = 0$, $\Lambda_2 = 0$, and $\Lambda_i < 0,\ \forall i \geq 3$. In this case Λ_3 is the major indicator of the rate of attraction/repulsion into the torodial region.

Additionally, in a torodial limit cycle, there are two frequencies of movement for an enclosed trajectory. The first frequency, f_1 is that exhibited around the major axis of the torus, while the second frequency, f_2 is that shown around the revolutional circle. If the ratio, $\frac{f_1}{f_2}$ is reducible to a ratio of two nonzero integers (*commensurate*), then the torodial limit cycle is *closed and periodic*. If the frequency fraction is not commensurate, the limit cycle is *quasi-periodic* and the phase trajectory covers the torus region *densely*, i.e., every point on the torus is eventually visited or arbitrarily closely visited by the trajectory. This condition is equivalent to the denseness of the trajectory orbits, the final condition for a chaotic system. In addition, no point in the region is visited more than once, hence the term quasi-periodic. A novel point of research would be to find if the bi-frequencies involved define any interpretive dynamics.

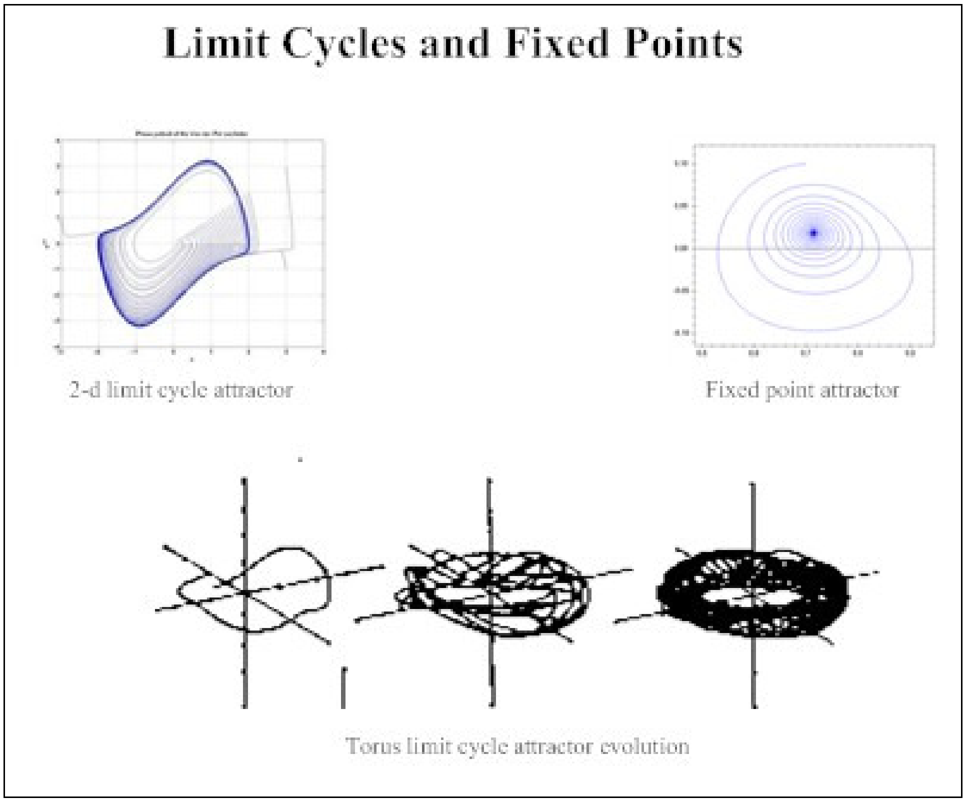


Figure 8. Limit cycles and fixed points
Adapted from "Limit cycle, fixed point, and torus attractor" By XaosBits, 2006.

If in a phase space region S , trajectories in a neighborhood of S diverge away from S , converging instead to exterior attractors, then S is called a *strange attractor*. In a torodial limit cycle, iterative bifurcations occur, producing basins for strange attractors and subsequent chaotic behavior.

One quantitative way of measuring how close the behavior of a trajectory is to being characterized as chaotic is the judicious use of the *Feigenbaum* number or constant. The Feigenbaum constant is proportional to the asymptotic limit of a series of differences between successive bifurcating points for a double-period limiting cycle and a natural chaotic limit (Feigenhaum, 1979). In more detail, let some function, *f*, generate a limiting cycle behavior and let μ_n denote the point in phase space where a *2n*-bifurcation occurs. In addition, let μ_∞ denote the limit of this process. Denote the distance between this limit and the point of *2n*-bifurcation by:

$$\left|\mu_\infty - \mu_n\right| = \frac{\Gamma}{\delta^n} \tag{3.156}$$

Here $\delta > 1$ is a constant known as the *Feigenbaum number* and Γ is the proportionality constant. Geometric convergence of this limit is assumed. In (3.156) one can solve for δ taking proper limits:

$$\delta = \lim_{n\to\infty} \frac{\mu_{n+1} - \mu_n}{\mu_{n+2} - \mu_{n+1}} \tag{3.157}$$

Feigenbaum numbers are universal for various categories of functions that describe dynamics exhibiting bifurcating behavior. In particular, they predict when a dynamic will exhibit chaotic-like behavior because of their limiting value with respect to the bifurcation points. In other words, the distance between a bifurcation point, μ_n and μ_∞ is compared to δ^n as a means for measuring the closeness to chaotic behavior. As an example of this convergence, using the logistic map (3.153) with $r = 2$, after four bifurcations ($n = 4$), a Feigenbaum number of approximately 4.6692 was produced and chaotic behavior ensued with small perturbations.

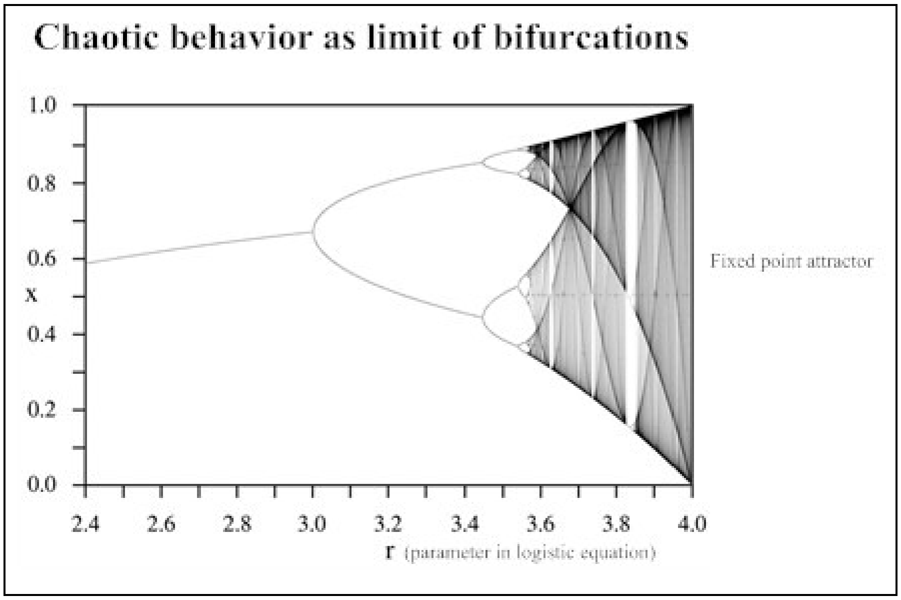


Figure 9. Chaotic behavior as limit of bifurcations
Adapted from "Bifurcation diagram as *r* increases" By PAR, 2005. Copyright 2009 PAR. Reprinted with permission under the GNU Free documentation license – Creative Commons Attribution-ShareAlike 3.0.

Chaotic behavior also exhibits *fractal dimensionality*, that is, a chaotic process fills space in such a way that a statistical fractional dimension can describe the spatial span of the *self-similar structure* of chaos. This is topological mixing in the sense that a phase space region will eventually intersect with a phase space regions in the time process. Fractal dimensions are special cases of a general Rényi dimension given by:

$$D_\alpha = \lim_{\varepsilon \to 0} \frac{\frac{1}{1-\alpha}\log\left(\sum_i p_i^\alpha\right)}{\log \frac{1}{\varepsilon}} \tag{3.158}$$

where p_i^α are the respective probabilities of each iteration defining a Rényi entropy. The weight α is defined as the relative number of times those regions of the support of the attractor are visited. In a *Mandelbrot fractal dimension*, $\alpha = 0$ the iterations are independent and $\sum_i p_i^{\alpha} = \sum_i p = N(\varepsilon)$, where $N(\varepsilon)$ is the smallest number of self-similar structures of size ε that cover the original structure. Chaotic limit cycles will then cover the space of a torus with Rényi dimension given by D_{α} (Mandelbrot & Van Ness, 1968). Values of α for a region will depend on the functional form of its network power-law. These regions are topologically close to attractor basins in various subregions of the chaotic regime. The larger the torus volume is, the more attractive the region.

An interesting aspect of phase spaces is that a phase curve (trajectory) may pass through multiple limit cycles. If the phase is repelled away from one limit cycle, it may be attracted into another limit cycle. The interpretation for a phase space region may be that multiple limit cycles represent different types of phase spaces or degrees of strength of attraction of phase spaces. A trajectory may wander into a permanent state of a weak phase space region until sufficiently perturbed. Phase spaces are then characterized by multiple regions of torus limit cycles and isolated fixed point attractors and singularities. No trajectory becomes extinct in phase space although fixed point attractors are akin to phased-out weak trajectories. It is only raised or lowered via new measurements and the nature of the phase space, i.e., the geometry of phase space, multiple limit cycle landscapes, fixed point attractors, saddle points, and strange attractors.

What is a measure of information from a source that exhibits chaotic behavior in a dynamical system? This question investigates the notion of information measure or entropy in dynamical systems. Pesin showed that certain deterministic nonlinear dynamical systems exhibiting chaotic behavior have Kolmogorov-Sinai entropy (KS-entropy) given by the sum of the positive Lyapunov exponents, that is $H_{KS} = \sum_i \lambda_i^+$ (Pessin, 1977). The KS-entropy is given for topological objects in terms of automorphisms and is formally defined as:

$$H_\Phi = \sup_\alpha \lim_{k\to\infty} \left(\frac{1}{k}\right) H\left(\alpha \vee \Phi\alpha \vee \ldots \vee \Phi^{k-1}\alpha\right) \tag{3.159}$$

for an automorphism Φ.

To connect an information theoretic measure with the KS-entropy, one can first describe an abstract dynamical system as a triplet $\mathcal{M} = (M, \mu, \Phi_t)$ where (M, μ) is a measure space with a one-parameter group of automorphisms $\{\Phi_t\}$. One then has a result given by Frigg for the KS-entropy in terms of Shannon entropy:

$$\begin{aligned} H_\Phi &= \sup_\alpha \lim_{k\to\infty} \left(-\frac{1}{k}\right) \sum_{j=0}^{k-1} \sum_{l_1,\ldots,l_k=1}^{n} p\left(\alpha_{l_1}^{t_1}\alpha_{l_2}^{t_2}\ldots\alpha_{l_k}^{t_k}\right) \\ &= \sum_{i=1}^{n} z\left[p\left(\alpha_i^{t_{k+1}} / \alpha_{l_1}^{t_1}\alpha_{l_2}^{t_2}\ldots\alpha_{l_k}^{t_k}\right)\right] \end{aligned} \tag{3.160}$$

where $z(x) = x\ln x,\ x > 0,\ z(0) = 0$ is the usual function in the Shannon entropy and partitions $\alpha = (\alpha_1, \ldots, \alpha_n)$ are taken over the phase space of the dynamical system in such a way that α_j^t is true if the trajectory $\Phi_t(x)$ is in the partition cell, α_j at time *t*, i.e., $\Phi_t(x) \in \alpha_j$ where the state of the system at the initial time, t_0 is *x* (Frigg, 2003). The probability, $p\left(\alpha_i^{t_{k+1}} / \alpha_{l_1}^{t_1}\alpha_{l_2}^{t_2}\ldots\alpha_{l_k}^{t_k}\right)$ is the condition probability that the trajectory, $\Phi_t(x)$ is in cell α_i at time t_{k+1} given that it was in cell, α_{l_j} at time t_{l_j} for $j = 1,...,n$. The supremum in (3.160) is taken over all partitions α of the phase space of the dynamical system.

There is no quantum mechanical analogue to these definitions and conditions of chaos, i.e., no direct quantum chaos exists because no such sensitivity to initial conditions in a quantum mechanical system exists. This is true because a quantum evolutional (time-dependent) system contains only periodic motions with definite frequencies, the exact anti-chaos. Instead, the question of whether quantum mechanical systems can explain or describe classical chaotic systems in the limit, that is, as the discretization internal given by Planck's distance approaches 0 or when large systems are considered, as quantum effects diminish, is the subject of quantum chaos (Berry, 2003). Put in yet another way, quantum chaos is the characterization of a quantum system that possesses a classical analogue that exhibits chaotic behavior (Millonas, 1993).

One approach to studying chaotic behavior in the limit of classical dynamic systems is the investigation of their eigensystems, i.e., through their energy level spectra. To this end, nearest neighbor distributions (NNDs), P_{NN} are utilized with Poison distributions of the energy levels, $P(s) = e^{-s}$. The probability, P_{NN} is that of finding a nearest neighbor at some given distance from a reference point in a many-body interacting system (Torquato, Lu, & Rubinstein, 1990). NNDs are utilized in densely packed particle systems which are treated as inpenetrable for approximations to their dynamics to be feasible (MacDonald, 1992). The NNDs of the energy spectra of classical chaotic systems are used to describe the extent of sensitivity of these systems in their eigensystems and as approximations to some quantum system in the limit. Chaotic classical systems may also be characterized by the statistics generated from the random matrix eigenvalue ensembles of these systems. Specifically, if a classical dynamical system is invariant under time reversibles, then their eigenspectra statistics closely follow those of an ensemble of Gaussian random matrices and can be described by the Wigner distribution:

$$P(s) = \frac{\pi}{2} s e^{-\pi s \ /4} \tag{3.161}$$

Now denote by $\mathcal{H}(\hbar)$ the (Hilbert) state space of all quantum systems that are characterized by the quantization (Planck) distance $\hbar$. Let $C(\Lambda)$ denote the space of all classical dynamic systems which are chaotic and have eigenspectra Λ. Since classical dynamic systems are the quantization limits of quantum systems, they can be expressed as trivial quantum systems. Classical chaotic dynamical systems in the limit are also quantum systems. Hence the space of all classical chaotic processes are dense in the space of all quantum processes. So $\overline{C}(\Lambda) \subset \bigcap_{\hbar > 0} \mathcal{H}(\hbar),\ \forall \Lambda$ where the closure operator is with respect to finer discretizations of the classical continuous space. It will be shown later that Zadeh GTU-inspired logic subsumes that of quantum logic under certain conditions. Hence, GTU processes, to be defined in chapter 4, are generalizations of fuzzy, quantum, and their hybrid processes, and so subsume all chaotic processes. Information in the sense of GTU processes is then realized through each specialization process, including chaotic processes.

We now visit the topic of stochastic chaos. For this part the setup is most demonstrable using diffusion processes which are quite general in describing stochastic systems. We follow the development from Millonas (1993). Consider the diffusion process q, described on $\mathbb{R}^n$ by the coupled stochastic differential equation:

$$dq^i(t) = -\partial^i \Phi(q)dt + \sqrt{g}W^i(t),\ i = 1,\ldots,n \tag{3.162}$$

where $\Phi(q)$ is a time-independent potential bounded from below, $W^i(t)$ are uncorrelated Weiner processes, and g is a diffusion coefficient. Let $\rho_t(q)$ be the evolutional probability density of q which is described by the Fokker-Planck equation:

$$\partial_t \rho = \frac{g}{2}\Delta\rho + \nabla\cdot\left(\rho\nabla\Phi\right) \tag{3.163}$$

One may then use the form (time separation ansatz) for ρ : $\rho_t(q) = \rho(q)e^{-\lambda t/g}$ and changing basis using the transformation $\rho(q) = e^{-\Phi/g}\Psi(q)$, one obtains the eigenvalue equation:

$$\mathcal{H}\Psi_\lambda(q) = \lambda\Psi_\lambda(q) \tag{3.164}$$

where $\mathcal{H} = -e^{\Phi/g}\mathcal{L}e^{-\Phi/g} = -\frac{g^2}{2}\Delta + \hat{\Phi}(q)$, $\mathcal{L} = \frac{g^2}{2}\Delta + g\nabla^2\Phi + g\nabla\Phi\cdot\nabla$ and

$\hat{\Phi}(q) = \frac{1}{2}\left(\nabla\Phi\right)^2 - \frac{g}{2}\nabla^2\Phi$. The solutions to (3.164) for small g are given by:

$$\Psi_\lambda(q) = \sum_\alpha c_\alpha \left|\nabla S_\alpha\right|^{-\frac{1}{2}} e^{\frac{i}{g}S_\alpha(q,\lambda)} \tag{3.165}$$

where $S_\alpha(q,\lambda) = \int_q p_\alpha \cdot dq$ are the solutions to the Hamiltonian-Jabobi equation:

$\frac{1}{2}(\nabla S_\alpha)^2 + \hat{\Phi} = \lambda$. The integration in the solutions, S_α are along the classical trajectories of the Hamiltonian equations of motion given by:

$$\begin{gathered} \dot{p} = -\frac{\partial H}{\partial q},\ \dot{q} = \frac{\partial H}{\partial p} \\ H(p,q) = \frac{1}{2}p^2 + \hat{\Phi}(q) \end{gathered} \tag{3.166}$$

The solutions of (3.163) are then given by:

$$\rho_t^\lambda(q) = e^{\left(-\frac{\Phi+\lambda t}{g}\right)}\Psi_\lambda(q) \tag{3.167}$$

The problem framed in this way gives a description of quantum chaos as the effect that any chaotic behavior as depicted in (3.166) affects (3.164) through the eigenspectra. The level spacing of the eigenspectra given as *S*, was then conjectured, as mentioned earlier, to follow the distribution, $P(S) = e^{-S}$ with level repulsion through the asymptotic behavior $P(S) \underset{S\to 0}{\longrightarrow} 0$, thus leading to the speculated Wigner distribution form (3.161).

We are now in a position to define stochastic chaos in the general case of diffusion processes with time-independent potentials as the properties of stochastic systems that are described by (3.162) when the equation of motion dictated by (3.166) exhibit chaotic behavior. This is very different from directly studying the possible chaotic behavior of stochastic systems described by (3.162). When $g = 0$ (no noise), there is no relevance. If one defines a family of potentials dependent on a parameter,

$\varepsilon, \{\Phi_\varepsilon\}$ and the respective descriptions of solutions given by the evolutions in (3.162), in such a way that as $\varepsilon \to \infty$ the solutions transform from regular globally integrable ones to chaotic ones, then the conjecture is that the behavior of the eigenspectra level spacings goes from being described by Poisson distributions, $P(S) = e^{-S}$, to Wigner distributions, (3.161). Under these conditions, $P(S)$ gives the distribution of the level spacings of the eigenspectra, $\{\lambda_i\}$ of the system. Utilizing (3.160), one gets a measure of the information entropy based on these $P(S)$ and hence, an information measure of a stochastic chaotic or quantum chaotic process. Generalizations to evolution systems, such as (3.162), in terms of GTU and general fuzzy and hybrid processes will be presented in chapter 4. Under certain conditions analogous to the development here for stochastic, deterministic, and quantum chaos processes, a notion of information measure can be developed for these GTU processes.

An interesting alternative to any differentiable system defined for spacetime, whether quantum or relativisitic is the fractal nature of spacetime, i.e., replacing differentiability with fractality. In the discrete theory of physics, e.g., the notion of Planck scale physics to be discussed in this study for information physics, fractility would offer a spectrum of dimensional solutions between that of differentiability and discreteness. In the case of a continuous but nondifferentiable function, a fractal approach would involve smoothing a nondifferentiable function *f*, in terms of a function, f_ε of resolution (scale), ε such that f_ε is differentiable for $\varepsilon \neq 0$ (Nottale, 2007). In this way, a fractal curvilinear system, $\mathcal{L}(x,\varepsilon)$ of scale resolution, $\varepsilon > 0$ is introduced that would replace the differentiable Riemannian manifold of spacetime represented by $\mathcal{L}(x,0)$. The systems of stochastic differentiability to be discussed in chapter 4 for the macroinfodynamics of info-holarchies can then be replaced by corresponding stochastic fractal systems. The GTU process differential systems are in turn replaced by GTU-fractal systems. These will be discussed and formed further in that development.

Adaptation and Evolution

Adaptation is the coevolution of at least two systems. Prokopenko, et al. propose the following definition of adaptation: Adaptation is a process in which a system's behavior changes in response to a stimulus so that the mutual information between it and a surrounding or enveloping nonstationary environment in which the stimulus emanates from, increases (Prokopenko, et. al, 2009). A system may be ignorant of its environment's structure, but may contain feedback mechanisms which continually modify its internal model of that interacting and possibly coevolving environment.

Adaptive systems exhibit at least three main characteristics: (a) generation of variety, so that its entropy decreases, (b) observation of feedback from the coevolving environment, and (c) a selection process in order to reinforce or inhibit interactions with the coevolving environment, which increases information content. From this perspective, this open system can be combined with the coevolving environment to construct a new super system. Adaptation, in general increases mutual information between a system and its stimulus environment and as such, the loss of information from variation is less than the increase in mutual information from the selection process. Formally, if S is a system and Z is the stimulus environment, then $I(S,Z) = H(S) - H(S \mid Z)$ is the mutual information shared between a system and its stimulus environment. The selection process increases $H(S)$ because the probability distribution increases its modality, while the diversity generating process decreases $H(S)$ because the probability distribution gets smeared, that is, is closer in probability divergence metric distance to a uniform distribution. This can be expressed as:

Proposition. If G_D and G_S are respectively, diversity generator and selection operators acting on the space of distribution (quantum density operators), $\mathcal{P}$, then for any $\rho \in \mathcal{P}$,

$$\begin{aligned} &D(G_D(\rho),U) \leq D(\rho,U) \leq D(G_s(\rho),U), \text{ and} \\ &H(U) \leq H(G_D(\rho)) \leq H(\rho) \leq H(G_S(\rho)) \end{aligned} \tag{3.168}$$

where D is a divergence measure on $\mathcal{P}$ and U is the uniform distribution ($\frac{1}{n}I$, the uniform quantum density operator of dim n where I is the identity operator). The operator, G_D , can be further decomposed into component operators representing mutation, G_M , and recombination (crossover), G_R , in the environment of an evolutionary machine. In the presence of an exogenous and stimulus environment Z, these operators are in turn, directly dependent on such systems. Hence, we can write these operators as G_R^Z, G_S^Z, and G_M^Z . More clearly, if an evolutional operator is built from a sequential composition of such operations, then

$$G_E^Z = G_i^Z \circ G_j^Z \circ G_k^Z,\ i, j, k \in \{D, S, R\}, i \neq j \neq k \tag{3.169}$$

represents a general evolution operator. Then the conditional entropy of the evolution of ρ , $H(G_E^Z(\rho) \mid Z)$ can be decomposed linearly as

$$H(G_M^Z(\rho) \mid Z) + H(G_R^Z(\rho) \mid Z) + H(G_S^Z(\rho) \mid Z) \tag{3.170}$$

under a certain class of evolution component operators. The condition of adaptability reduces to the case where

$$H(G_M^Z(\rho) \mid Z) + H(G_R^Z(\rho) \mid Z) \le H(G_S^Z(\rho) \mid Z) \tag{3.171}$$

so that in an adaptive system, *S*, $I(S,Z)$ increases. In order to define adaptability, one must define what a stimulus to a system is. Using a probabilistic approach, an event, denoted by A, is a *stimulus* for a system, *S* to change to another system S_{new} , if

$$p(S \to S_{new}) > p(S \to S_{new} \mid A) \tag{3.172}$$

If one defines a system changing to another system as a state change, from ρ to ρ_{new} then, in terms of state spaces (quantum densities), this condition may be rewritten as:

$$p(\rho_{new} \mid \rho) > p(\rho_{new} \mid \rho, A) \tag{3.173}$$

A quantum system *S* whose state is represented by the density measure, ρ_t at time *t*, is said to *adapt* during the time increment $[t_0, t_0 + h]$ with respect to a stimulus *A* and change to the quantum system $S^{'}$ represented by the density measure $\rho_t^{'}$ at time *t* if:

$$p(\rho^{'}_{t_0+h} \mid \rho_{t_0}) < p(\rho^{'}_{t_0+h} \mid \rho_{t_0}, A) \tag{3.174}$$

and $\rho^{'}_{t_0+h} \ne \rho_{t_0}$. Furthermore, in this case, *S* is said to be *adaptive* and

$$\lim_{t\to\infty} p(\rho_t^{'} \mid \rho_{t_0}) = \lim_{t\to\infty} p(\rho_t^{'} \mid \rho_{t_0}, A) \tag{3.175}$$

If the stimulus event A is causally linked internally to the system, then S is called *self-adapting*. The types or orders of adaptability are recognized. First order adaptation refers to the changing of predictive probabilities while holding stimulus sensing operations consistent. Second order adaptation applies changes to the mechanisms that control feedback, variety generation, and the selection procedure for inhibition or reinforcement. Lastly, third order adaptation applies to changing those mechanisms in second order adaptation for each agent in an autonomous multiagent system (Prokopenko, et. al, 2009).

Holarchies

Complexity and related information measures have been described and expanded upon. However, a question remains: "what kinds of organizations do natural laws construct from information flow and complexity?" We now turn to the relatively novel topic of holonic organization and its constituent parts named holons. Arthur Koestler is credited with first coining the phrase holons to refer to nested self-similar whole-part organizations in his attempt to describe self-organizing open hierarchical order (SOHO) systems (Koestler, 1967/1990). In his definition he laid out three major requirements for holonic behavior: (a) stability, (b) autonomy, and (c) cooperation. These requirements in and of themselves do not uniquely define holons as certain CASs in general satisfy them. The nested self-similarity of holons requires that they each exhibit behaviors of whole organizations while retaining the individuality of parts of an organization. In other words, a holon is a whole consisting of parts (each of which are holons) and is simultaneously a part of a larger organization that is also a holon. Hence a nested self-similiarity is exhibited in holonic organizations. Organizations depicting these structures and behaviors are referred to as *holarchies*. If one adds to the structure of a holarchy that of a CMAS, then one has a holonic multiagent adaptive system (HMAS).

The prototypical HMAS is a generic immune system. Smith illustrates a chain from the biosphere as a holarchy and closely examines how holarchies are built from constituent parts (Smith, 2009). This will be revisted after this study's micromodel of information has been developed. However, the most anthropomorphically ostensible example is that of a noösphere—the collective group of consciousness and energy-matter in the universe (Vernadsky, 1945; de Chardin, 2003). To this add the extension of exogeneous intelligence, that is, the phenomena of general (artificial) intelligence and consciousness through media outside the norm of the biosphere. This extension is, of course, the stuff of the universe—the concept of "universe as computer", a natural postulate of a potential holonic calculus. Wilber has made a societal holonic connection more explicit by introducing a general 2 x 2 matrix map covering the levels of the noösphere in his All Quadrants All Levels (AQAL) framework (Wilber, 1996). Additionally, Wilber uses the term *transorganizational* as the prototypical extension of orgranizations that depict holonic structure—capable of transforming and reinventing their whole-parts (holon-components) to form new types of organizations. Transorganization as holonic structures and story-telling narratives (narrativistic organization) is further developed in Boje (2000).

In this study, the properties of self-assembly and adaptation used within the framework of info-holarchies and their generalized processes form the basis for transorganization in info-holarchies. This is an attempt to construct an abbreviated catalog of holarchies in the noösphere. Laszlo has extended these concepts to an interconnected web held together by an *Akashic field* which is akin to a universal abstract information field (Laszlo, 2004). These structures and their action spaces are phenomenologically holarchical because they näively point to the holonic infrastructure connecting the quantum SM of physics with the biosphere, extending on to the geosphere and cosmological structures.

Sheldrake (1984, 1995) has proposed his *morphogenic resonance* as a nonlocal feedback mechanism that is manifested by *morphic fields* which are the fundamental evolvers of organisms through the spatio-temporal propagation of information and amplification of similarity in pattern. Essentially, organisms are formed and evolve based on an imposing patternization mechanism from these fields which is constructed by ancestor organisms. Natural laws, to Sheldrake, are more like habits that are morphogenetically translated (taught and learned) between generations through their respective species-specific morphic fields. Objects in the universe that possess (connect with) these morphic fields are referred to as *morphic units,* which are fundamental units of form, organization, and arrangement. When these objects are living entities they are subject to the processes of morphogenic resonance and specialized biological morphic fields called *morphogenic fields*. Morphic fields are not traditional fields in the sense of physical field theory. Instead they are posited to cause interaction between different holonic levels in organisms through relational strenghening of patterns. Sheldrake crafts this thesis around the main concept of Darwinian evolution – natural selection.

The most well known and studied of physical fields are the electro-magnetic (EM) fields that resonate to create most of the energy that humans detect on the mesoscopic scale. In an attempt to correlate fields in this spectrum to human consciousness and the translation to motor neurons in the brain, McFadden (2002) has theorized that human brains emanate EM fields in order to construct consciousness and action potential – his conscious electro-magnetic information (CEMI) field theory. This action potential is basically the stimulation of motor neurons which in turn, communicate intent to external bodies and organisms. This is a controversial thesis that has not been experimentally proven nor that has a basis in neurobiology as pertains to ionic chemical flow in dendritic and axonic dynamics. These will be discussed in detail in chapter 4 when an application of the info-holarchy will be made to neural-brain dynamics and structure.

The idea that a physical field contains an information ensemble that is received by another neuronal subsystem is not new, nor does it add any more merit to the physics of biologics. All biologics carry information via genetic spaces, memory mechanisms (epi-genetic spaces), and resultants from neuronal processing. The brain emanates electrical activity via its biochemical neurotransmitter activity. Attaching such mechanisms to a field is not particularly problematic, but implying that that field is received in a specific fashion so as to construct all of consciousness and action potential is a leap. Again, the field concept is what is important as a lesson in this study's attempt to construct an information field theory for all organisms, biological and otherwise.

While the ideas of morphic, akashic, and CEMI fields are pedestrian, not establishable in physics, and are not falsifiable, they nonetheless establish motivation for a possible existence of an interconnected framework in the form of a (virtual or placeholder) field for the phenomenology of organizations as informational holarchies in the universe. It remains to establish a rigorous mathematical and physical schema to construct verifiable holarchies as organisms.

An early informal attempt towards this goal was made by Koestler (1967/1990) in his example of the human brain. More recently, an informal systems-theoretic notion was presented that shows how a holarchy can be constructed based on multiagent systems and holonic relationships between their layers (Pichler, 2000). This study discusses rigorous generalized information-theoretic metamodels towards this goal, based on (a) a newly proposed information particle, the *informaton*, constructed as a generalized holon, (b) an information field theory, and (c) information holarchies or info-holarchies, as scaffolds for information flow. The scaffolding is glued together using concepts from spinfoam networks of loop quantum gravity (LQG), an information signal field theory, general uncertainty causaloids, and topos theory, a category-theoretic generalization to mathematical logic and set theory.

Applications of this metamodel will be presented in terms of inference machines, specifically, the neuronal structure of biological brains, and to a novel holographic multidimensional dashboard representation of the dynamic information in business organisms, as they frequently will be referred to (organizations are societal organisms and hence the use of the more general description - organisms). Analogies will also be sparingly made in passing to socio-economic and cosmological systems. Koestler's supreme vision of a holarchic universe will be the underlying foundation of these constructs. The novel injection of an information skeleton for each holarchy is the main contribution of this study to his insight. This study proposes that a sufficiently powerful and universal mechanism for information structures, such as the info-holarchy, will greatly benefit the description of the evolution of business organisms and spaces with respect to an underlying and correspondingly powerful information management metaprocess.

In the ensuing discussion, basic ideas from Rodriguez, Hilaire, and Koukam (2003, 2008), and Smith (2009) will be used to construct a computational notion of holarchies while we frame a separate more rigorous and convenient notation and causaloid-stochastic model for HMASs. Systems merge with other systems to form holonic structure based on their individual affinity toward each other. Holarchies have various levels of organization. Each level consists of a set of holons that share communication and functional similarity. At each level, a peer holon at that level is capable of interpreting the actions of that level. However, its view of other levels of the holarchy becomes progressively vaguer or different as the distance between its level and the viewed level increases. This is the Law of Perspective for holarchies.

There are three categories of structures for holarchies: (a) federated autonomous, (b) moderated group, and (c) unitary merged. In the federated model, a group of agents coalesce into a connected web where each agent has complete autonomy with respect to its actions. In a unitary merged model, all the members of the group are merged into one holon, losing the notion of individuality. Finally, in a moderated group, the agents form a connected group where one of the agents acts as a moderator for the group, essentially, a gatekeeper of communications and signals. The moderated group acts as a hybrid of the frist two types, federated and unitary merged, whose behaviors exhibits extremes in the degrees of agent autonomy. To this end, for this discussion, the moderated group will be examined such that parameters will be presented as control knobs in defining the identity of the agents. In this way, a general holarchy can be reprseented by the model and its parameter settings.

In the following model of holarchies, holon inter-level communication is evidenced by the transfer of information between representive heads at each level and not directly to the viewing or requesting holon. Added to the requirements for a holarchy of Koestler are the following four properties: (a) assimilation, (b) adaptation, (c) communication, and (d) reproduction. These actions are manifested by holons at different stages of interaction with other holons. Communication ensues between holons that are at the same level. Assimilation happens when a holon of a higher level interacts with a holon of a lower level. Adaptation results when a holon of a lower level must change in accordance with the changes occurring in a holon of a higher level. Emergence appears when holons at one level interact to form a higher order holon with some new properties alien to and preservation of old properties from its constituent holons. Moreover, these emergent phenomena are manifested in higher dimensional spaces. So, as an emergence occurs, a new morphology develops that necessitates extra spatial or temporal dimensions. We will introduce a causal formalism that will generalize temporal measurement, the causaloid framework and so, in this sense, dimension expansion in emergence in holons will be generalized.

How do holons interact in the abstract? The Role-Interaction-Organization (RIO) model of Hillaire, et al. (2000) will be used in framing the holarchy model. In general, adaptive agents have evolutional goals and services. They have sensorial inputs and exude actions based on a space of rules that they adhere to. Because of their adaptation, these agents can change their respective rule spaces. Holons possess these properties. Some definitions will clear the way for a cleaner approach to framing holarchies for our use.

Definition. (Role). A role, R_i, is an abstraction of a behavior in a certain context and confers a status within an organization. The Role gives the playing entities the right to exercise its capacities. Roles may interact with other roles defined in the same organization.

Definition. (Interaction). An interaction, I_{ij} links two roles, R_i and R_j in a way that an action in the first role produces a reaction in the second. This may be a causal or societal linkage.

Definition. (Organization). An organization, O, is defined by a set of roles, $R^O = \{R_i^O\}$, their interactions, $I^O = \{I_{ij}^O\}$ and a common context that defines a specific pattern of interactions, $P^O(I^O)$ to form a triplet, $O = \{R^O, I^O, P^O\}$

Definition. (Holon Model) A *holon* of level *n*, is a triple $\mathcal{H}_n = \{H_{n-1}, O, \pi\}$, where:

H_{n-1} is the set of subholon members of the super-holon $\mathcal{H}_n$

O is the set of organizations (organisms) that govern the actions of the super-holon $\mathcal{H}_n$

$\pi : H_{n-1} \mapsto 2^{roles(O)}$ is a set function that assigns to a subholon a subset of roles in $\mathcal{H}_n$ defined by the governing set of organizations O such that $\forall h_i \in H_{n-1}, \pi(h_i) \neq \varnothing$.

The spectrum of autonomy of the agents in a holon is thus actualized by the governing organizations O of that holon. As such it controls the eventual structure of the holarchy. The moderated group model will be discussed here because one can control for the autonomy of the agents and hence of the structure category of the holarchy through the rules from the sets of O.

Two systems have a high affinity to each other if they closely share goals and if their services (actions) are highly complementary. Affinity then becomes a measure of the attraction between two or more systems as measured by the similiarity between their respective goals and interlocking services offered.

Definition. (Affinity) Affinity is a measure of the compatibility of two holons to work together towards a shared goal(s).

To make this more precise, let $D(h_i \,||\, h_j)$ denote an affinity divergence between two holonic systems, h_i and h_j. In this scenario, holon h_i has taken on the role of a HEAD in its containing holon level in an HMAS H. Holon h_j is vying to join this holon level through its affiliation with h_i. As a rule, one then chooses a threshold, ε_i, in which the condition, $D(h_i \,||\, h_j) < \varepsilon_i$ implies that h_j joins h_i in the same holon level in H and assumes the role of PART. In this discussion, a holarchy will be managed by roles of its constituent parts. Each part will have four possible stationary roles (a) HEAD, (b) PART, (c) MULTIPART, and (d) STANDALONE, and one transitory state, MERGING. A HEAD will be responsible for communicating with other holons on behalf of its occupying holon members and structure. It is the only agent holon that can reside in more than one level at a time and performs its actions independently on each level. It is thus a moderator for its holon level. In this respect, holon level *i* can only directly interact with holon level *i-1*. Higher order inter-holon level interaction is done via a chain of HEAD holon interactions from adjacent holon levels. A STANDALONE will be autonomous with respect to its holon members and structure-it does not belong to a holon within its holon level and can only interact with the head(s) of the holons on that holon level. A PART will be subject to requests from its fellow parts in the occupying holon and to either requests or commands from the holon head. It can only interact with agents from its holon container.

If a part requests a service from a part of another holon, it will send that request to the head who will request that of a part in its connected holon. A MULTIPART holon is part of more than one holon at the same part level (See Figure 6). The action of merging (state of MERGING) takes on different definitions for each role. MERGING is the state in which a holon has reached an unsatisfying measurement and requests to change its role within a holon level. For a STANDALONE agent such requests may be to join a holon via that holon's head. If accepeted the STANDALONE agent becomes a PART agent of that holon. If not the STANDALONE may endeavor to form its own super-holon and annoint itself as the head. It may also simply choose to remain a STANDALONE. PART agents may request to opt out of membership in a holon (fission process) either through self decision or via a command from the holon head. It does this through the measurement of its satisfaction in a certain setting. It may also request to becoming a part for another holon on the same holon level. If no conflicts occur when a part has become a part for more than one holon, then it proceeds to becoming a MULTIPART. Conflicts can occur in at least three ways: (a) interest conflicts in which sup-holons do not share similar goals or have contrasdictory services, (b) authority conflicts in which heads of the super-holon request contradictory requests from the multipart, (c) unbalanced authority conflicts in which one of the super-holon's head has more power than the others over the multipart, and (d) a combination of the first three types of conflicts. Agent or holon satisfaction is the measure used to build rules for these actions. Satisfaction will be defined for a holon later.

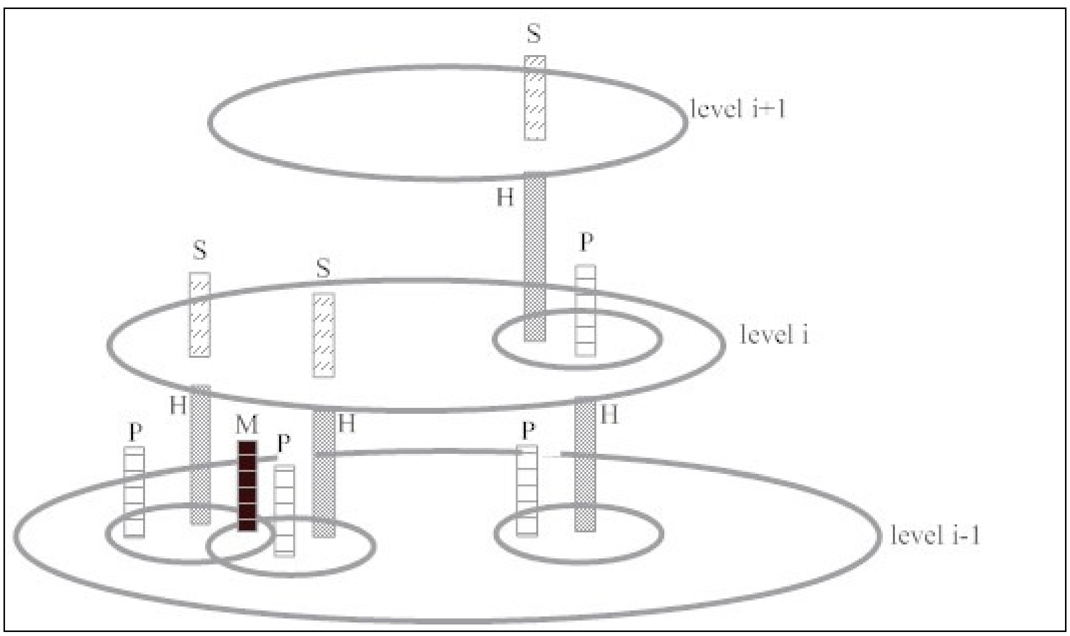


Figure 10. Holarchy and holon agents

In defining an affinity divergence, a measurement is taken. If that measurement requires a certain amount of resources, then the act of calculating a divergence reduces the resources left for a holon. If calculating (measuring) $D(h_i \,||\, h_j)$ requires a resource allotment of $R_D(h_i, h_j)$, and holon h_i has resources of $R(h_i)$, then the act of calculating the truth value of the proposition, $D(h_i \,||\, h_j) < \varepsilon_i$ will be carried out only if $R_D(h_i, h_j) \leq R(h_i)$ where h_i is the requesting holon. So the action rule, $a_h(h^{'})$ for accepting an agent $h^{'}$ into the holon represented by the head h_i will be:

$$a_{h_i}(h^{'}) = \begin{cases} 1 \text{ (accept), if } R_D(h_i, h^{'}) \leq R(h^{'}) \text{ and } D(h_i \,||\, h^{'}) < \varepsilon_{h_i} \\ 0 \text{ (reject), otherwise} \end{cases} \tag{3.176}$$

This may be generalized so that a decision on accepting members is dictated by a group of member holons in a weighted scheme. These will then dictate the *archy-hoods* of the decision process, i.e., monarchy, oligarchy, polyarchy, or apanarchy. Define the divergence measure to be a weighted average of the single divergences between all of the members of h and the requesting holon, $h^{'}$:

$$D(h \,||\, h_j) = \sum_{h_i \in h} \omega_{ij} D(h_i \,||\, h^{'}), \ \sum_{h_i \in h} \omega_{ij} = 1 \tag{3.177}$$

This can be generalized to a functional, F defined on the cross-product space, $H^v \otimes H_h^C$ where $|h| = v$, H^v is the $v-$dimensional space of vectors whose components are subholons of h and $H^v \otimes H_h^C$ is the space of subholons exogeneous (nonmembers of) to the holon h:

$$D(h \,||\, h^{'}) = F(\overline{h}, h^{'}), \ \overline{h} = (h_1, ..., h_v) \tag{3.178}$$

In order for D to qualify as a divergence measure, the form of F can be quite general, as in the requirements for an (h,ϕ) -divergence discussed in (3.88). The resource functions can be similarly generalized to accommodate all the member subholons:

$$R_D(\bar{h}_i, h') \leq R(h') \tag{3.179}$$

The actions can thus be rewritten as:

$$a_{\bar{h}}(h') = \begin{cases} 1 \text{ (accept)}, R_{D_{(\phi,h)}}(\bar{h}, h') \leq R(h') \text{ and } D\left(\bar{h} \,\|\, h'\right) < \varepsilon_h \\ 0 \text{ (reject), otherwise} \end{cases} \tag{3.180}$$

Dynamicism in the roles of holons within the holarchy will be dictated by the merge states and the degree of satisfaction for each. To this end, different measurements of satisfaction will be introduced for holons. First a general definition.

Definition. (Satisfaction). Satisfaction is a measure of the progress of a holon towards the accomplishment of its current goal(s).

Let $\mathcal{H}_i \in \mathcal{H}$ depict the set of holons that holon (agent) i belongs to where $\mathcal{H}$ is the set of all holons in a holarchy, H. Let $R_i \in \{\text{H (HEAD), P (PART), M (MULTI-PART), S (STAND-ALONE)}\}$ denote the role state of holon i. Now define the different levels of holon (agent) satisfaction. The following three satisfaction measures are with respect to the agent i:

Self-satisfaction: Ss_i denotes the satisfaction that is self-produced (without influence from others).

Collective satisfaction: Cs_i^H denotes the satisfaction that is produced through the collaboration with other agents of the holon H, that it belongs to.

Leadership satisfaction: Ls_i^H denotes the satisfaction produced for the head of the holon *H*, that it belongs to and is attributed to it alone.

Then the *accumulated satisfaction* produced for the agent *i* is:

$$As_i = \sum_{h \in \mathcal{H}_i} Cs_i^h \tag{3.181}$$

The *leadership satisfaction* produced for the agent *i* if it is the head of the holon *h* is:

$$LHs_i = \sum_{\substack{j \in h \\ j \neq i}} Ls_j^h \tag{3.182}$$

Denote the *necessary instant satisfaction* produced for the agent *i* to finish a task assigned to that agent with constraints *K* as: Ns_i^K. Finally, the *instant satisfaction* produced up to the moment (both individual and accumulated) is:

$$Is_i = \begin{cases} As_i + Ss_i, & \text{if } R_i = P \text{ or } R_i = M \\ LHs_i + As_i, & \text{if } R_i = H \\ Ss_i, & \text{if } R_i = S \end{cases} \tag{3.183}$$

Rodriguez, et al. model these roles and actions in a deterministic algorithm for multiagent simulation, we here label as a holonic engine. This role rule space can be enumerated as a state transition matrix, where H_1 and H_2 are competing holons for multiparts on the same level:

Table 1

Holon State Transition Matrix
Transition events for traversing from one holon state to another

old \ new	**S**	**P**	**MP**	**H**	**M**
S	$Is \geq Ns$				$Is < Ns$
P	$\left[Is < Ns\right]$ AND $\left[Ss > Cs\right]$		$\left[Is < Ns\right]$ AND $\left[\mathrm{A}s^{H_2} > \max(Ls, As^{H_1})\right]$	C_{PH}	
MP		$\left\|As^{H_1} - As^{H_2}\right\| >> 0$			
H	$Is < Ns$				
M		$Ls < As^{H_1}$		$Ls > As^{H_1}$ OR $a_{H_1} = 0$	

Here C_{PH} represents the condition that the current head of the super-holon has relinquished its role as such because for it, $Is < Ns$ and for the part subholon potentially assuming the head role, $As^{H_1} = \max_{H_1} As^{H_1}$ (the current most satisfied and hence resourceful part in the super-holon). Other rules may apply for choosing a new head for a super-holon, i.e., there may be coalitions of subholons that form to choose such. One may also consider the application of a general game-theoretic framework for the dynamics of holon development. This will be investigated in the next section on quantum games adapted for HMASs. Utility can be equated to satisfaction, while cost may be akin to resource consumption. Since holarchies are normally open systems, a nonzero sum game setting would be more suitable to a practical construction.

Higraphs have been shown to be a useful graph-theoretic tool in describing holarchies (Rodriguez, 2005). Higraphs (Hierachical graphs) describe hierarchical relationships in terms of graph components. By mapping holarchies onto graphs, various mechanisms for exploration can be opened. In particular, graph networks can be constructed for holarchies so that dynamics can be laid upon them in a more rigorous fashion.

Definition. (Higraph). A higraph H is a quadruple $\{B, E, \rho, \pi\}$ where B is a set of universalized elements called *blobs*, E, a set of edges which is a subset of $B \times B$, $\rho : B \to 2^B$ a hierarchical function that defines the direct descendents of a blob, and $\pi : B \to 2^{B \times B}$ a partitioning function on B.

In general, one can define a recursion for the hierarchical functions of a holarchy. Define recursive hierarchical functions for a super-holon, H with the set of subholons, $\mathcal{H}$:

$$\begin{aligned} &\rho_i(H) = \rho_{i-1}(H) \cup \left[\bigcup_{k \in \rho_{i-1}(H)} \rho_{i-1}(k) \right],\ i > 0 \\ &\rho_1(H) = \rho(H) = \mathcal{H} \end{aligned} \tag{3.184}$$

The level i of the holarchy with respect to a starting holon H, is then given by $\rho_i(H)$. In the case of multipart members, one uses the directed acyclic graph (DAG) of the higraph that represents the holarchy H. A multipart link is represented by multiple edges from the multipart to its super-holon vertex. To include a legitimate description of a HMAS, one uses the notion of a configuration in a higraph. A blob in a higraph can be an abstraction such as a logical operator. For holarchies that employ logical gates so that alternative super-holons can be composed, we consider a more general setting.

Definition. (Higraph Configuration). A higraph configuration, *C*, in a higraph *H*, is a subset of blobs in *H* such that: (1) for any nonelementary (nontrivial) $b \in C$ that contains *r* orthogonal components, $|\pi_i(b) \cap C| = 1$ holds for each $1 \le i \le r$, and (2) for any $b \in C$, if $\rho^{-1}(b) \neq \varnothing$ then $|\rho^{-1}(b) \cap C| = 1$.

Higraph configurations legitimize a hierarchical set in the sense of guaranteeing semantic structure. One can now form legal (semantic) holarchies that use logical operators as vertices (super-holons) by finding the higraph configurations of such higraph representations of holarchies. Finally, the roles for super-holons in a holarchy can be defined using the hierarchical function in the following manner:

A holon *h* has the multipart role in the holarchy represented by $H = \{B, E, \rho, \pi\}$

$$\Leftrightarrow \exists h_1, h_2 \in B \ni h_1 \neq h_2,\ h_1 \notin \rho(h_2),\ h_2 \notin \rho(h_1), \text{ and } h \in \rho(h_1) \cap \rho(h_2)$$

A holon *h* has the part role in the holarchy represented by $H = \{B, E, \rho, \pi\}$

$$\Leftrightarrow \neg\exists h_1, h_2 \in B \ni h_1 \neq h_2,\ h_1 \notin \rho(h_2),\ h_2 \notin \rho(h_1), \text{ and } h \in \rho(h_1) \cap \rho(h_2)$$
$$\text{and } \exists h_3 \in B \ni h \notin \rho(h_3)$$

Here, we generalize the higraph representation of a holon and in so doing, generalize relationships in a holarchy. We label these holarchies as hyper-holarchies. Consider a higraph that contains hyperedges, that is, multinary edges-edges connecting multiple vertices (Harel, 1988). A DAG mapped to a higraph was sufficient to represent a simple holarchy. Suppose now that super-holons can be related to other super-holons by an arbitrary, nonheirarchical function. Furthermore, suppose super-holons are mapped in this way to multiple other super-holons. One now has a hyper-holarchy that is represented by a higraph with hyperedges. These hyperedges from the viewpoint of hyper-holarchies, connect seemingly disparate super-holons with each other. Consequently, a version of virtual holarchies appear that generalize the notion of hierarchies of holons.

Definition. (Hyper-higraph). A hyper-higraph, H^N is a higraph which contains hyperedges, $E^l, l = 1, \ldots, N$ of multiconnectivity up to N.

Definition. (Hyper-holarchy). A hyper-holarchy, $\mathcal{H}^N$ is a holarchy that contains arbitrary, possibly nonheirarchical relationships with up to N multinary connectivity between holons and is representable by a hyper-higraph H^N.

Stochastics and fuzziness can enter into the structure of a holarchy by considering edges of the higraph representing that holarchy to be probabilistic or fuzzy. Hence, GTU inspired rules for the existence of edges in a higraph representation of a holarchy define a GTU holarchy. In this manner, one defines a GTU-holarchy of which, one instance is a quantum entanglement version of edges and thus if holarchies.

Definition. (GTU Higraph). A GTU higraph is a hyper-hygraph that has edges, E that are governed by a GTU constraint as defined by a Zadeh GC. This, in turn, defines a GTU holarchy.

Consider a further stochastic approach to holon dynamics. The state of a holon may be modeled as a general stochastic process. A specialization could be a Markov process. Let ${}^{h}p^{i}_{XY}(t, \Delta t)$ denote the probability of agent (subholon) i transitioning from state (role) X to state (role) Y with respect to the super-holon h during the time interval $(t, t+\Delta t)$. For each subholon, i in super-holon h, a stochastic process, $S_t^{i,h}$ modeled by a series of random variables indexed by t, following such a probability law can be constructed. In this regard, all the above mentioned definitions of subholon satisfaction can be parameterized using time t. Then the probability rules can be made explicit for each state transition. For example,

$$\begin{aligned} &{}^{h}p_{HS}^{i}(t,\Delta t) = \\ &p[sub-holon\ i \text{ is head of super-holon } h \text{ at time } t,\ \mathrm{I}s_i(t+\Delta t) < ns_i^C(t+\Delta t)] \end{aligned} \tag{3.185}$$

Quantum Games For Multiagent Systems

In our later consideration of informatons as bipartite entangled particles, the question of agent (particle) coordination and cooperation is germane to the possibility of building higher order organizations such as holarchies with quantum entanglement. We have investigated a system of rules and actions for holarchy development. Using the resource and satisfaction measures of the prior section, a game-theoretic slant can be put on this framework. In this setup a quantum game is initiated for holons and their respective agents. The agents utilize entangled and unentangled pairs of particle information. Satisfaction and resource measures will be converted to utility measures for a game. Game coalitions are then formed akin to holon development, i.e., an agent enters into membership into a holon as it would for a coalition in a game setting.

Quantum games are games in which players may share in the resource of qubits (qudits) supplied by a central authority (attached quantum system). POVM operators are applied by each player to the qubits received in order to extract information (measurement) from a central authority (game quantum system). The payoffs are then distributed based on these measurements. Nash equilibriums may be calculated based on this distribution of quantum superposition and entanglement. We investigate two-particle entangled states for quantum games with a mixture of unentangled pairs. This has been shown to reduce player collusion and general computation issues (Zhang & Hogg, 2003). Consider an *n*-player game. The set of choices for the *n* players in a given turn is given by $c=\{c_1,...,c_n\}$, $c_i \in C$ and the payoff function for the i^{th} player is given by $A_i(c)$. The state of the game is given by the superposition, $\omega = \sum_c \psi_c \left|c\right\rangle$, where the sum is taken over all choices from C for each player in the game.

Each player has access only to their particular part associated with the choice c_i. They apply an operator only to their corresponding part in the superposition state, An entangled state $J\omega$ is produced where J is an entanglement operator that commutes with the classical single-player operators J_i. A player then selects an operation to apply to their part of ω, resulting in the state $v' = \left(\bigotimes_{i=1}^{n} V_i\right) J\omega$. Now they undo the initial entanglement, which results in the state $\psi = J^{\dagger} v'$. The final superposition state is used to give a definite choice for each player. This is accomplished by the application of a joint operation followed by a measurement. For a given (choice) ω and J, the final superposition is a function of their choices, that is, $\psi_F = \psi(V_1,...,V_n)$. Finally, a measurement of ψ_F is taken. The probability of producing a particular choice, c, is $\left|\psi_c\right|^2$. The expected payoff of player i is then given by $A_i^*(c) = \sum_c \left|\psi_c\right|^2 A_i(c)$. Players know, in advance, the initial and final operations to be applied to the superposition. The original game payoff structure is to be preserved by this quantum game. For details on the specific setup information for particular game types see (Zhang & Hogg, 2003). A quantum game can then be given by the tuple $G = (n, C, A, J, \omega)$ where $A(c) = (A_1(c), \ldots, A_n(c))$. Nash equilibria are then dependent on the choice of the pair (ω, J). In the case of a qubit game, players make binary choices represented by a bit. In a qudit game, a multibit is used to represent d choices. Most commonly, for a qubit game, the initial setup state is $\omega = |00...0\rangle$ and

$$J_n = \frac{1}{\sqrt{2}}\left(I + i \bigotimes_n \sigma_x\right),\ \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},\ I \text{ the identity} \tag{3.186}$$

Under this setup, the initial game state $J\omega = \frac{1}{\sqrt{2}}\left(|00...0\rangle + i|11...1\rangle\right)$ is maximally entangled. If players are restricted to the use of the two operators, I and $U = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}$ then G is reduced to a classical game. In this case, the reduced classical game is a subgame of G. The case of mixed entangled states will be reviewed next. Here, two players share two qubits maximally entangled with probability p and not entangled with probability $1-p$.

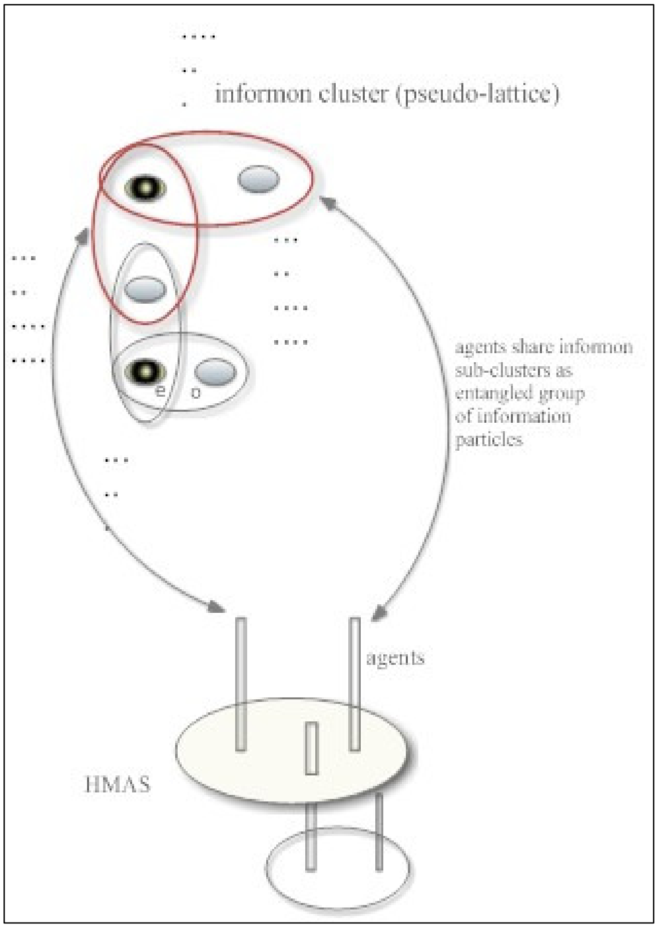


Figure 11. Informaton sharing in a HMAS

The probability p may or may not be publically announced. Now consider two games, G_0 and G_1 with the same strategy space, and let G_p be the game where G_0 is played with probability $1-p$ and G_1 is played with probability p. The respective payoff functions are, $A^{(0)}$, $A^{(1)}$, and $A^{(p)}$ where $A_i^{(p)} = (1-p)A_i^{(0)} + pA_i^{(1)}$. Now let G_0 be the game in which the qubits are not entangled and G_1 the game in which they are maximally entangled. We then have the following consequences:

1. If (S_0, S_1) is a Nash equilibrium for (G_0, G_1) then it is a Nash equilibrium for G_p for $0 \le p \le 1$.

2. The strategy (S, S) is a Nash equilibrium for both G_0 and G_1 and so is for G_p for $0 \le p \le 1$ where S is the strategy where the operators are of the form:

$$U(\theta,\phi,\alpha) = \begin{pmatrix} e^{-i\phi}\cos\frac{\theta}{2} & e^{i\alpha}\sin\frac{\theta}{2} \\ -e^{i\alpha}\sin\frac{\theta}{2} & e^{i\phi}\cos\frac{\theta}{2} \end{pmatrix} \tag{3.187}$$

are used. In particular, $U_0(\pi,0,0) = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ and $U_1(\pi,0,\frac{\pi}{2}) = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}$

are used with probability *p* and *1-p* respectively for two-qubit entanglement pairs. These results can be extended to the case were each player has access to multiple qubits. Let $B_e = \left(b_i, b_j\right)_{i \in E_1, j \in E_2}$ be the set of maximally entangled pairs and $B_u = \left(b_i, b_j\right)_{i \in E_3, j \in E_4}$ the set of unentangled pairs. Let P_i denote the i^{th} player. In a general situation, for a given P_i, it will share r_i pair elements in B_e with other players, $\left\{P_k\right\}_{k \in I(i)}$ for an index set, $I(i)$ where $\left|I(i)\right| = r_i$. Now let $P_e = 2^{\{P_i\}_{i \in E}}$ denote the different subsets of players in which mutually shared pairs have an overlay of qubits, so that each subset of players in P_e is relevant to each player's probability of contribution in that subset. Let the operator $V_e^{(i_1,\ldots,i_l)}$ be applied to the $(i_1,\ldots,i_l)-th$ qubit tuple corresponding to a subset with l relevant qubits in P_e and the operator $V_u^{(i_1,\ldots,i_m)}$ be applied to the unentangled tuples. The final game state can be expressed as:

$$\left|\psi\right\rangle = \hat{J}^{\dagger}\left(\underset{(i_1,\dots,i_l)}{\otimes} V_e^{(i_1,\dots,i_l)} \otimes \underset{(i_1,\dots,i_m)}{\otimes} V_u^{(i_1,\dots,i_m)}\right)\hat{J}\omega \tag{3.188}$$

where $\hat{J} = \sigma^T\left(\underset{n}{\otimes} J_2\right)\sigma$, $J = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 0 & 0 & i \\ 0 & 1 & i & 0 \\ 0 & i & 1 & 0 \\ i & 0 & 0 & 1 \end{pmatrix}$, and σ is a permutation matrix of size n^2

that permutes vectors of the form $\left(x_0 x_1 \dots x_i x_{i+1} \dots x_{n-1}\right)$ to $\left(x_0 x_1 \dots x_{i+1} x_i \dots x_{n-1}\right)$.

The case for a QG game with relativistic effects and generalizing to the Zadeh GTU process will be developed for the info-holarchy model in chapter 4. In addition, a semiotic version of this game will be constructed based on the Piercean model of semiosis to be reviewed and expanded upon later in this chapter. In this class of games, the observer as an interpretant, is considered in the reception of strategy information between players (agents). This means that the utility of a particular payoff will be observer-dependent, not absolute across the player spectrum.

Bridging Information Scales

Through the structures of general heirarchies, such as holons and other organization frameworks, information exists at various levels and interacts to create the dynamics of movement and creation from one scale or level to another. How is this dynamic manifested? We have iterated on structure and axiomatic laws for behavior, but the dynamic of action has been limited to the afterthought of obedience of law.

Lerner has proposed an inter-level stochastic model utilizing the stochastic calculus and a variational principle applied to information entropy to patch together a picture for the dynamics that intertwine at different levels of organization (Lerner, 1998, 2003, 2006). He labels this formalism *informational macrodynamics* (IMD). The premise of this framework is as follows. In complex systems, and invariably, in CMASs, interactions occur simultaneously between agents (particles). Because of the large conglomeration of this dynamic, an amount of emergent collective indeterminancy is created not possible through independent relations only. This dynamic appears at all levels of the organism. At a given observer level, these are manifested in the relative macro and microlevels of that observer level. In this respect, for a given dynamic organizational structure, there exists many relative micro and macrolevels, and hence many relative mesolevels of existence.

Even in a discrete representation of an organization, an infinite number of observer levels are possible and hence an infinite number of relative macro, meso, and microlevels may exist. Moreover, given an observer level, the may exist an infinite number of macro, micro and mesolevels. What this really means is that a chain of intermediate ordered levels exist for an observer level. Therefore, it would not make sense to declare a certain level as meso, micro, or macro without referring it to a relative observer level and without reference to its predecessor and successor levels within that observer level. In this dissertation, relative to a given observer level, the term microlevel will refer to a predecessor level, macrolevel to a successor level, and mesolevel to a proximate level.

Lerner posits that the microlevel dynamics are more closely dictated by irreversible stochastic processes, while those at the macrolevel are more akin to thermodynamic processes. His proposed model is a macrolevel functional that consists of a Markov chain evaluated using a path integral (Feynman sum of histories) in which contributions from local functionals, which are the interacting agents, are made via an application to information entropy using a variational principle (VP). The microlevel models are dictated by Itô stochastic processes that satisfy an Itô stochastic differential equation. The VP is the responsible mechanism for inter-level dynamics. The key operation in the exchange of information between levels is the application of the VP in an informational form in a macrolevel Shannon entropy functional. This process chooses an optimal macrolevel or macrodynamic model based on the transfer of information from the Itô stochastic processes that govern at the agent microlevel.

Microlevel processes, $X = \left(X_t : 0 \le s \le t \le T \le 1\right)$ where each random variable is n-dimensional, are to be modeled by an Itô stochastic differential equation of the form:

$$\begin{aligned} dX_t &= a_t(X_t, U_t)dt + \sigma_t(X_t)dW_t = F_t X_t \\ X_s &= \eta \text{ on } \Delta_s^T = \left[s, T\right],\ 0 \le s \le T \le 1 \text{ (initial conditions)} \end{aligned} \tag{3.189}$$

with drift function, $a_t(X,U)$, $U = U_t(X_t)$ feedback control functions (control knobs), and W_t a Weiner process with diffusion operator, $b_t(X) = \frac{1}{2}\sigma\sigma^T$. Underlying this is a probability space, on the interval, $\left[0,T\right]$, $(\Omega, \mathcal{F}, P)$, $\left(\mathcal{F}_t\right)$ a nondecreasing family of sub-σ-algebras of $\mathcal{F}$. The Wiener process, W_t is *w.r.t.*

$\left(\mathcal{F}_t\right)$, $t \le T$ and $X = \left(X_t : 0 \le s \le t \le T \le 1\right)$ are continuous functions on $\left[s,T\right]$ with the σ-algebra $\mathcal{B} = \sigma(X)$. In general, set $\mathcal{B}_t = \sigma(X_s : s \le t)$. Additionally, $a_t(X)$ and $b_t(X)$ are $\mathcal{B}_t$-measureable functionals and η is $\mathcal{F}_0$-measureable. Assume that a solution to

(3.189) is $\tilde{X}_t$. $\tilde{X}_t$ will be compared to the Weiner process, W_t using a measure of (neg)entropy between two processes. This will give a measure of how far $\tilde{X}_t$ is from disorder, i.e., a measure of information about the microprocess solution. To this end, define the entropy functional on solutions of (3.189):

$$S\left(X,(s,T)\right) \stackrel{\text{def}}{=} M_{(X_s,B)}\left[\ln(q^{-1}_{(s,T)}(\tilde{X}))\right] \geq 0 \tag{3.190}$$

where

$$M_{(X_s,B)}\left[y\right] = \int_B y p_{(X_s,B)} d\omega \tag{3.191}$$

and

$$q_{(s,T)}(\tilde{X}) = \frac{d\tilde{p}_{(s,T)}}{dp^W_{(s,T)}}(\tilde{X}) \tag{3.192}$$

Here, $q_{(s,T)}(\tilde{X})$ is a measure of the difference between the probability density of the solution, $\tilde{X}_t$, $\tilde{p}$, and the probability density, p^W of the transformed process W_t in (3.189). B is a Borel-measureable subset on $[s,T]$. One may see that S is a divergence measure between solution processes of (3.190) and the transformed process, W_t. Furthermore it is based on the Shannon entropy, i.e., S is the *KM*-divergence between solutions of (3.190) and its underlying transformed process W_t. One may proceed to generalize this divergence by the use of classical versions of the divergences of the form (3.88).

Next, a formal Hamiltonian operator will be defined for the solution processes. Consider the positive additive functional defined on the space of solutions, $\tilde{X}_t$:

$$\omega_{\Delta_s^T} = \int_s^T \sigma_t^{-1}(\tilde{X}_t) a_t(\tilde{X}_t, U_t) dW_t + \frac{1}{2}\int_s^T \left|\sigma_t^{-1}(\tilde{X}_t) a_t(\tilde{X}_t, U_t)\right|^2 dt \geq 0 \quad (3.193)$$

The Radon-Nikodym derivative *w.r.t.* the solution and transformed probabilities can then be written as:

$$\frac{dp^W_{(s,T)}}{d\tilde{p}_{(s,T)}}(\tilde{X}) = e^{-\omega_{\Delta_s^T}} = q_{(s,T)}(\tilde{X}) \quad (3.194)$$

Define the Lagrangian operator:

$$L_t(\tilde{X}, U) = \tfrac{1}{2}\int_s^T \tfrac{a_t(\tilde{X}, U) a_t^{\dagger}(\tilde{X}, U)}{2b_t(\tilde{X})} dt \quad (3.195)$$

Then an entropy functional, *S*, can be expressed as:

$$S\left(X, (s,T)\right) \overset{\text{def}}{=} \int_s^T M_{(X_s, [s,T])}\left[L_t(\tilde{X}, U)\right] dt \quad (3.196)$$

Optimization of this entropy leads to a "regularization to macro order" established for the random observations, X_t. This divergence can be viewed as a functional that is dependent on the diffusion operator, b_t and one writes $S\left(X, (s,T)\right) = S_{b_t}$. Next, one utilizes the variation minimax principle on S_{b_t}. The extremal trajectories of X_t will dictate the macrolevel process, which will produce a proper probability for the entropy functional, hence producing a proper entropy functional S_{PF} of the macroprocess. The canonical form of a *conservative system*, i.e., one in which $div[G_t] = 0$, is a Hamiltonian system obeying the Hamilton equations. Let (X_t, Ψ_t) be the phase space coordinates, where $\Psi_t = \dfrac{\partial S_{b_t}}{\partial X_t}$. Then the Hamilton equations are:

$$\frac{\partial X_t}{\partial t} = \frac{\partial \langle H \rangle}{\partial \Psi_t},$$
$$\frac{\partial \Psi_t}{\partial t} = \frac{\partial \langle H \rangle}{\partial X_t} \tag{3.197}$$

Together with the differential constraint condition, $C(X_t, \Psi_t) = \dfrac{\partial \Psi_t}{\partial X_t} + 2\Psi_t \Psi_t^{\dagger} \geq 0$, the macrodynamic model is regularized. Here $\langle H \rangle$ is the ensemble average value of H over the trajectories, X_t. The solution to this Hamiltonian system is:

$$\langle H(t) \rangle = \Psi_t^{\dagger} \frac{dX_t}{dt} = \frac{1}{2} b_t \frac{\partial \Psi_t}{\partial X_t} \tag{3.198}$$

At this point, a time discretization is performed so that the differential constraint condition and the Hamiltonian above, are imposed at a lattice of discrete time points. These points then define nodes and form an information network in time. Evolutional operators become matrices. Information transfer from microlevels to macrolevels is optimized when information loss in H is minimized (Lerner, 1998). Now, construct a double-coping feedback control mechanism (vector), v and conjugate vectors possessing a macro model process, $A(t)$ defined on the discretized points $t \in [t_1,,,t_n]$ as:

$$\begin{aligned}
&\Psi_t = \frac{A(X_t + v(t))}{2b_t}, \\
&A(t)v(t) = u(t), \\
&A(t) = \lambda_i(t), \text{ where } \lambda_i(t) = \alpha_i - j\beta_i,\ i = 1,\ldots,n \\
&v(t) = -2X_t
\end{aligned} \tag{3.199}$$

This leads to the formulation: $A(t) = \dfrac{b_t}{2\int_0^t b_t dt}$, which in turn, leads to the macromodel differential operator:

$$\frac{dX_t}{dt} = A(X_t + v(t)) \qquad (3.200)$$

Using a variational principle, if $\phi = \sum_n a_n \psi_n$ is a normalized wavefunction (mixed quantum state) and H the Hamiltonian, then $\langle \phi \mid H \mid \phi \rangle \geq E_g$ where E_g is the ground state (lowest energy state). Hence, if $\varepsilon(\Psi) = \dfrac{\langle \Psi \mid H \mid \Psi \rangle}{\langle \Psi \mid \Psi \rangle}$, then $\varepsilon \geq E_0$. This is the target for an approximation to a ground state and the variational minimax problem of finding a greatest lower bound ground energy.

Topoi and Categorification of Information Systems

In the development of every subfield in mathematics, a specific population of objects and relationships is normally built out of an axiomatic system of expression. Mathematical logic systems are not immune from this pattern. In an attempt to bridge this reoccurring phenomenon, category theory was developed by MacLane (1945) and others. Categories are objects that consist of general patterns for set-like objects and mappings called morphisms between these set-like objects. They are more general than the usual set-theoretic notions taught in elementary math. Moreover, they set patterns for all mathematical am dlogical systems. Appendix D encapsulates the basic elements of category theory leading to the specialized version called topos theory that emulates our physical notions of set theory. These will then be generalized for systems representing quantum gravity and information. These generalizations will then help in the overarching description of info-holarchies in this study.

Definition. Topoi (plural topos) is a mathematical generalization to "things" that possess the following structures:

1. an initial empty object, such as would be the null or empty set, $\varnothing$,
2. terminal objects which are akin to singleton sets,
3. colimits which generalize binary coproducts which, in turn, generalize to the disjoint union of sets,
4. limits which generalize binary products which, in turn, generalize to Cartesian products of sets,
5. equalizers which generalize to sets where mappings agree on,
6. coequalizers which generalize to quotient spaces so that for functions, $f, g : Y \to X$, $f(y)$ and $g(y)$ are identifiable,
7. set exponentiation in which the set of all functions mapping one set to another is identified,
8. generalized subset functions mapping a set into $\{0,1\}$ which is a way of identifying its subsets and when generalized maps into a more general truth value space, Ω which may consist of more than FALSE and TRUE values, such as in fuzzy and probabilistic, and degrees of belief systems (Baez, 2006).

Presheaves are now to be defined as categorical structures needed to represent physical systems in quantum gravity and fields. A *presheaf* is a covariant functor, f , that maps a category, C to the set category, SET. Let Set^C denote the category of such presheafs on C with the following behavior for its maps:

$$\begin{tikzcd} X(A) \arrow[r, "X(f)"] \arrow[d, "N_A"] & X(B) \arrow[d, "N_B"] \\ Y(A) \arrow[r, "Y(f)"] & Y(B) \end{tikzcd}$$

Figure 12. Presheaves

where X and Y are presheaves of C, N_A and N_B are natural transformation between X and Y such that for A, B objects in C and f an arrow in C, the above diagram commutes. Intuitively, the maps N_A and N_B give a notion or "picture idea" of what X is within Y for objects in C (Isham, 1999). There is a more functional definition of a presheaf on C using the useful notion of sieves. Define a *sieve* on an object of C, A, as a collection, $\mathcal{S}$ of arrows, $f : A \to B$ in C with the following property: if $f \in \mathcal{S}$ and $g : B \to C$, an arrow in C then the composition arrow $g \circ f : A \to C$ belongs to $\mathcal{S}$. Sieves draw in arbitrary arrows in composition. A presheaf can now be defined as:

Definition. A map $\gamma : C \to \mathcal{S}et$ is a presheaf on C if for any A an object in C with $\gamma(A)$ being defined as the set of all sieves on A, and if $f : A \to B$, then $\gamma(f) : \gamma(A) \to \gamma(B)$ is defined as $\lambda(f)(S) = \{h : B \to C \mid h \circ f \in S\}$, $\forall S \in \gamma(A)$.

A consequence of these definitions is the following:

Definition. The set of all arrows in C whose domain is an object $B \in dom(C)$ is called the *principle sieve* on B denoted by $\uparrow B$ and can be described as:

$$\gamma(f)(S) \equiv \{h : B \to C \mid h \circ f \in S\} = \{h : B \to C\} \equiv \uparrow B$$

The set $\gamma(A)$ of sieves of A has the structure of a Heyting algebra and hence can represent a general distributive lattice with null and unit elements that is relatively complemented. Heyting algebras, in turn can then represent an intuitionistic semantic logic structure first developed by Kripke (1965), generalizing a Boolean algebra and logic for quantum and fuzzy structures. More to this point, it has been shown that quantum probability logic structures are special cases of fuzzy probabilistic logic structures (Pykacz, 1994; Mesiar, 1995). This will be revisited after a review of a concept of generalizing uncertainty and its expressions and language representations.

To tie into the previous review of the physical discrete model to be used in this discussion for information systems, namely that of spinfoams and spin networks, a final aspect of category and topos theory with respect to topological models of quantum physics, topological quantum field theories (TQFT) will be presented. Define an *n*-dimensional manifold, $M : S \to S'$, that describes that portion of spacetime between two (*n-1)*-dimensional manifolds representing two different spacetime states of a system, S and S'. M is called a *cobordism* from S to S'. M acts as a time surrogate operator in the sense of being representative of the time that has passed between the spacetime states S and S'. With this definition of state transition, time is a topological change. The set of cobordisms between S and $S^{'}$ form a noncommutative group under the operation of composition. This group represents the various ways in which a time-like sequence can proceed between states in spacetime. In the following, Baez is used in the discussion of spinfoams, TQFTs and their category theoretic aspects (Baez, 1999). Spinfoam models represent versions, albeit promising models of TQFTs for quantum gravity, the attempt to unify general relativity (GT) and quantum mechanics (QM). TQFTs map structures from differential topology, i.e., smooth manifolds, and hence structures from general relativity to corresponding structures in quantum theory.

Cobordisms display something about how the macrodynamics of large scale relativistic objects react or relate to the microdynamics of small scale quanta. TQFTs map Hilbert spaces of the spacetime state vectors of a spacetime manifold S, denoted by $H(S)$ to S. In doing so it maps Hilbert spaces to other Hilbert spaces representing two different manifolds of spacetime. It uses the cobordism between two spacetime manifolds, $M : S \rightarrow S^{'}$ to do this assignment. Let $H(M) : H(S) \rightarrow H(S^{'})$ denote this mapping so that if ψ is the state of S. Then $H(M)\psi$ is the transformed state of $S^{'}$. This mapping of the TQFT preserves compositions and identity of the cobordisms. In 4-dimensional quantum gravity represented by spinfoams at the Planck scale, the network is separated by tetrahedra with faces that represent the connecting spacetimes S and $S^{'}$. The topological mapping, $H_{SF}(M) : H(S) \rightarrow H(S^{'})$ corresponding to the spinfoam TQFT is defined via transition amplitudes for each cobordism, M, between S and $S^{'}$. These complex-valued amplitudes are summed up over all such tetrahedron to get the total transition amplitude from a state ψ of S to a state ψ' of S'. By requiring that $H(1_S)$ be the identity in the subspace $H(S) \subset S'$, then the spinfoam TQFT mapping preserves a subspace identity. This TQFT mapping is preserved without regard to how the tetrahedra are developed. It is invariant with respect to this discretization.

In 4-D quantum gravity, the triangle faces are tetrahedra and the tetrahedra are hyper-tetrahedra. Denote the topoi of cobordisms between *(n-1)*-dimensional manifolds by ***nCob*** and that of Hilbert spaces by ***Hilb***. Then the TQFT $H : nCob \to Hilb$ is a functor between the two topoi. Most importantly, a TQFT maps aspects of quantum theory with those of geometric spacetime. To consider the structure of binding these two paradigms of physical reality one uses the relations between these functors, that is, functors of TQFTs and other functors of morphisms between physical systems.

Consider now an iterative process of defining functors of functors, that is, a continuing categorification of functors, leading to *n*-categories. This iterative categorification leads to a way of examining higher order structures of maps between system models, in this case, those of quantum theory and general relativity. This can be generalized further to investigate relationships or maps between any two formal systems of logic, set theory, or component systems in general. This would include the specifics of HMASs and other complex adaptive multiagent systems. In other words, a formalism of a general system can be represented as a system topoi, denoted by **Sys**, under appropriate operations acting on subsystems of and component systems within it (Doring & Isham, 2008). Propositional and higher order type languages dependent on the system *S*, denoted by $\mathcal{PL}(S)$ and $\mathcal{L}(S)$ respectively, can be attached to such topoi in order to give them the rigor of formalized deductive logical systems such as the intuitionistic and paraconsistent logics mentioned before. Proceeding up the ladder of *n*-categories of an original system, Sys_m of dimension *m*, each successive higher level of morphisms and objects adds one dimension. For example, a *0*-category would be a regular set. The 1-category above this would be a category with the sets as objects and set functions. The 2-category above this would be a 2-category of morphisms of set functions and in so doing would define how those set functions in the 1-category are isomorphic. These successive constructions continue upward. Hence, in order to more closely study the structure of an *n*-category, one must go to the *(n+1)*-category. In the study of physical systems, proceeding as such to the next categorification leads to a clearer picture of the morphisms of the prior physical system. This is the key to melting together a comprehensive structure for quantum gravity from QM and GR, and hence, in general between any two representations of reality or of systems, including logic information systems. This is a tool for the study of information systems as field and particle theories in this discussion. Information is influenced and driven both by quantum and geometric relativistic effects.

Generalization of Uncertainty

In the preceding discussions on information measures, entropy, and complex adaptive systems, uncertainty was introduced most ostensibly through a probabilistic axiomatic system. In this section a review of an attempt to generalize concepts of uncertainty that subsume probability will be done. This construct will then be applied in an analogous attempt to generalize uncertainty in quantum probability and quantum systems in the next chapter.

Zadeh has given a proposal for a generalized theory of uncertainty (GTU) in which notions of uncertainty including: (a) probabilistic, (b) possibilistic, (c) veristic, (d) usuality (fuzzy probability), (e) random, (f) fuzzy graphic, (g) bimodal, and (h) group types of uncertainty are modeled through a generalized constraint model. Complementatry to this, a generalized constraint language (*GCL*) consists of all generalized constraints coupled with the rules for qualification, combination, and propagation. A generalized constraint (*GC*) is a triplet of the form (X, r, R) where *X* is a constrained variable, *R* is a constraining relation, and *r* is an indexing variable which identifies the modality or type of constraint semantics (Zadeh, 2005). The index list consists of the following pneumonic: $r = blank$, possibilistic, $r = p$, probabilistic, $r = v$, veristic, $r = u$, usuality, $r = rs$, random set, $r = fg$, fuzzy graph, $r = bm$,bimodal, and $r = g$, group variable. A formal uncertainty language such as a *GCL* calculates precisiations (the mapping of a vague measure into a precise number) more readily than formalized logics. Constrained variables, *R* can take the form of: (a) a general *m*-vector, (b) a proposition, (c) a function, (d) a function of another variable, (e) a conditioned variable, (f) a structure, (g) a group variable, or (h) another generalized constraint. Bi-valent conjunction, projection, and propagation operators, $\otimes_c, \otimes_{proj}, \otimes_{prop}$ respectively act on two (possibly different) *GC* objects, $(X_{k_1}\ is\_i_1\ R_{j_1}), (X_{k_2}\ is\_i_2\ R_{j_2})$ to generate a third (possibly different) GC object $(X_{k_3}\ is\_i_3\ R_{j_3})$.

A *GC* object, $g = (X, r, R)$, is associated with a test-score $ts_g(u)$ which associates an object *u* (which the constraint is applicable to), a degree to which *u* satisfies the constraint. The test score defines the semantics of the constraint that is associated with *g*. The value of $ts_g(u)$ may be a point in the unit interval, $[0,1]$, a vector, or other mathematical structure such as a member of a semi-ring, lattice, poset, or bimodal distribution. The relation, *R* from *g* is allowed to be nonbivalent, as in a fuzzy equivalence. In this way, a *GC* generalizes a fuzzy set and so, a *GCL* can lead to a generalized fuzzy system of generalized constraints.

Zadeh presents a precisiation natural language (*PNL*) as a means of assigning precise meaning to a proposition drawn from a natural language (*NL*) through a GC. The *PNL* is then a mapping, $\Gamma_{PNL} : p \to g$ from a proposition, *p* to a *GC*, $g = (X, r, R)$. Hence, information, in general, is representable as a *GC* because a proposition is a carrier of information, being a potential answer to a question. Let *S* be a system. Let S_{P} be the space of all propositions in *S*, S_{Υ} the space of *GC*s in *S*, and $\Gamma_{PNL}(S)$, the mapping assigned to a *PNL* for *S*. Then $\Gamma_{PNL}(p) \in S_{\Upsilon}$ for a precisiable proposition *p* in *S*. Denote the space of precisiable propositions of *S* by $S_{\mathrm{P}}^{'}$. In general, $S_{\mathrm{P}}^{'} \subset S_{\mathrm{P}}$ for *NL* systems. Let S_{GCL} be the space of all *GC*s of *S*. Then S_{GCL} is more expressible relative to *S*, than a first order logic, modal logic, Prolog, and LISP is to *S*, if *S* is a *NL*. Because quantum logics can be derivable as a family of subsets of fuzzy probability structures (using Lukasiewicz operators), a *GCL* can be formalized for it, though quantum probability may be framed as a generalized probability theory as well (Cohn, 2007). The importance of this is that any quantum logic (logical system), *L*, with an ordering set of probability measures, *S*, are isomorphic (representable) in the form of a family of fuzzy subsets of *S*, $\mathcal{L}(S)$, satisfying certain conditions, including the use of Lukasiewicz operators instead of Zadeh's operators on fuzzy sets (Pykacz, 2007). Hence *L* is representable by a *GCL*. In this sense, quantum logics are special cases of (and isomorphic to) fuzzy probability logics and so are in the realm of a *GCL* representation.

Does this *GTU* represent a kind of generalized logic in the taxonomy of algebraic logics? In other words, can the GTU transcend a spectrum of the algebraic hierarchy of logics, which include fuzzy and quantum logics, and of other duals to these, notably referred to as dual-intuitionistic logics or paraconsistent logics? Paraconsistent logics are logic systems that formalize inconsistent nontrivial logics in the sense of the rejection of the principle of explosion (noncontradiction), the premise that anything follows from contradictory premises (Béziau, 2000). The principle of explosion is as follows: for a proposition *p*, and an arbitrary claim *A*,

$$
\begin{aligned}
&p \wedge \neg p \text{ (premise)} \\
&p \text{ (conjunctive elimination)} \\
&p \vee A \text{ (weakening for any A)} \\
&\neg p \text{ (conjunctive elimination)} \\
&A \text{ (disjunctive syllogism)} \\
&\Rightarrow A \text{ (conclusion)}
\end{aligned}
$$

Paraconsistent logics overcome Gödelian limitations, i.e., incompleteness of axiomatic systems. In addition, because they are accepting of the truth or falsehood of both a premise and its negation, they are flexible in overcoming other seemingly paradoxical physical theories such as the quantum nature of long range gravitational influences or macroscopic and mesoscopic entities. In this regard, this discussion places interest in this aspect of a paraconsistent logic in the formation of a new physical information theory based on possibly inconsistent, but nontrivial fields and particles of information.

In the following discussion, a topoi is developed for fuzzy sets. By using the *GC* structure, a topoi for a *GC* can be built around this procedure . The general direction given by Plotkin will be followed for the Higgs topos, *SET(H)* defining a fuzzy set-theoretic topos (Plotkin, 1994, pp. 108-112). Let *P* be an arbitrary linearly ordered set with first element 0 and last element 1. *P* equipped with such elements can be considered a lattice and in particular a special algebra called a Heyting algebra which is a sound and complete multilogic intuitionistic and fuzzy (IF) generalization to a Boolean algebra (Clote & Schwichtenberg, 2000). IF logics are a basic form of more general logic structures based on *t*-norms. A *t*-norm is a function, $t:[0,1]\times[0,1]\rightarrow[0,1]$ satisfying the axioms:

1. commutativity, $t(a,b)=t(b,a)$,
2. monotonicity, $t(a,b)\leq t(c,d)\Leftrightarrow a\leq c$ and $\mathrm{b}\leq d$,
3. associativity, $t(a,t(b,c))=t(t(a,b),c)$, and
4. existence of an identity, i.e., $\exists$ element 1_t, $\ni$ $t(a,1_t)=a\ \forall a$.
5. continuity in $[0,1]\times[0,1]$, although left-continuity suffices for most fuzzy systems.

The structure $H_t=\left[[0,1],t,1_t\right]$ is then defined as a commutative totally ordered monoid. More deeply, the *t*-norm induces a natural residuation, denoted by $\rightarrow_t$, so that the updated residuated structure, $H_t^{'}=[H_t,\rightarrow_t]$, becomes a commutative naturally ordered residuated monoid, also known as a *hoop (*Agliano, Ferreirim, & Montagna, 2007). A *residuation*, denoted by the symbol $\rightarrow_t$, is a binary operation defined

as: $\forall x, y \in [0,1],\ x \rightarrow_t y = \sup_{z \in H_t'} [z \mid t(z,x) \leq y]$. The residuation, $\rightarrow_t$ is the pointwise largest

function such that:

$$\forall x, y,\ t(x,(x \rightarrow y)) \leq y$$

the *t*-norm logic version of the modus ponens rule of inference for logic systems. Recall that the *modus ponens rule of inference* for classical logic states that:

If P then Q.
P is true
therefore Q true

The residuation is therefore the weakest function that would imply that its generated *t*-norm logic structure has a valid truth function for implication in a generalized fuzzy logic system. It legitimizes the *t*-norm generated hoop structure as a functional fuzzy logic system. An equivalence property of a residuation is: $\forall x, y, z \in [0,1]\ \ t(x,y) \leq z$ if and only if $\mathrm{x} \leq \mathrm{y} \rightarrow_t \mathrm{z}$. This is a generalization of the two-valued (Boolean) logical conjunction.

Using this general notion of a *t*-norm based logic, multivalued logics are formed. Fuzzy systems are one such specialization of these *t*-norm systems. An *n*-ary propositional connective is a function, $F_c : [0,1]^n \rightarrow [0,1]$ that generalizes the *t*-norm for multiple propositional operations. For generalizing Heyting algebras, a *t*-norm can be introduced to generate semilattice structures, the so-called *t*-norm algebras. More specifically, in a Heyting algebra, H, (i) every pair of elements, (a,b) has a pseudo relative component (*rpc*), that is, a greatest element, c in the ordering of *H* such that $a < c \leq b$ where $<$ and $\leq$ are the ordering and strict ordering operators of *H* respectively, (ii) *H* possesses a zero element, and (iii) *H* is a lattice. Define a residuation operation $\rightarrow$ on *H* by:

$$p \rightarrow q = \begin{cases} 1 & \text{if p} \leq \text{q,} \\ q & \text{if } p > q \end{cases}$$

Equiping *H* with this operator results in an *rpc* lattice. Let Ω be the universe of discourse space of truths, which could be the interval $[0,1]$. Define $H = ([0,1], \leq)$ where $\leq$ is a natural ordering. Then H is a *complete Heyting algebra* because greatest upper and lower bounds exist for every subset of *H*. The definition of fuzzy equality of elements will be defined. Assume that $H = (\Omega, \leq)$ is a complete Heyting algebra. Let *A* be a set, which in a category is an object of the category of a set, SET. A *fuzzy* Ω-equality *i*n *A* is a mapping, $A \times A \rightarrow \Omega$, denoted by $(x, y) \rightarrow [x \approx y]$, where $[x \approx y] \in \Omega$ such that the inequalities (i) $[x \approx y] \leq [y \approx x]$ and (ii) $[x \approx y] \wedge [y \approx z] \leq [x \approx z]$ hold for every $x, y, z \in A$. The pair $(A, [\approx])$ is then a fuzzy set. Let $E(x) = [x \approx x]$ for each $x \in A$ and define an equivalence operator, $\cong$ by $[x \cong y] = [E(x)] \vee [E(y)] \rightarrow [x \approx y]$. In this sense a fuzzy equality is the same as a fuzzy equivalence relation.

The Heyting algebra, $H = (\Omega, \approx)$ can be regarded as a fuzzy set with the membership map $\Omega \times \Omega \rightarrow \Omega$ given by $[p \approx q] = p \rightarrow q$,for $p, q \in \Omega$ and $p \rightarrow q = (p \rightarrow q) \wedge (q \rightarrow p)$. Now define a category using $H = (\Omega, \leq)$. Let $\mathcal{A} = (A, \approx), \mathcal{B} = (B, \approx) \in \Omega$-set and define a morphism $f : \mathcal{A} \rightarrow \mathcal{B}$ as a mapping $f : A \times B \to \Omega$ satisfying the conditions:

1. $[x \approx x'] \wedge f(x, y) \leq f(x', y)$,
2. $f(x, y) \wedge [y \approx y'] \leq f(x, y')$,
3. $f(x, y) \wedge f(x, y') \leq [y \approx y']$,
4. $[x \approx x'] = \bigcup_{y \in B} f(x, y)$

for $x, x' \in A$ and $y, y' \in B$. Use the notation, $f(x, y) = [f(x) \approx y]$ to give a measure of the equality potential of $f(x)$ and y. Next, define the equality of morphisms. Let the morphisms, $f, g : \mathcal{A} = (A, \approx) \to \mathcal{B} = (B, \approx)$ be equal if their respective "fuzzy graphs", $f, g : A \times B \to B$ coincide. Define the composition of the arrows $f : \mathcal{A} = (A, \approx) \to \mathcal{B} = (B, \approx)$ and $g : \mathcal{B} = (B, \approx) \to C = (C, \approx)$ by a mapping $g \circ f : \mathcal{A} = (A, \approx) \to C = (C, \approx)$ where: $g \circ f(x, z) = \bigcup_{y \in B} f(x, y) \wedge g(y, z)$ for $x \in A, y \in B, z \in C$. This composition implies $\exists y \in B$ such that $f(x) = y$ and $g(y) = z$. It has been shown that $H = (\Omega, \approx)$ is a topos (Higgs, 1973).

The GTU employs a formalism that accommodates known fuzzy, probabilistic, and classical uncertainty measures. A question arises about how one could or should compare each type of uncertainty. Klir (2006, pp. 387-388) proposed the notion of the principle of information invariance. This is a study in the space of transformations between the various frameworks of uncertainty. In order for one theory of uncertainty to be comparable to another a space of invariant transformations must be found between the two. Geometric and topological evolutions between the two may then be found in a quest to compare informational prowess of each. A notion of generality can also be constructed based on these transformations. More specifically, let T_1 and T_2 be two uncertainty theories. Suppose that T_2 is less general than or incomparable to T_1. Now let $u_i, i = 1, 2$ be corresponding uncertainty functions of the respective theories. These functions measure the uncertainty of a given phenomena to be estimated. The function, u_2 will be used to estimate the function, u_1.

The principle of information invariance then posits that the amounts of uncertainty in u_1 and u_2 must be the same or to an approximate equality. Let F be a transformation from T_1 to T_2 and let $U(u_i)$ be the amount of uncertainty measured for function u_i. Then for F to be $\varepsilon-$invariant *w.r.t. U*, $\left\| F(U(u_1)) - F(U(u_2)) \right\|_T < \varepsilon$, for some sufficiently small $\varepsilon > 0$ and suitable metric norm, $\| \; \|_T$ on the space of transformation. In an ensemble theory of uncertainty, multiple runs of an experiment are collected so that *n* trials are present. If both theories employ ensembles, then ε will depend on the sample size, *n*. Hence, one can utilize an asymptotic process and then $\left\| F(U(u_1^n)) - F(U(u_2^n)) \right\|_T < \varepsilon_n \to 0$ for comparable theories. Examples of these comparisons are done between probability measures and graded possibilities and others in Klir (2006, pp. 390-398).

General Semiotics

To this point Shannontype information theories have been reviewed and discussed as the foundation for objectively measuring information content and flow in channels.

The approach of Shannon and others in that genre have concentrated on the physical transformation of data. This definition of information does not approach the problems of semantics, linguistics, and pragmatics of information. Indeed, the receiver has not been given any avenue for feedback or interpretation on the stream of information involved. This is an incomplete framework for a generalized theory of information and prevents a systemic definition of information from being formed (Callaos & Callaos, 2002).

Admittedly, a relativistic measure, semantics plays an important role in the transformation of information from one quasi-mind to another. Here the term *quasi-mind* is used as a generalized computing device that is a self-aware system (SAS). Human brains and their consciousness are an example of such. This study will discuss other even more powerful examples of quasi-minds. Without a measure of semantics, no meaning can be attached to an information stream from source to receiver. Additionally, the pragmatics of such streams is left waning for the receiver interpretation of purpose. With these dilemma came the study of semiotics. Semiotics has been obligatorily defined as the study of generalized signs. Signs are a placeholder for the conveyance of meaning between a source and receiver. Specifically, de Saussare first defined a sign as consisting of two components, a significant and a signifier (de Saussure, 1916). The significant is the thing that stands for something other than itself. A signifier is the mechanism that gives meaning to the representation of the signifant. This is a dyadic representation of semantics. Injecting more rigor into the idea of signs, Peirce introduced a triadic system of signs. In his model, a sign or semiotic triad consisted of (a) an icon or representamen, (b) an object, medium or index, and (c) an interpretant or symbol (Peirce, 1931). Peirce categorized entities according to these three components of a triad as having the properties of (a) Firstness - representamen, (b) Secondness - object, and (c) Thirdness - interpretant.

In this triad, a representamen, denoted as R_i where *i* depicts the ith triad, is an entity that represents another entity as a potentiality. The object, denoted by, O_i, is an instance or actualization of that representamen. Finally, the interpretant, I_i is the manifestation or patternizer of the representamen, a metapattern for the sign of the traid.

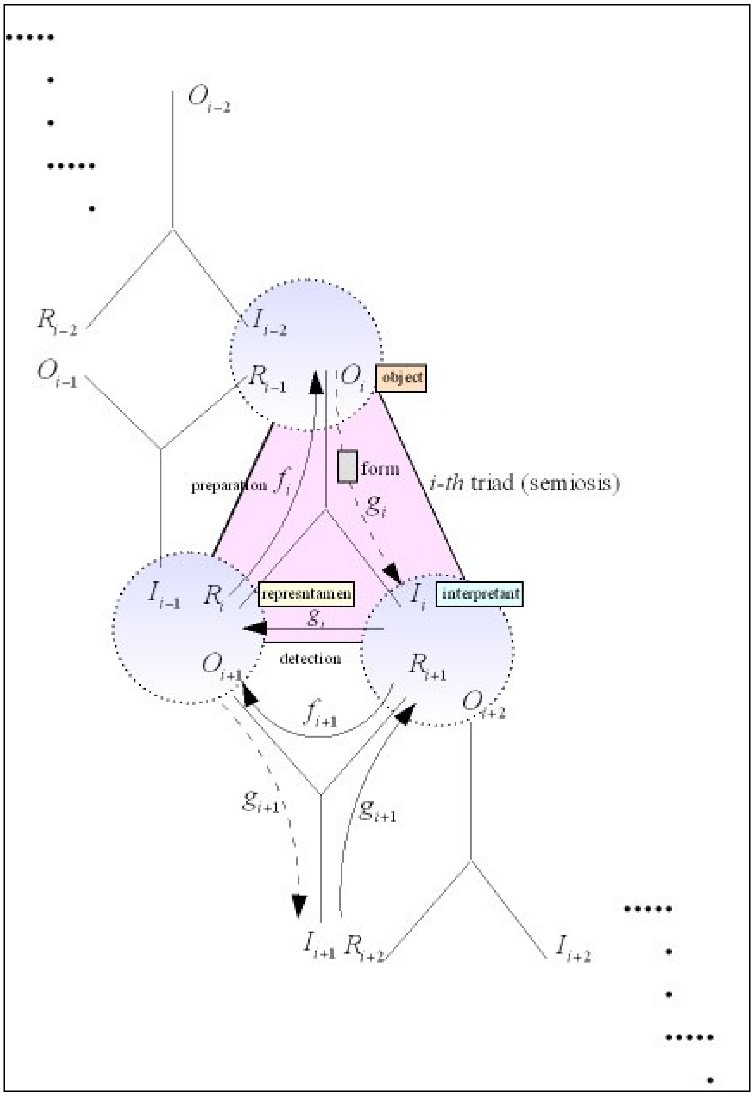


Figure 13. Semiosis chain

Peirce viewed each of these entities as processes rather than structures. A triad in isolation has no particular importance because an information piece is propagated through a chain of semiosis or semiotic chain which is a sequence of correlated semiosis triads. One triad is linked to another by the following transformation: the interpretant of the *i-th* triad, I_i is the representamen of the *(i+1)-th* triad, R_{i+1} and the representamen of the *i-th* triad, R_i is the object of the *(i+1)-th* triad, O_{i+1}. Two triad functions are defined, f_i and g_i. The function f_i acts as the preparation operator for creating an object instance from a representamen. The function g_i serves as a detection operator for developing an interpretant, the idea presented to the quasi-mind, the interpreter of the sign via a form pattern from the object. This form entity is a pre-pattern that is auxiliary to the object instance in forming an interpretant.

Semiotic models have been developed for biological systems (Queiroz, Cklaus, & El-Hania, 2007). These may be generalized to nonliving entities including quasi-minds such as digital processors. Semiotic models may also be built for quantum systems in the following manner. In a general quantum system, the representamen is given by the Schrödinger equation:

$$i\hbar\partial_t\psi(t) = H_0\psi(t) \tag{3.201}$$

where H_0 is the Hamitonian operator corresponding to the energy of the system and $\psi(t)$ is the system state variable at time *t*. Alternatively, the Heisenberg equation may be used:

$$\frac{d}{dt}A(t) = -\frac{i}{\hbar}\left[A(t), H_0\right] \tag{3.202}$$

where *A* and H_0 are operators defined over a Hilbert space $\mathcal{H}_0$. Here the operator, *A* evolves in time. Both equations are governed by probabilistic dynamics and as such play out the potential for the quanta. They are therefore the representamen for the quantum system. The measurement operators, given by $\{M_\omega\}_{\omega\in\Omega}$ acting on the quantum system serve as the object of the semiosis triad since they instanciate a realism for the quanta. Finally, the set of eigenvalues, the eigenspectrum $\{\lambda_i\}$ corresponding to the quantum eigenstates $\{|\varphi_i\rangle\}$ of the system generalize or present with a metapattern for the quantum system, the interpretant.

In this study an abstract physical basis for generalized information will be constructed based on (1) an information particle, referred to as an informaton, (2) an accompanying information field theory, (3) an information process described by an Itô diffusion, under the premises of generalized uncertainty (GTU), and (4) a new causal framework for probability—the causaloid. In contrast to this, in situation-semantic studies of information, relative situations involving a physical condition in an exchange of information using natural languages between entities are analyzed (Barwise & Perry, 1980; Barwise, 1989). Attempts at representing (situational) information as abstract nonphysical particles have been attempted, most notably by Devlin (1995) through concept objects called *infons*. In situational logic, object types are given by individual entities, a_i, relations between these individuals, *R*, spatio-temporal coordinates, (x,t), situations, *s*, and *v*, a representative of classical truth values, T or F. Situations, $s=\langle R,\{a_i\},(x,t)\rangle$ are given by a relation, *R* between individuals, those individuals, $\{a_i\}$ and a spatio-temporal coordinate, (x,t). Each of these type objects can have parameter spaces, basically specific instantiations of these types. Formally, *x* is of type *T*, written as $x:T$.

Infons are then represented as tuples, $\phi = \langle\langle R, \{a_i\}_{i\in I}, v\rangle\rangle$. Accordingly, an infon, ϕ states that among the individuals $\{a_i\}$, the relation R holds or does not hold according to the truth value *v*. Infons are categories for situations. While there may be abstract and concrete types of situations (i.e., nonactualized versus actualized) for infons, they are treated as concrete. The situation *s supports* the infon, ϕ, written as $s \models \phi$, if $v = T$ in ϕ when *s* dictates the individuals and relation in ϕ. Essentially *s* factualizes the infon, ϕ. There is an accompanying infon logical structure that satisfies closure under conjunction, disjunction, and bounded quantification over its parameter elements. Negation is not closed under infon structures. Infons are semantic animals that have situations as support mechanisms. Unless there are physical communication channels between infons, they do not have physical analogies.

This abstract definition of information is where this study's version of information particles— informatons, parts from. However, the advantage of the use of infons is in their independence from language or coding schemes. Indeed, an infon, ϕ, is said to be a *fact* $\Leftrightarrow \phi$ has been actualized or is real. For a real situation, *s*, one then has that $s \models \phi$. However, what is judged to be real in the universe, if not by physical or sensorial manifestation? In this study, informatons are by contrast, described from first principles of nonclassical physics and both subjective (observer-interpretive) and objective information theories. The descriptions of both semiotic chaining in Peircean (1931) semiotic logic and Devlin (1995) situational logic infons, can then be augmented to the structure of informatons, thereby correlating observer-based meaning and situations to the objectivity of informatons in quantum gravity. This will be investigated and constructed in chapter 4.

Stonier (1990, 1992, 1996) in his trilogy on information physics proposed a basis for the existence of information particles he unfortunately also referred to as infons. Stonier's infons differ dramatically from Devlin's in that while both versions are hypothetical entities, Stonier uses analogies of energy and matter particles, bosons and fermions respectively, to propose his information particle—the Stonier infon. Stonier refers to infons as massless particles that are pure forms of information. Stonier extensively utilizes interesting analogies—heat is to energy what organization is to information. However, information is not directly defined in his infon structure except as transient, appearing in and out of existence depending on the speed at which other particles, theoretical gravitons, photons, etc., are propagated at (Stonier, 1990, 141-145). According to Stonier, his infons are trans-convertible between fermions and bosons. Nonetheless, no mention of fields of information is made or developed. In this study, informatons have structure, inhabit all of spacetime, and are given a field-theoretic basis, as well as a physical content transcending quantum gravity. Their dynamics are also described mathematically and applied to form a theoretical foundation for organization evolution. Informatons are persistent and exist as the basis of information flow for particles and forming organization. This information flow dictates energy-mass-force.

Evolutional Information

In an intriguing treatise on the universality of information as a means of describing all structure and process, Gershenson (2010) proposed eight axioms or laws of information. These eight laws together develop a picture of information as a universal language for constructing and describing the world and its dynamics. Although Gershenson does not provide a rigorous or technical definition of information, he uses it as a lingua omnimodus to propose building all phenomena. In this study, a similar notion will be posited with technical details on the composition of a particulate of information. Firstly, we review Gershenson's notions for this universality of an information vocabulary.

Gershenson begins by using a definition of information and its observers from Umwelt (von Uexküll, 1957), expanding them to fit biologics and cognition, resulting in the construction of five overlying notions of abstract information and its carriers:

(1) *information* is anything that an agent can sense, perceive, or observe,

(2) an *agent* is a description of an entity that acts on its environment

(3) the *environment of an agent* is the sum total of all the potential information it can perceive

(4) *living information* is the ratio of information created by itself over information created by its environment

(5) a system is cognitive if it knows something

Using these fundamental notions, Gershenson then defines his two main categories of information: (1) first-order information – that information which is perceived directly by an agent, and (2) second-order information – that information which is perceived indirectly by an entity through other entities. Here we generalize this notion by simply defining information of *order n* as information perceived by an agent indirectly through a chain of *n-1* prior information perceptions carried in sequence leading to the direct perception to that agent. Additionally, this study describes this notion more technically through the concept of the info-holarchy – each holonic level in the info-holarchy perceives first order information within its own level. All other perceptions are of higher ordered information with the order dictated by the number of levels needed to propagate and capture that information. Gershenson continues by classifying types of transformations of information into the following four groups:

(1) dynamic – information which changes itself – objective-internal change
(2) static – information which is perceived by an agent where the agent changes, but the information does not – subjective-itnernal
(3) active – information which is perceived by an agent in which the agent changes the information – objective-external
(4) stigmergic – information which is perceived by an agent in which the agent makes a change to the information before propagating it to other agents – subjective-external (intersubjective)

These are the laws of information transformation. The laws of information propagation are as follows: (1) autonomous – most information is self-propagated, (2) symbiotic – different information cooperate, (3) parasitic – information exploits other information for its own propagation, and (4) altruistic – information promotes propagating other information. Obviously, a random information chunk could have a combination of these traits to varying degrees, under varying environments. Gershenson's third law of information is that of requisite complexity – the ability to propagate succeedingly more complex information evolutionarily.

The fourth law is that of information criticality – the balancing between information stability and variability, in terms of nonlinear dynamics, that of approaching the *edge of chaos* regions in phase space, producing self-organized criticality or noise complexity. The fifth law is posited as the law of information organization – the propensity to produce self-constraints when building structure in order to form organization that is advantageous to the pooled structure. Organic systems display such properties in homeostatis. The sixth law of information is that of self-organization – the propensity to organize in the most preferred or highly probable states. This law is akin to the attractor basins in nonlinear dynamics. The seventh law is potentiality. This law promotes the idea that agents of information may produce diverse amounts of perceptions, i.e., different meanings to the same information. Evolutionarily, this may mean that the agent's mutation and combination operators act on information to produce different spectrums of meaning. The eighth law of information is that of perception – each agent possesses a unique repertoire of operators to apply to some information in order to produce a unique spectrum of meanings.

Gershenson posits that these laws generalize evolutionary processes which include cognition and life dynamics. Scales at which information is perceived are important because as the spatio-temporal scale decreases, information increases while as the scale increases, information decreases. More succinctly, let $A(I_s)$ be the amount of information potentially present at spatio-temporal scale *s*. Then

$A(I_s) \underset{s \to 0}{\to} \infty$ and $A(I_s) \underset{s \to \infty}{\to} 0$. In other words, to occupy the whole universe, is to have no information, while to occupy nothing means to know all information. In terms of an info-holarchy, descending down levels one gains information potential, while ascending up levels one loses information potential.

This is a deceiving and oversimplistic viewpoint. Holarchies are self-similar. Additionally, a holon-agent in one holonic level is relativistic, i.e., another view of the info-holarchy produces a different holonic level occupied by that holon-agent. Hence the scale may be relative to the process holonic level view, so the amount of information potential is relative - changing or gyrating similar to a gestalt perception. Theoretically, through the notion of semiotic-chaining, a holon-agent or more precisely, an informaton in an info-holarchy, can be succinctly part of an *info-chain* that spans from Planck-scale levels to super-galactic or edge-of-universe levels, both physically and semiotically. In info-holarchies, holon-agents and their information are part of the same entity, the informaton – consisting of an effective particle dual-pair, an observer entity, *o*, and an entity that generates information fields, *e*. This structure attempts to describe information in terms of physical dynamics without a separate syntactic interpretation or license of what information is in relation to other characteristics of matter, energy, perception, cognition, or organics.

Asymmetry as a Unification of Information Concepts

The various concepts of information as put forth by Shannon, Kolmogorov, Chaitin, Solomonoff, Carnap, Von Mises, Jaynes, and others, while holding to the general dictum of counting states of existence, nonetheless are different, emphasizing varying points of contention. In an attempt to unify these concepts Muller (2007) proposes their unification using the symmetry groups of transformations acting on an information set. This achieves a central concept of asymmetry as a common description of information. One starts with Muller's description of a system, *S* that is capable of distinguishing states of an object, *Q*; his information gathering and using system (IGUS):

Definition: Let Θ_Q denote the state space of an object Q, $\Phi_{Q_{t_0}|S} \subseteq \Theta_Q$ denote the set of states of *Q* that can be discerned by a system, *S* at time t_0, and $\Phi_{Q_{t_1}|S} \subseteq \Theta_Q$ denote those states of Q that are discerned by *S* at time t_1. Then *S* is an IGUS iff $\Phi_{Q_{t_0}|S} \neq \Phi_{Q_{t_1}|S}$ (Muller, 2007, p. 950-57).

IGUSs are capable of distinguishing states of an object and hence of conceptually measuring information of that object. This is a subjective definition of a distinguishing system, nonetheless it posits when a system is fine-grained enough to separate and distinguish information events. Finally, Muller using an application of the Cauchy-Frobenius Lemma, Frobenius (1887), to the finite group of symmetric transformations, *G* acting on a finite object set, Q, quantifies information content with respect to an IGUS *S*:

$$I(Q) = \log\left(\sum_{g\in G}\left|Q^g\right|\right) - \log\left(\left|G\right|\right), \qquad (3.203)$$

where Q^g is the subset of points $q \in g$ such that $g(q) = q$,i.e., the fixed points of g on Q (Muller, 2007, p. 1128-33,1128-34). This equates the log of the number of orbits of Q acted upon by G to the information on Q. Symmetries on a set show how information can be transformed invariantly, i.e., indistinguishable states. Hence, the universal measure of information of an object set would be the asymmetries on that object set. This is a unifying feature of probabilistic and algorithmic-based information complexity measures. This concept can be applied to quantum and GTU based information measures in an analogous manner, replacing the classical group symmetries, G with the group symmetries of the Hilbert operator transforms or of the more general GTU operators.

Review of Methods

This study and theoretical development of a novel model for information and organization was constructed by utilizing hybrid grounded theory. It included developing abstract mathematical concepts based on generalizations to prior models reminiscent of theoretical mathematics and physics. This development was achieved through the use of induction and the expansion and generalization of models from contemporary studies in information theory, complexity sciences, network theories, organization theories, general uncertainty, theoretical physics, and higher order mathematical constructs from category and topoi theory. The traditional methodologies from quantitative and qualitative social science research were abandoned in favor of a more holistic and pattern-based approach based on simulations and generated data sets that were compared to various natural and societal phenomena. Simulation-based research methodologies are a relatively novel approach to social research and are better suited to answer the "what if" scenarios of complex nonlinear and uncertain multiagent adaptive systems than traditional quantitative and qualitative methods (Dooley, 2002).

Discussion and Conclusion

In this chapter a thorough review with discussion and expansion to suggestions was given on general information-theoretic concepts, various nonclassical approaches to such, as in the quantum, fuzzy, and generalized uncertainty theories, and bridges that connect microscopic theories of information to the macroscopic and mesoscopic approaches of complexity, specifically complex adaptive systems (CASs). The Holonic approach to CASs, the holonic multiagent system (HMAS), was discussed in the context of generalizing organisms and self-aware systems (SASs). The loop quantum gravity (LQG) approach to the physics of information adapting the spinfoam network methodology was introduced. The further digitization of these models for quantum computation and information models of qudits was expanded upon as a possible avenue to approach a generalized information theory, along with the concept of an information field theory utilizing Bayesian methods for quantum signal processing via fields. Complexity was reviewed and approached from the lens of information theory, a prescribed unifying methodology in complexity studies. This approach blended with the overall theme of information as a forerunner to the physics of matter/energy and organization.

Finally, semiotics and situational semantics were reviewed as tools for adding semantic, pragmatic, and linguistic structure to the Shannonesque information content-only approach. An observer, a generalized brain or self-aware system expands the definition of human interpretation, necessitating a part in any physical theory of information. A classical communication channel is incomplete without a receiver. Physical amd mathematical realities are incomplete without generalized self-aware systems, a Copenhagen quantum-theoretic mantra. However, quantum information, in and of itself was an incomplete approach to completing this requirement. LQG-spinfoams and networks, laced with the mechanisms for conveying super-quantum or generalized uncertainty information was suggested as a means to bridge this gap.

There is no known physical particle that is a pure qubit or qudit. They are abstract manifestations of information containers for quanta. They do however serve as a general effective model for containment of information. This is where this discussion defers by offering an effective particle-field model for generalized information containers, what will be labeled as *gu*-bits (generalized uncertainty bits) and in the tradition of particle-like names, *informatons*. The review of topoi and category theory serves as a means to broaden the general mathematical description of these entities. Indeed, the application of higher-order algebras or *n*-categories may be fundamental to understanding information spaces in LQG spinfoams that are self-aware semiotic structures and that serve as morphogenetic mechanisms for CASs and the info-holarchy. These motives, among others, are the goals of chapter 4 to follow. This study on the notion of generalized information building complex systems that may effectively model patterns organization evolution is the ultimate result being pursued. Evolutional patterns will replace traditional business analytics and approaches to displaying these patterns in holographic dashboard-caves can supplant classical statistical graphing techniques and their flat-screen variants.

Chapter 3: Research Method

Introduction

The research methodology utilized in this study was that of a hybrid grounded theory for developing abstractions and generalized models based on prior paradigms. No experimental numerical data was collected as the data, in this study, are models not numbers. In this chapter a theory for information metamodels was constructed based on the abstraction, generalization, expansion, and modification of other more isolated, albeit grounded and successful physical, information, and organization theories and models. The theory was not constructed based on traditional statistical data analysis used in quantitative studies or in certain data-centric qualitative studies and their mixed hybrids.

This hybrid approach is the predominant methodology utilized in abstract mathematics and physics research (Brown & Porter, 2004). Abstract models are developed for the application of physical information theories to the social sciences: quantitative in the mathematical review, generalization, and expansion of existing information and physical theories, and qualitative in the specialization of these abstractions to certain techno-socioeconomic and physical components in nature and societies. Rather than gather data from particular phenomena and apply general quantitative statistical tools, this study focused on reviewing well defined and established classical, contemporary, and postquantum theories of physical and logical systems, then generalizing them to construct a novel approach to information dynamics. Traditional quantitative studies were not used because of the scope of generalization of organizations – no static or sufficiently predictive dynamic model suffices to describe the evolution and emergence of information in forming abstractions of organization, that is, patterns are better tools than models for prediction and description.

Hybrid Grounded Theory: Research Design and Approach

Organization is the aftermath of creation. Without material, organization is a hollow concept. Without organization, material is devoid of meaning, but more importantly, it is devoid of informational structure, the basis for entropy. Modeling and prediction are the stalwarts of finding the informational structure of organization and its material. However, classical modeling and prediction of emergent phenomena are illusions of grandeur. Patternization of emergent organization is a more apt tool for neo-post-modern, computational, and simulation-based research.

Twentieth century nonclassical paradigms were technically and philosophically built and extended from 18th century mechanical approaches (Kuhn, 1996). However, nonclassical thinking such as that emanating from the ideas of non-Euclidean geometries, thermodynamics, relativity, quantum mechanics, physical field theories, probability, evolution, and rudimentary machine computation, were seeded during the later century. Specifically, general relativity, quantum mechanics, and evolutionary studies bore these fruits. Simultaneous to and alongside these shifts in scientific thought, the more conventional studies of organization and management of humans and machines were developed, albeit maintaining the determinism inherit in conventional 18th century science.

These scientific paradigms were postmodern hallmarks because they directly involved the observer as an active influence in shaping reality. However, as these frameworks more adequately described phenomena, they also exposed their weaknesses through the lens of complexity and information. Nonlinearity was the enemy of such systems. Almost all events in the universe are complex and nonlinear because nonorganic entities are eventually influenced by some adaptive, living organic entity and because they are involved in some emergent manifestation of other entities. Organized phenomena required an understanding of multiple agent behavior. In the spectrum of organization scale, from the microscopic interactions of subatomic particles to the ultra macroscopic forming of mega-clusters of supergalaxies and the possibility of multiverses, emergence arose has the newest magical property of matter. To this end, this development necessitated experiments showing the realization that multiagent systems are superior metamodels of organization, Shohan and Leyton-Brown (2009), and that a unifying theory of physical information and spacetime can more adequately model the evolution of those organizations, Chaisson (2009) and Kaufmann (1995). It is in this context of postmodern paradigms of information and matter that this study is motivated and supported.

The development of these postmodern ideas of organization precipitated the introduction of more revolutionary nonclassical thinking, the neo-modernization of QM, GR, and evolution. This study proposed a metamodel based on the common property of information in all materials and of a calculus for organization of those materials. This metamodel framework takes the form of an information particle, the informaton, an information field theory, and the introduction of an archetypal and prototypical structure—the info-holarchy—based on the constructed first principles of physical information. A reformulated organization theory can then be based on a form of the information physics introduced in this study, subsequently applied to tools for measuring business dynamics.

The hybrid grounded theory approach utilized in this study is rooted in the traditions of abstract mathematical physics research. In this paradigm, conventional theories are exposed as incomplete or inadequate in describing phenomena that have been scrutinized by instrumentation in experimental studies. Einstein's development of GR was as a result of explaining gravitation by a physical field theory, spacetime curvature conforming to special relativity, and faithfulness to his equivalence principle. These supporting constraints emanated from a realist philosophy of nature. Nonetheless, GR did not immediately supplant Newton's inverse distance gravitational law which assumed a force acted upon mass from a distance without any physical explanation. It was not until Dyson, Eddington, and Davidson (1920) observed light ray deflections during a solar eclipse, thus revealing the curvature of its path, as predicted by GR, did the scientific community accept GR. Einstein did not himself provide direct experimental evidence prior to this. He relied on the merits of his model: a more consistent and complete mathematical and physical description of gravity.

As a less abstract example, Taylor (1911) developed the emphasis on task reductionism, task time optimization, and heirachical organizational structure in conveyor-like industries in the early 20th century, the beginning of the management and organization sciences. Taylor's stop-watch management was a linear and rational methodology, mapping the human worker to the architecture of a machine that fed on monetary needs alone. Because humans and their societies are at best, boundedly rational, cognitively dissonant, display properties of nonlinearity and chaos, and are motivated by a myriad of factors in the workplace, Taylor's paradigm left much to be desired in describing dynamic organizations (Olsen & Eoyang, 2001).

Adjacent to Taylor's opus was the more rigourous work of Shewhart (1939), who developed the statistical process control theoretic aspects of management science. While Taylor's management paradigm was slowly replaced by more flexible and holistic organizational theories of dynamic structure and function, Shewhart's process control ideas were expanded on by many, including most famously, Deming (1952). Group and network dynamics replaced the dictum that individuals could be controlled and their work throughput optimized. Statistical process control homed in on more precise measurement of linear phenomena in business. Again, this new paradigm of organizational diversity was not automatically embraced – no new and consistent evidence was immediately produced with their proposals. However, these novel models of management eventually made their way into management studies and experiments. Analytical results followed.

Holistic theories started to surface through the view of organizations as systems. This was initiated by von Bertalanffy (1951) through his early thesis on adaptive feedback systems theory. Nonetheless, linear and sequential views of management and organization prevailed because of a lack of real-world evidence and concrete examples that could be sampled and consequently analyzed. Systems theory is about the whole of the organization, not individual parts. However, even in this vein, systems theory was still a linear idea. The ability to simulate complex organization was not yet available and theoretical notions had to suffice to adequately generalize organization behavior.

This same incompleteness in describing organization dynamics held true in the arena of performance metrization in business. Balanced scorecards (BSC), key performance indicators (KPI), Six Sigma, Deming's 14 points, and the behavorial spectrum of theories X, Y, and Z based on Maslow's hierarchy of needs, are a few of the paradigms introduced into management science in the past decades in an ever-changing attempt to optimize organization performance (Deming: 1982; Fitz-Gibbon, 1990; Maslow, 1970; Schneiderman, 1987). All these business concepts continued to espouse classical thinking and tool building that tended to linearize and simplify the complex dynamic of organizations. In these paradigms, processes were elevated to be multidimensional within the organization, but are sequentialized for control purposes.

Following and parallel to these developments, the models of bounded rationality, chaos, and emergence of diverse spectrums of behavior (self-organization) within an organization were piecewise introduced into the business sphere (DeShon & Svyantek, 1993; Dooley & Johnson, 1995; Simon, 1957). Subsequent to this, holistic business research studies were conducted. However, even currently, these theories are considered as classical thinking as indicated by the aforementioned performance measuring paradigms. Emergent phenomena need to be studied utilizing the emergent sciences rather than the optimization of a classical model of behavior, notwithstanding the intent to expand the understanding of those phenomena. This study's research methodology depends on the generalization power of its theoretical constructs and on new perspectives of how to observe and describe patterns of behavior in organization evolution. To measure the robustness of these generalizations, their reduction to specializations must be well grounded. This study's major construct, the info-holarchy, will be proposed to represent specialized organizational structures, such as information-based businesses and their evolutional patterns and highly complex, adaptive inference-based organisms, such as neuronal structures in mammals and nonclassical computational devices.

This study proposed the development of a novel information theory based on first principles of physical spacetime that encompasses the emergent sciences of complexity, causaloid-based loop quantum gravity, generalized uncertainty, nonclassical logics, and evolutional adaptation. These paradigms are categorized as postmodern, but are founded on the neo-classical paradigms of quantum mechanics, general relativity, Darwinian evolution, and nonlinear studies. Additionally, the utilization of topos theory from the high-level representations of category theory that generalize the classical mathematical constructs of sets, points, and functions will be applied to this information-based notion: the idea of an abstract particle of information and an accompanying physical field theory based on general uncertainty principles that usurp classical probabilistic approaches. This framework will then be used as a calculus for the organization of entities, the info-holarchy.

The methodology that presents this metamodel is one of generalization and synthesis of many different paradigms of physical and mathematical realizations of matter and its ensuing organization. The immediate applicability for this study based on a formal framework for information and organization is to techno-socioeconomic organizations and inference machines of which the most ostensible examples are neural structures and intelligent nonclassical computational devices.

The Role of the Researcher

The role of the researcher in this discussion is that of reviewing the existing state-of-the-art of theoretical information theories, generalizing them to construct a novel nonclassical approach to information physics as applied to the morphology and dynamics of organisms and organizations in nature and society. Quantitative physical theories and their mathematical abstractions are used exclusively in this development. Human studies are absent in order to better objectify a study that, in the end, explores the far edges of how observer subjectivity may dictate physical information. However, the established models of information and physics reviewed in chapter 2 were generalized, synthesized, and expanded. I have relied more heavily on the applications of the proven abstractions of contemporary physical and logical systems than on an isolated quantitative studies of one phenomenon or group thereof.

Because no human or organizational studies involving quantitative sampling were done in this study, there was no need to gain access to potential participants or organizations. Nonetheless, I applied for and received Walden University IRB approval (Approval # 11-04-10-0319302). Applying the info-holarchy metamodel to dynamic organization structures and artifacts in chapter 5 was achieved by the process of reducing a general law to specific parameterizations. No researcher-participant working relationship was necessitated by this methodology of abstract construction and applied reduction. Additionally and consequential to this task, no ethical protection of participants or business organizations was necessary past the anonymity of any particular product, business name or private organization and its proprietary properties. Industries and objects were referred to in generality. Inference machines, neural structures, and general information-laden business organization profiles were chosen as potential applications of this study's metamodel based on the commonality of their communication, evolutional, and historical patterns and lifecycles (i.e., these organisms depict high-level patterns of evolution that involve similar cateogories of information dynamics).

Ethical considerations in this discussion are limited to the application of its proposed abstract model for information to human-inspired and natural processes reviewed and expanded upon in chapter 4: (a) the organization of the brain-mind system and (b) a foundation for viewing dynamic information flow and patterns using holographic principles and the info-holarchy structure in a socio-economic organism, the business entity. No human studies were performed that required the collection of private information or that apply methods of collecting infomation from individuals or of any application of psychological or physical controls or constraints. The researcher maked a sincere attempt to construct a novel model for information dynamics, not as a means to an ultimate model, but as one small tool in further understanding a more powerful idiom for the newly energized study of information in all physical theories. The researcher did not require a researcher-participant working relationship nor required particular ethical protection mechanisms for research participants in the absence of such.

This chapter will introduce more questions than posited answers. These questions will take the general form of existence of evolutionary rules of organization through a new theory of information and organization physics. If information is the calculus of organization and makes possible the creation of particles of the physically perceived universe, is it consistent in other universes of perception (i.e., usurps evolutionarily formed physical laws)? Is information a first principle of existence? Are there other more powerful generalizations to information and entropy? More concretely, can socio-economic organizations such as business entities be treated as specialized organized inference machines, more robust versions of computational devices with a collective generalized intuition? Is it possible to sufficiently peer into and manipulate the dynamic evolutional workings of a business entity via a new kind of interactive holographic representation – a hyper performance dashboard? Finally, is the information holarchy presented in this study, an appropriate and accurate metamodel for patternizing the neural substrate of biological brains, another specialized inference machine?

Setting and Sample

In this study, the data collected were not numerical or intuitive measurements, but were paradigmatic theories of (a) information, (b) uncertainty, (c) physical spacetime, (d) mathematical representation, (e) complexity, (f) networks, (g) games and decision, and (h) organization. This study is an exercise in Kuhnian philosophy, specifically, a prerequisite for positively effecting normal science information and organization paradigms. This study was centered on a developing hypothesis involving the microlevels of information and the macrolevels of organization and their connecting bridges that exist in our mesoscopic world. This hybrid grounded theory methodology manifests the abstraction of a new metamodel from well founded classical and neo-classical paradigms of science outlined above. No sampling of measured phenomena or experimental design was involved in these abstractions of the reviewed paradigms. The design of this study is adequately described as the attempt to generalize and synthesize well founded theories of information, cosmological and particle physics, organization, complexity, and nonlinear studies. Some of these theories are currently being investigated in well formed particle physics experimentation in particle accelerators such as the Large Hadron Collider and in computer simulation studies of multiagent systems that obey certain rule regimes.

Data Collection and Analysis

The data collected were in the form of appropriate information and physics paradigms, not sampled results of experimentation. Chapter 4 generalizes and synthesizes these paradigmatic theories by constructing an information-based metamodel. The applications of this metamodel were presented as specialized models to inference and business organizations in chapter 4. Patternization is utilized in the absence of numerical modeling and prediction. Emergence is better described by these patterns than by a limited number of experimental results – a type of phenomenology of individualization of these models. If patterns predicted by the info-holarchy metamodel for these applications differ in a significant manner from their respective real-world manifestation (i.e., bifurcation may ensue where none were predicted), then the info-holarchy may not be appropriate or adequate at the scale of the organization being specialized. For example, within a particular business, will an info-holarchy adequately describe how information flows and how it may create or morph new substructures on the scale of departments, facilities, groups, individuals, and global extensions? If the workable scales are limited then the generalization of the metamodel is deficient. If the metamodel predicts new organization substructures that have not yet emerged in the observed lifecycles of businesses, then its prediction can be queued based on time-epochs.

The development of a new information model for organization in this discussion is, in a sense, an exploratory study. It points to the possibilities of patternization as a replacement for classical modeling and prediction in the study of phenomena. In this way, follow up studies are encouraged that attempt to apply the info-holarchy to describe real-world organizations and organisms in a dynamically scaled manner, that is, simultaneously in as many scale levels as is computationally possible. It is posited here that such a view of the lifecycle of organizations is more in tune with emergent phenomena.

Protection of Human Participants

In this study, while no inclusion of human participants was done, to protect the possibility of mentioning any particular organization by name or by proprietary property, I did not generalize or stereotype societies, cultures, or industries for the pure motivation of simplifying a phenomenon to fit this study's model for organizational dynamics. This study was not an attempt to limit the potentiality of societies, their members, or pigeon-hole their cultures into a box of models. Quite the opposite, this study posited that societies are hyperdynamic, whose behavior approaches that of emergent laws of form.

Societies are instantiations of organisms that follow evolutional physics and information dynamics.

Dissemination of Findings

The info-holarchy and the presentation of its application to specialized models of organization were given as a preliminary poster paper at the Walden University Winter Residency in Dallas, Texas, in 2008. It is the my intention to follow this study with a series of discussions in the form of research articles in journal print and other Walden residency poster papers with respect to further applications and a new computational methodology based on the info-holarchy metamodel as discussed in chapter 5.

Summary

In this chapter, a review of this study's (a) research methodology, (b) researcher role, (c) ethical considerations, (d) methodology of data collection, sampling, and analysis, (e) considerations for the protection of potential human and organization participants, and (f) dissemination of findings were presented. This study was manifested through the use of a hybrid grounded theory of mathematical and abstract physics research – the generalization and synthesis of established and proposed contemporary theories and models into a unifying metamodel. No data was collected. Instead, models from contemporary theories of physics, information theory, complexity science, organization theory, network theory, and game theory were utilized to generalize and subsequently construct a novel information-theoretic metamodel for organization. No human participants were used. Use of specific names of organizations was avoided.

Chapter 4: Results

Informatons and the Info-holarchy

Organisms and complex organizations are examples of collective emergent information flows. While the term, information, is used in a ubiquitous manner to mean a general relevancy of data and knowledge propagation and retrieval, the apparatus for its structure and flow between event and observer is ambigious. Most classical definitions of information do not entail the existence of an observer or self-aware system. Thermodynamics, QM, and GR were to challenge that proposition, along with the notion of semiosis. Nonetheless, the duality of a source-receiver based information theory as a doormat to a generalized information theory for physical existence (PE) remains controversial and further ambiguates that thesis. Regardless, uncertainty is common in any facet of information measurement.

There is no larger indication of this than in the uncertainty principle in QM. The measurement problem in QM is manifested from the famous Heisenberg's uncertainty principle:

$$\Delta x \Delta p \geq \frac{\hbar}{2} \tag{5.1}$$

where Δx and Δp are the respective RMS of the simultaneous measurement of position and momentum of a quanta and $\hbar$ is Planck's constant (Heisenberg, 1927). Once position is measured with adequate precision, momentum or any function of it thereof loses that precision in its measurement (i.e., one measurement disturbs the other, an observer effect). The truly important result of this phenomena is that quanta have no definite simultaneous position and momenta, independent of measuring devices in standard nonlogical positivist QM. Things are not as clear in the interpretation of QM using entanglement where measurements on a quanta that are entangled with another effect or limit that of the other.

Information in the form of these measurements also loses objectivity. It remains to show if the collapse of the generalized Schrödinger wave equation dictates such measurement or is done separately by nature through the property of entanglement (Thaheld, 2007; Zeilinger, 2010, 285-288). Everett's many-worlds interpretation says otherwise. In this model, parallel universes exist simultaneously. Each universe takes a uniquely different reality history. No collapse of the wave equation ensues because every "known" particle and corresponding particle history is in one such universe and hence, all conceivable histories exist simultaneously (Everett, 1957; Tegmark, 2003). From an information-theoretic viewpoint, all potential information exists. If each possible particle state exists in this parallel scenario, then each possible information state exists. Information has been theorized to propagate from the properties and observables of physical systems alone. In other words, no information exists without a physical phenomenon.

This discussion departs from that philosophical premise and instead adopts the "it from bit" directive of Wheeler and followers. Here I have generalized this directive to an "it from *g-bit*" philosophy, where a *g-bit* is this study's hypothetical information container, the informaton particle, that is based on notions of generalized uncertainty (GTU), LQG-spinfoams, semiosis, an ensuing concept of a generalized information (informaton) field theory, and finally, a generalized information holarchy structure—the info-holarchy. An earlier attempt at formulating an information field theory based on classical field techniques and reviewed in chapter 2 assumed this reductionist approach (Enßlin, Frommert, & Kitaura, 2008). Quantum mechanics adds to this condition by implying that no information may exist if both an observer and a physical event or presence does not exist. In this study information was introduced as an epi-phenomenon. Information in the form of abstract particles that each consist of an observer and event-generator entity pair was constructed. It was hypothesized that mathematically abstract subquark particles such as the helon and preon models can be represented by these information particle systems. These bi-partite systems will be called *informatons*.

In the tradition of physical field-theoretic methods, a new generalized information field theory will be attempted utilizing the structure of the informaton and its holonic nature in building macroscopic clusters. These clusters will then be used to construct a class of complex organization – the complex adaptive multiagent system (CAMS). More specifically, the class of HMASs will be utilized as the target construct of informaton-based self-organization and assembly. The abstract mathematical structure of such informaton systems will be investigated utilizing the information-theoretic properties of LQG spinfoam network models and their higher order mathematical structures and descriptions via *n*-categories.

The duality in quantum mechanics—the phenomena of observing and analyzing either a particle or wave interaction—and the property of complementarity—the inability to simultaneously measure both position and momentum of a particle—are cornerstone implications of the Heisenberg uncertainty principle. This principle also implies that the subjectivity of an observer (i.e., the application of a measurement operator [POVM] on a quantum state), potentially interpreting multiple versions of physical presence, is at the center of how information is perceived. Following quantum mechanics, a model of information particles is presented in which an event entity (event-generator source) and an observer entity (receiver) are entangled (super-correlated) and paired to form a novel unit of information—the *informaton*.

Reductionism exists in the physical sciences because the detailed study of the connected constituent parts of a whole leads to a better understanding of the mechanisms of composite organisms and organizations, not withstanding the role of holism and emergence. Nonetheless, in order to grasp and extend a physical theory of presence, both reductionism and holism must be applied in a coordinated, equitable, and consistent manner within the dictums of scientific research. This approach to modeling reality is a prototypical methodology in legitimate post-normal science (Funtowicz & Ravetz, 1993). Particle physics is a unique example of a transformation to such a paradigm. For example, in a quest towards further reductionism in the representation of physical particles in the universe, the helon (rishon) and preon models in physics abstract the composition of the smallest experimentally verified actions for quarks, into ever smaller compositions.

Helons are abstract particles that consist of ribbon twists representing fractional charge (Bilson-Thompson, 2005). When these ribbons are combined using a physically constrained rule set, they mathematically form quarks and account for the subsequent charge and spin structures (presently in fermions only). However, there remains great debate about the existence of any particle as a point-mass in any theory of quantum gravity (Halvorson & Clifton, 2001; Nikolic, 2009). Bohr, Mottleson, and Ulfbeck (2004) interpret quanta (general particles) in quantum mechanics as phantasms—distribution holders or fields for the description of particle dynamics. Indeed, the most successful rendition of quantum mechanics has been as a field theory, the quantum field theory (QFT). QFT is a most holistic approach to quantum mechanics because it assumes that a field is the central tenet of any plausible way of identification.

With respect to the abstraction of information, this same push and pull of paradigms and frameworks persist. However, no mechanism for localization or field exists explicitly for information. This may be because of the ambiguous and flippant way in which information is sometimes used and defined for convenience. Mathematical abstractions of information posit that only the physical presence of matter and/or energy exudes information in the universe. This information is made plausible only by a suitable measurement apparatus. However, the proposition that an abstraction of information is the source of presence in particles has not been taken seriously except by the proponents of digital physics. One reason for this seems to be the lack of a universal model for information both as a mathematical abstraction and a physical source. In the spirit of this micro structuring, this paper proposed a general robust information particle, the informaton. Informatons may act as constituent parts in building physical particles, eventually leading to higher order constructs, organisms, and general organizations. Furthermore, the glue that fits and molds these organizations will be supplied by a generalization to the principles of uncertainty, one specialization of which is the quantum information model.

Each informaton will consist of an event-observer pair of entities. These entities are information abstractions existing only in pairs. In this sense, an informaton is a type of supersymmetric particle. The event entity of an informaton can be the event entity of another informaton consisting of a different observer entity. Similarly, an observer entity of an informaton can be the observer entity of another informaton consisting of a different event entity. Informatons are thus a type of permanent entanglement of event generator (source) and observer (receiver) entities.

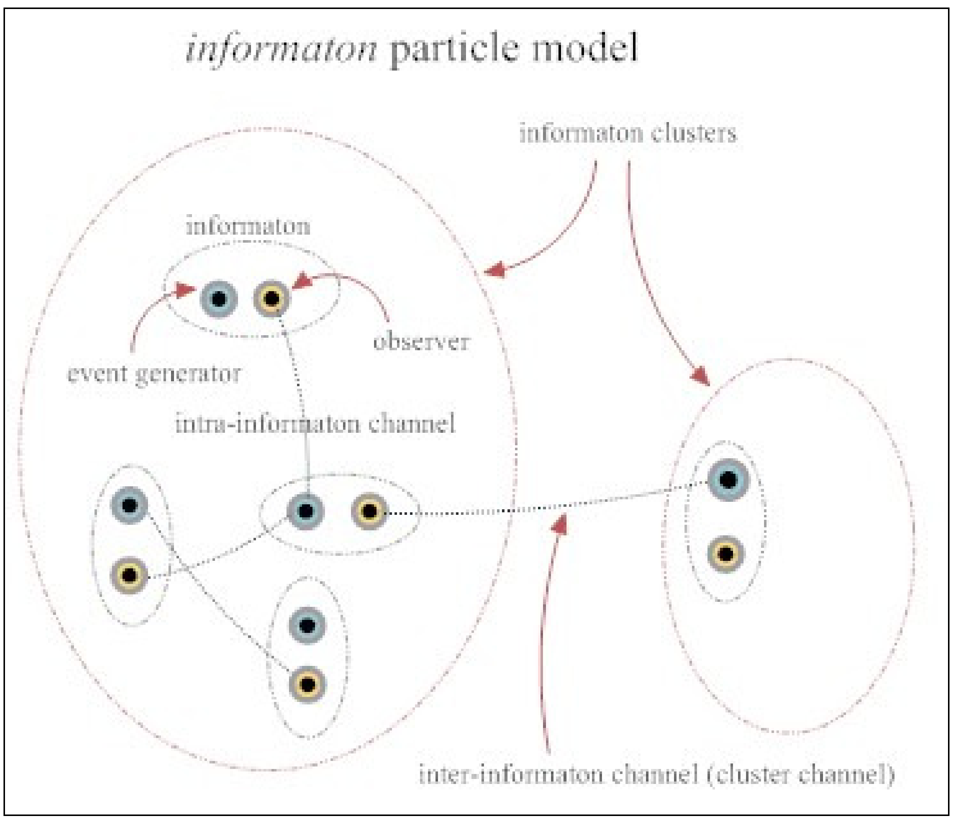


Figure 14. Informaton particle model

Event entities are propagators of information flow while observer entities are recipients, observers, and measurements of information flow. The mechanism for receiving information flow in an observer consists of the composition of a finite (or infinite, given that the processing or computation time is finite) number of signal filtration operators on the stream,

$$s = s_{i_1} \circ s_{i_2} \circ \ldots s_{i_n} \circ \ldots \tag{5.2}$$

The mechanism for event generation of information is given by the propagation of a set of observables, $\{o_i\}_{i \in I}$, of a physical particle, such as subquarks. Inter-informaton fields are abstractions of the stochastic ensemble theory of field theory constructed for information transfer between and within informatons. Fields are essentially a stochastic way out of dealing with the combinatorially explosive computations involved in considering individual interactions of particles.

Consider the differences between a particle from the standard model (SM) of quantum physics and an informaton. Informatons are bipartite subsystems but cannot exist in isolation of their constituent parts. Standard particles are reductionist entities that exhibit certain behavior, have properties, and possess observables. When measured they propagate as information to separated self-aware systems (SASs) (i.e., observers). In the quantum model, observables are manifested by the application of POVM operators acting on the state-ket of the entity. Informatons generalize this action by combining the phenomena of propagation of observables (information) and the detection of such measurement by an intertwined SAS observer into one system. Hellman, Mondragon, Perez, and Rovelli (2008) have shown how when observer and observed quanta are combined into one general relativistic-quantum system, probability amplitudes can be calculated without the explicit use of time. In the case of an informaton, the event and observer are combined into one system. Their joint probability amplitudes can now be calculated in a fashion not involving time and as such constitute a candidate for a physical entity. Quantum entanglement and its generalizations formed from GTU here act as the communication QG-channel between event and observer components of an informaton. In a QG channel, the constraints of both GR and QM dictate the fidelity. With this in mind, the capacity of a QG channel via a GR-QM Unruh-DeWitt channel, a generalized information channel that supports both relativisitic and quantum effects, will be discussed later in this section. Moreover, informatons in this framework are constructed to build observables. It is posited that because of this, matter-energy particles can be "informationally" constructed from informatons. They are the information conduits of reality.

Informatons communicate by the exchange of generalized bits of information. The "it from bit" paradigm of Wheeler (1990) is elevated, in this discussion to "it from *g*-bit", where a *g*-bit generalizes a quantum computational unit to any information unit based on very general notions of intuition, uncertainty, and superquantum phenomena, the *s*-bits. SM particles follow quantum mechanical rules. Informatons follow general uncertainty theoretic (GTU) rules. I will investigate what this means in terms of information containers (i.e., the construction of *g*-bit dynamics). In quantum mechanics, a general pure qubit state can be expressed as a superposition of two distinct bit states:

$$|\psi\rangle = \alpha_0 |\psi_0\rangle + \alpha_1 |\psi_1\rangle, \text{ where } |\alpha_0|^2 + |\alpha_1|^2 = 1 \tag{5.3}$$

For a multistate qubit, or qudit, with *d* possible states, this superposition can be expressed as:

$$|\psi\rangle = \sum_{i=1}^{d} \alpha_i |\psi_i\rangle, \text{ where } \sum_{i=1}^{d} |\alpha_i|^2 = 1 \tag{5.4}$$

One may consider the quantity, $|\alpha_i|^2$ (squared amplitude), as being the classical probability of finding the qudit in the state, $|\psi_i\rangle$. Consider, next, an informaton-type qudit. The ability of an informaton to communicate a bit of information from event to observer is dictated by the event propagating one bit and the observer being able to cognitively recognize and receive that bit. The states of the informaton are therefore bi-states consisting of the coordinated event and observer states of propagating a bit and receiving that bit respectively. Propagating a bit is equivalent, in this discussion, to possessing a discrete observable at its constituent root, i.e., spin, components of momenta, mass, etc. If both entities in an informaton follow quantum mechanical and general relativisitic rules, then each will possess observables that are governed by the nonlocality and causality of an integrated physical theory, such as LQG spinfoam networks, a canonical quantization of General Relativity on a 3+1 dimensional decomposition of space-time.

Spinfoams represent path histories of systems that translate from one spacetime state to another using this canonical quantization (quantizing or discretizing a classical theory). The surface of foam networks represents qubit or ½-spin observable systems (Terno, 2006). For an informaton-modeled system, the event generator will propagate a bit of information within its lightcone and quantum causal loop. The receiving observer entity will receive that bit of information according to those same rules of engagement. The state of the receiver *w.r.t.* the bit propagated by the event-generator is then given by a rule whose parameters include the event-generator's observables. In quantum information terms, this rule is equivalent to the probability of being in a state that has received a bit of information from a particular event-generator. The number of eligible event-generated bits is constrained to the lightcone and quantum causal loop of the observer. Denote this space by, E_o for the observer o. Simply put, if ε is an event-generator and $\varepsilon \in E_o$, then the potential informaton denoted by the pair (o,ε) has a nonzero quantum probability. However, what does it mean for a bit of information to be generated and then received by a particular pair $\iota = (o,\varepsilon)$ in a relativistic-quantum manner?

This collaboration between source and receiver can be defined as a quantum channel, under QM rules. However, under the further geometric restrictions of GR, this quantum channel canot be simply described, as it is under a nonrelativistic quantum channel, that is, as a completely positive operator (Peres & Terno, 2003). The vacuum between event and observer acts as a noisy quantum-relativistic channel at best. Hence, an informaton contains an inherent quantum-relativistic noisy channel. Channel capacity can be calculated based on the premise that the inter-informaton channel behaves as Unruh-DeWitt detectors, that is, as point-like 2-state quanta (qubit) interacting under a scalar quantum field (Cliche & Kempf, 2009). By those results, no further noise is introduced by relativistic effects. Channel noise is manifested through quantum perturbations. These results generalize to qudit and GTU-inspired entangled systems.

LQG spinfoam networks describe qubit/qudit dynamics and evolution under the assumptions of path history mechanisms and their ensuing hypersurface geometries. Spinfoam networks (SFNs) act as quantum-relativistic computers using their computational representation of ½-spin systems. To view this, consider a patch area in a SFN. This patch is abstracted as a quantized pixel, that is, a pixel that represents a ½-spin particle observable or qubit,q_i, when a puncture, p_i is made by an open SFN's edge in a superimposed quantum state of the form: $\left|\psi_{q_i}\right\rangle = \frac{1}{\sqrt{2}}\left(\left|\frac{1}{2}\right\rangle \pm \left|\frac{1}{2}\right\rangle\right)$. A set of N such punctures, $\{p_1,..., p_N\}$ is associated with a stream of N qubits, $\{q_1,..., q_N\}$ and their N surface pixels respectively on the SFN (Zizzi, 2000). Furthermore, by replacing the algebra of polynomials, $\mathcal{P}^2$ acting on the surface of the 2-sphere, S^2, whose surface represents the states of a qubit, with the noncommutative algebra of complex *n x n* matrices, $\mathcal{M}^n$ acting on S^2, via the mapping, $x_i = kJ_i, i = 1,2,3$ where $[J_i, J_k] = i\varepsilon_{ijk}J^k$, $k = \frac{r}{\sqrt{n^2-1}}$, r the radius of S^2 and $n = 2j+1$ the number of pixels (discrete cells), the *fuzzy sphere* for computation on a discrete LQG space is constructed. The elements, $\{J_i\}$ form the *n*-dimensional irreducible representation of the algebra of $SU(2)$ (Zizzi, 2005a). Additionally, by utilizing noncommutative C* algebras of complex positive linear functional (operators) for representing quantum states, the algebra of logic quantum gates is constructed.

Computation of qubit pixels on fuzzy spheres is the basis for an approach to QG using quantum computation on SFNs (Zizzi, 2005a). By considering bi-partite qubit or qudit systems on SFNs using the qubit pixilation and computation on fuzzy spheres, one can accommodate for informaton-inspired computers and memory registers in LQG. Specifically, consider how an *n*-register from a sequence of *n* qubits represented in a surface *S* computes a binary function, $f:\{0,1\}^n \rightarrow \{0,1\}^m$ where $m \le n$. The unitary operator, $U_f\,|x\rangle|y\rangle = |x\rangle\big|y \oplus_{2^m} f(x)\big\rangle$ where $\oplus_{2^m}$ is addition mod 2 is constructed. Here $|x\rangle$ is a register of size *n* to store the arguments of *f(x)* and $|y\rangle$ a register of size *m* to store the values of *f(x)*. Then the computation can be implemented by $U_f : \sum_x |x\rangle|0\rangle_y \rightarrow \sum_x |x, f(x)\rangle$ where $|0\rangle_y$ is the initial zero-state of the argument store register *y* and $|x, f(x)\rangle$ is the 2-qubit state of the argument and function value (Zizzi, 2005b).

This approach also accommodates the natural realization of the Holographic Principle in which the information content (entropy) of a spacetime *(D+1)*-dimensional volume is bounded above by a proportion of the area of its *D*-dimensional boundary Bousso (2002), i.e., $S_V \le \frac{A}{4}$. LQG (spherically symmetrical version) naturally accommodates the holographic principle and so satisfies a crucial component for a unifying theory in physics and information (Gambini & Pullin, 2008). The relevance of this is that as a surface patch of the SFN is punctured, only its surface area needs to be measured in order to calculate an approximation to its information entropy. The pixel surface areas in SFNs are then taken to be on the Planck scale. The larger implication of this is that for a black hole, *B*, the number of Boolean degrees of freedom, R_n, in a region *R*, completely surrounding *B*, is given by:

$$R_n = \frac{B_S}{\ln 2} = \frac{4\pi B_m}{\ln 2} = \frac{B_A}{4\ln 2} \tag{5.5}$$

where B_s, B_m, and B_A are the surface, mass, and horizon area of the black hole respectively. This gives an upper bound on the number of qubit pixels representable in B and so, puts a limit on the discreteness of B and on any volume in the universe under severe conditions of blackhole evaporation. This is clearly a measure of the discreteness of spacetime without sacrificing the accuracy of calculating observables of reality modulo the Planckian scales. For this discussion label this environment the Planckian LQG computer (PLC). Zizzi refers to this as the quantum computer view (QCV) of discrete quantum gravity on the Planck scale (Zizzi, 2005b). In a PLC, the 3+1 dimensions of spacetime gravity break down (‘t Hooft, 2009). This is both the physical and philosophical weakness and strength of the PLC approach to QG and hence, to generalized computation and information. This dimensionality reduction implies the plausibility of an infinite correlative model for PLCs at the Planck scale, while exhibiting the emergence and irrelevance of its inside dynamics, information-theoretically. Informatons fundamentally existing at the Planck scale, utilizing generalized uncertainty principles, while honoring the constraints of QG, will be a candidate for the atoms of information and the constituent parts of physical information fields.

Informatons can be statically positioned as quantum (GTU) cellular topological lattice automata in spacetime. What this means is that informatons occupy every discrete and malleable point position in spacetime regions (eventually expanding to the universe), connected by their unique automaton rules and a topological lattice structure. Topological lattices are lattices, L in which the operations of join, $\vee$, (the supremum), and meet, $\wedge$,(the infinum), are continuous maps from $L \times L \to L$ (Strauss, 1968). In Planck-scale LQG points on the lattice L are Planck-volumes. Topological lattices are then ways of approximating ever smaller volumes (limit points) in the eventual discreteness of lattices that can also have exotic topological structures, e.g., topological fields and other invariant spacetime curvatures. Informaton lattices can then curl into themselves in various ways, as is depicted by m-branes and spinfoams. An informaton substructure can then be constructed by its information patterns to form a topology in a sublattice leading to the generation of exotic entities such as those m-branes and spinfoams.

Information exchange patterns are made possible by the lattice connections (lattice fields) of informatons which are Planck-scale communication channels. One then may apply quantum (GTU) channel limitations and rules reviewed earlier to these lattice connections. Information fields (to be clarified and justified later) can also be generated through these Planck-scale channels producing abstract information channels. Additionally, fermionic, bosonic, supersymmetric, and quantum dot structures for quantum cellular automata can be built to generate scaffolds for energy-matter (McGuigan, 2003). In a similar manner, lattice cellular automata can generate GTU logics and hence information flow from GTU-based processes. Lattices automata are generalizations of lattice structures with automaton-like processing nodes and edges in which general logics can be built from their structural dynamics (Kupferman & Lustig, 2007). In our prior discussion on quantum logics, lattices were a generic generator of uncertainty and nonAristotelian logic systems. Unique information automaton rules (e.g., quantum and GTU-based strategies) and lattice structures based on GTU processes and the general relativistic-quantum spinfoams introduced for informatons define the generation of energy-matter particles. LQG spinfoams are in their continuous nature, limts of lattice structures and as such topological lattices coupled with GTU logics can enrich the spinfoam formalism. Informatons can then be considered generalized spinfoams (Ding, Han, & Rovelli, 2010).

Physical energy-matter fields are dictated by the information rules of these lattices. Informatons remain in their static positions while the generated energy-matter is manifested through motion, i.e., particles are generated and propagate through the scaffolding of static informatons. All forces and motion of energy-matter are generated by the exchange of generalized information bits—g-bits, within the informaton lattice. Put more strongly, informaton lattices manifest forces and energy-matter. Information exchange patterns of informaton lattices (soon to be idealized as info-holarchy organizations) form force, energy-matter, and their ensuing fields. This is the main premise of informaton-based reality. Informaton lattices are abstract mathematical metamodels in the sense that all energy-matter and force are manifested by certain patterns of information exchange between and shared with other informatons.

There are chains of unknown energy-matter and force constituents that would exist between informatons and the current taxonomy of SM and hypothesized particles from QG theories. For example, preon theory is the abstraction of subparticles named *preons* that construct quarks, leptons, and gauge bosons (D'Souza & Kalman, 1992). Further refinement of preon theory was the triplet Rishon model of Harari (1979) and

Shupe (1979) and refinements from the Helon model of Bilson-Thompson (2008) in which constituent SM particles are decomposed into triplet combinations of new abstract particles called *rishons* and *helons* respectively. These were attempts at more efficiently and accurately categorizing SM particles while predicting unknown particles that could emanate from a creditable theory of quantum-gravity.

In order for informatons to patternize higher order particles (and their constructed waves and fields), they must take into account all the properties of SM particles, including charge, chirality, strangeness, spin, and well as any property of predicted particles, such as gravitons and the Higgs boson. At the least they must construct such abstractions in principle by distinguishable and unique patterns of information exchange, i.e., patterns of flow and organization. The entropic gravity concept from Verlinde (2010) hypothesizes that gravity is a second-hand force emanating from information exchange amoung particles in a small volume, i.e., gravity is the effect of a higher probability of particles of mass being closer to each other as a result of the combinatorics of particles occupying nodes of spacetime polyhedron.

Cellular automata have been documented to have patterns emerge forming distinguishable categories (Wolfram, 2002). However, the topological lattice structure and GTU processes of informatons add new threads of complexity to these cellular automata dynamics. Starting with the premise that informatons reside in Planck-scale spacetime and information is digitized from gu-bits, no further reductionism is necessary. This is reminiscent of Leibnitz's indivisible monads without the theological and spiritual implications that were thrust upon natural philosophers of post-Renaissance epochs (Leibniz, 1992).

A novel abstract approach to developing possible QG theories using a form of causal probability, the causaloid, will be modified as a general conduit for launching the informaton model. The causaloid is an attempt at constructing a framework that embodies a probabilistic theory adaptable to an indefinite causal structure (Hardy, 2008). Quantum Theory is a probabilistic approach to state computation in physical systems. It requires a causal structure in spacetime because the evolution operator acting on quantum states is a linear time evolution. General Relativity, on the other hand, does not require a definite causal structure because of the natural of its global spacetime curvature and metric. However, it is a conservative theory because it is deterministic.

The approach to QG from a causaloid framework is to not assume or start from the premises of one theory and adapt to the other. What is therefore, an indefinite causal structure? Causaloids endeavor to form probability statements about observations in an indirect correlative manner. This means that if two regions of spacetime, R_1 and R_2, are considered in an experiment, in which a correlation is to be established between the two, one region will be probabilistically tied to a third, R_3 or series of other regions, $\{R_i\}$ that are only indirectly tied to the other. In this way a theory can be constructed using causaloids for spacetime structures that depict an indefinite causal framework with a probability calculus, exactly what a QG structure would have to consist of. For details on the causaloid construction and its implications for computation in a QG structure see Appendix B.

Consider a causaloid structure for bi-partite informatons utilizing the LQG spinfoam formalism at Planck scales. An informaton contains two g-bit entities, each with multiple possible states, i.e., qudits with general uncertainty. In the causaloid formalism, the pseudo-lattice structure of a quantum (classical) computer will serve as the background structure for informatons. In this setup, each node will represent generalized computational gates. Each node is linked by a scaffolding devise, the spatio-temporal manifold.

Two generalized qudits that will make up an informaton interact along spacetime curves that parallel the node structure of the pseudo-lattice computer. Refer to Figure 21 for a view of a QG computer pseudo-lattice of gates, node holders, interacting qubit flow and the causaloid framework. In a modification, generalized qudits replace (qu)bits, and the probabilistic statements of the likelihoods in the causaloid are replaced by GTU-based propositions that generalize fuzzy quantum gravity. Some aspects of GR computation and information will be injected into the structure of the informaton causaloid. Let Λ_G depict the causaloid for this informaton-based QG and GTU structure. Λ_G may then be utilized to calculate uncertainty propositions (statements) about the observables of GTU particles and hence of informatons.

Next, entanglement will be utilized to further define the dynamics of informatons, along with a generalization using the GTU paradigm of Zadeh. Entanglement is a stronger-than-correlative manner in which quanta can interact in a quantum mechanical sense, as reviewed earlier. Maximally entangled quanta entertain further surprising behaviors, including violations of Bell-type inequalities and supra-causal effects.

Consider the postulation that maximum entanglement of ε and o entails the existence of an informaton $\iota = (o, \varepsilon)$ quantum mechanically. Partial entanglement gives a relative existence of ι w.r.t.a measure of entanglement, a version of partial entanglement membership or fuzziness. For bi-partite systems, any measure of entanglement entropy is equivalent. Hence, any entanglement entropy measure may be used without loss of generality (WLG). For informatons, mutual entropy would give a measure of receptability between *e* and *o* and hence of the information binding strength of $\iota = (o, \varepsilon)$.

In particular, for qubit informatons, one may utilize the Bell-states of ε and o.

Let $\mathcal{H} = \mathcal{H}_\varepsilon \otimes \mathcal{H}_o$ denote the subsequent four-dimensional Hilbert space of possible states for the informaton, $\iota = (o, \varepsilon)$. If ε and o are maximally entangled then the four base states of $\mathcal{H}$ are:

$$
\begin{aligned}
|\psi^{\pm}\rangle &= \frac{1}{\sqrt{2}} |a\rangle|b\rangle \pm |b\rangle|a\rangle \\
|\varphi^{\pm}\rangle &= \frac{1}{\sqrt{2}} |a\rangle|a\rangle \pm |b\rangle|b\rangle
\end{aligned}
\tag{5.6}
$$

where *a* is the nonpropagating (resp. nonreceptive) state and *b* is the propagating (resp. receptive) state of ε (resp. o). The informaton states $|\varphi^{\pm}\rangle$ can be interpreted as the only classical communcation states and the states $|\psi^{\pm}\rangle$ the quantum informatons. The existence of such a decomposition is guaranteed by the Schmidt decomposition of elements in a Hilbert space from a basis. More generally, if there are *n* and *m* modes of gate operations (states) for the observer and event entities respectively then $\mathcal{H} = \mathcal{H}^n \otimes \mathcal{H}^m$ would be an *nm*-dimensional Hilbert space representing the possible states with *nm* entangled base states, the entangled qudit base states. When the coefficients of the decomposition are $\frac{1}{\sqrt{d}}$, where *d* is the dimension of the joint Hilbert space, then the state is said to be maximally entangled, Further generalization leads to an uncertainty model being imposed on the gate states. In particular, for this discussion, a general quantum stochastic Itô process can be modeled on the gate operation of each entity of an informaton. Note that quarks such as fermions and bosons impose certain constraints on these states through experimental observation. Abstract informatons are constrained by the information gate states of their abstract event-generator and observer entities respectively.

One may now consider a generalization of a prototypical quantum-relativisitic informaton. Let the rules of information engagement between ε and o be governed by the dynamics of a Zadeh generalized constraint, $g = (X, r, R)$. The quantum probabilistic rules of superposition in the first example of an informaton are a special case. Let *X* be the state variable, $|\psi_\iota\rangle$ of the informaton $\iota = (o, \varepsilon)$, *r* the probability operator, and *R*, the quantum superposition 2-state space relationship for two entities ε and o. Then $g = (X, r, R)$ dictates quantum states for informatons. In particular, the Bell states for qubits are generated using the amplitudes, $\pm\frac{1}{\sqrt{2}}$, being fully entangled. More general models can be generated by other generalized constraints. For example, by considering fuzzy or granular operators for *r*, fuzzy quantum logics can be induced for the states of informatons and other systems of particles. Using a precisiated language (*PL*) for the space of *GC*s, generalized constraints, $g_0 = (X_0, r_0, R_0)$ and $g_1 = (X_1, r_1, R_1)$ may be combined to form a novel *GC*, $g_3 = (X_3, r_3, R_3)$ using the logical operations-connectives (Zadeh, 2005):

1. $\wedge$ (conjunction),
2. $\vee$ (disjunction),
3. $\Rightarrow$ (implication),
4. $\Leftrightarrow$ (bi-condition),
5. $\neg$ (negation),
6. qualification,
7. projection,
8. constraint propagation through composition

Other operations are possible that preserve closure within the logical structure *(GC,PL)*. Conceptually, this implies that two fundamentally different logical systems for physical presence can be logically combined. In particular, two informaton conceptual spaces, where the rules of engagement for events and observers are different, may be logically combined to form a third different logical system for informatons. Utilizing Zadeh's generalized uncertainty theory (GTU) in the formation of informaton-based structures, discrete information units are formed and are labeled as *g*-bits. A *g*-bit is an abstract discrete unit of information that propagates within an informaton-based structure generated using the rules of GTU. Since informatons are *e*-bits by definition, *g*-bits generalize *e*-bits. In this scenario, disparate informaton subsystems can coalesce into novel ones. This sets the stage for the potential to form macrostructures from different microsystems of informatons. In the causaloid formalism, statements of the form (8.1) are modified to fit the GTU constraint formalism for intuitionalistic calculus, that is,

$$g(X_{R_1}, p, \rho) = p(X_{R_1} \mid F_{R_1}, X_{R_2}, F_{R_2}) \qquad (5.7)$$

where ρ is the density operator for the likelihood and p is the mneumonic for the probability calculus. The probability calculus can be replaced by any intuitionistic one, such as a granular measure that fuzzifies the value of X_{R_1} given fuzzy or crisp renditions of F_{R_1}, X_{R_2}, and F_{R_2}.

Informatons potentially cluster to form super-informatons, i.e., organisms of informatons interconnected via shared event and observer entities (entanglement) or through the coalescence of GCs via a PL in the GTU paradigm. This sharing process may not be complete in a finite volume, *V*, in the universe, that is, each formed informaton may not share its entities with all other entities of the complete set of informatons in *V* because entanglement is not guaranteed for all entities within *V*. Nonetheless, the singularity of the initial conditions of the Big Bang model point to the high probability of this occurrence. As in quantum entanglement, the spacetime between entities of an informaton span their respective lightcones and quantum causal loops. An informaton is therefore an abstraction for a generalized relativistic-quantum particle. Not withstanding the wave (field) properties of entities, this presents a higher level abstraction for discrete computation as well. The aforementioned QG computers in Appendix B are generalizable to the GTU inspired physics of the informaton.

Informatons become g-bits using the causal structure of GCs from Zadeh in place of quantum probability. Since granularity defines the fuzziness in GTU, GTU logic equates (is isomorphic) to quantum logic in a general setting by using Lukasiewicz operators, as mentioned in chapter 2 and from the results of Pykacz (2007) and others.

Generalized Information Field Theory

In an attempt to quantize and form a generalized field theory of information centered on the informaton model, one is first confronted with the problem of identifying a general signal apparatus. In the tradition of an information signal theory and as in the development of Enßlin, et al., a general signal response model, *R*, can be devised such that the data, *d*, propagated from an event is subjected to a filter operator, *s*, representing the measurement of the interested observable, with noise operator n_s. This model may be expressed as:

$$d = D(R(s), n_s) \tag{5.8}$$

where *D* is a functional on the product space of response and noise operators. *D* is normally simplified to be linear and separable into response and noise components as in the model of Enßlin et al. In this more general setup, *D* is constructed using Zadeh's GTU. Here quantum, chaotic, fuzzy, belief systems, and multilogic uncertainty versions of operators may be utilized in the model. In addition, $\langle n_s, s \rangle \neq 0$, i.e., the noise and signal operators may be correlated or even quantum entangled. The signal is to be maximized, while minimizing the noise, *w.r.t.* the receiver. In order to extract the best possible signal from *d*, a suitable transformation *T* is approximated as $T(d) \cong s$. In the form of a general *D*, there may be many such operators *T*. In fact, a space of *T* operators may exist such that $\|T(d)-s\| \leq \varepsilon_{(d,s)}$ for every $\varepsilon_{(d,s)} > 0$. It should be noted that the signal captured will be affected by nonlocal entanglement and relativistic influences in our LQG framework. This is the main departure from the epistemology of Enßlin et al. The Lagrangian-Hamiltonian formalism developed in Enßlin et al. was from a consideration of quantum fluctuations and field theory dynamics. In the quantum gravity case, the LQG spinfoam formalism will serve us better. We define the information field via the signal generation, s, as in Enßlin et al. The form of the signal operator will depend on the response, noise, and signal model operators, denoted here by the triplet (R,n,D). We write this dependence as $s_{(R,n,D)}$.

These observables are then assigned on $\partial\Gamma$, the boundary of a quantum spacetime spinfoam, $\Gamma_{i(e,o)} = \Gamma_e \bigcup \Gamma_o$ representing the tensor product state of an informaton with event entity *e* and observer entity *o*. We define g^s_{ed} as the holonomy of $s_{(R,n,D)}$ along the edge, *ed* of the dual to the tetrahedron, Δ_4, Δ^* (two-skeleton) of the informaton spinfoam $\Gamma_{i(e,o)}$. Next, we define a field, ϕ_s on the 4-copy space G^4_s, where G_s is the holonomy group of the above defined holonomies on $\Gamma_{i(e,o)}$. We pick ϕ_s so that it is symmetric and group-invariant in *G*. In the spinfoam development for 4-D spacetime LQG the Hamiltonian action reduced to the form:

$$S_H(\phi_s) = \frac{1}{2}\int (P_G[\phi_s])^2 + \frac{\lambda}{5!}\int (P_G P_H[\phi_s])^5 \qquad (5.9)$$

where P_G and P_H are projection operators defined as before in chapter 2. This establishes the spinfoam, $\Gamma_{i(e,o)}$, for the information (signal) field, s on the informaton particle $i(e,o)$.

At this point, we may substitute the Zadeh GC for the quantum logic in the above spinfoam formalism by virtue of the isomorphism between a quantum logic and a generalized fuzzy logic implementing Lukasawiez operators. The GC defined by $G = (X, r, R)$ defines a logic embeddable in the spinfoam formalism.

Complex systems represented as either n-dimensional, $x = (x_1, ..., x_n)$, countably infinite-dimensional, $x = (x_1, ...)$ or continuum, $x = (x_\alpha)_{\alpha \in \Omega}$ vectors over a field are inherently dynamic as state information must be accounted for in the analysis of a general organism. Hence, the dynamic system is represented as a stochastic process, $x(t) = (x_\alpha(t))_{\alpha \in \Omega}$. A general dynamic information model involving quantum microdynamics at microlevel organization, thermodynamics at mesolevel organization and the variation minimax value of an entropy functional represented by a Hamiltonian-Lagrangian at macrolevel organization is presented. The information meso and macrodynamics are governed by the imposition of the variational principle (VP) applied to higher-level Hamiltonian-Lagrangian systems describing the mentioned meso and macrodynamics with sustained contribution from the microlevel quantum dynamics.

The quantum microlevel dynamics will be supplemented with general uncertainty (GTU) extensions from Zadeh. The classical version of the information macrodynamic interplay is an idea from the concept of information macrodynamics (Lerner, 2003). An info-holarchy generalization of information macrodynamics will manifest the interplay of organization levels in an info-holarchy. In this process model, informatons (information particles) eventually dictate the macro manifestation of matter, energy, and spacetime flow from their collective microprocesses.

Evolutionary Model For Informatons

Zadeh's GTU process operators will now be constructed for informaton-generated organisms. Informatons as bipartite (fused source and receiver) entangled quantum gravity systems are proposed to follow relativistic laws such as Lorentz invariance as in the application to spinfoams in LQG. Stochastics come in the form of quantum noise. In this model, our generalization using Zadeh's GTU will be attached to such formalism, followed by general evolutional rules. Hence, a GTU-LQG-evolutional model will be built for informatons. As the *e* and *o* components of informatons are abstract entities that do not exist in isolation, as far as information conveying bodies are concerned, informatons will be treated as single abtractions of information or "particle placeholders".

First, a continuous relativistic quantum-stochastic time evolution model for the microdynamics of an informaton will be constructed, followed by the application of the GTU. This will serve as an approximation to the discrete Planck-scale LQG model for information setup earlier as a model for informatons. Then a mechanism for clustering informatons in an attempt to bridge to macrostates of larger informaton-based organizations will be built. The general concept of info-macrodynamics (IMD) through a micromodel of stochastic equations for small grain systems and macrodynamics of clusters following a Shannon entropy law and utilizing a variational principle applied to an information entropy functional, is adapted in part from Lerner (2003). This was reviewed and discussed in chapter 2. Lerner constructs a classical model for both the macro and microstructures. Here, an attempt will be made to construct a chain of progressively more general models, starting with a nonrelativisitc quantum version, proceeding to a relativistic-quantum model and then generalizing towards a Zadeh GTU model.

Nonrelativistic quantum stochastic calculus is an attempt at modeling continuous nonrelativistic quantum particles that have quantum noise modeled analogously to Weiner processes (Hudson & Parthasarathy, 1984). Especially directed toward Bosonic particles, this calculus engenders the description of irreversible quantum processes through the use of generalizations to Brownian and Poisson processes for semi-martingale models. Consider the solution of the quantum Itô stochastic equation:

$$\begin{aligned} dU_t &= -\left[(iH + \frac{1}{2}L^*L)dt + L^*WdA_t^+ + LdA_t + (1-W)\Lambda_t\right]U_t = F_tU_t \\ U_0 &= 1 \end{aligned} \tag{5.10}$$

where for each $t \geq 0$, U_t is a unitary operator defined on the tensor product space $\mathcal{H} \otimes \Gamma(L^2(\mathcal{R}^+, \mathcal{C}))$, where $\mathcal{H}$ is the Hilbert space containing the possible component states, Γ is the Fock space containing the possible component noise elements (general uncertainty), and $H, L, W \in \mathcal{B}(\mathcal{H})$, are bounded linear operators on $\mathcal{H}$, with W and H being self-adjoint. The solution of (5.10) is interpreted as the evolution of a system composed of a particle (in this case an informaton) whose Hilbert space of states is $\mathcal{H}$, interacting with a a noise process residing in the Fock space $\Gamma(L^2(\mathcal{R}, \mathcal{C}))$. More generally, for nonUnitary operators, this can be written as a homogeneous linear quantum stochastic differential equation:

$$\begin{aligned} &dU_t = \left[L_3 dt + L_2 dA_t^+ + L_1 dA_t + L_0 \Lambda_t \right] U_t = G_t U_t \\ &U_0 = 1, \end{aligned} \tag{5.11}$$

Hudson and Parthasarathy have shown that if the quadruple, $\{L_0, L_1, L_2, L_3\}$ are *strongly admissible* in $\mathcal{H}$ then the system has a unique solution, U_t^* which is an *adapted process* and strongly continuous in the tensor product space $\mathcal{H} \otimes \Gamma(L^2(\mathcal{R}^+, C))$ (Hudson & Parthasarathy, 1984). L_i is strongly admissible over $\mathcal{H}$ if $\mathcal{H} \subseteq L_i(\mathcal{H})$. A (symmetric) Fock space, $\Gamma(\mathcal{H})$ over the space $\mathcal{H}$ is defined as $\Gamma(\mathcal{H}) = \sum_{n=0}^{\infty} \otimes \Gamma_n(\mathcal{H})$ where $\Gamma_0(\mathcal{H}) = C$, and $\Gamma_n(\mathcal{H}) = \overset{n}{\underset{i=1}{\otimes}} \mathcal{H}$. U_t^* is an *adapted process* in $\Gamma(\mathcal{H})$ if for any $t > 0$, $U_t = U_{t]} \otimes I_{t]}$ where $U_{t]}$ is an operator that acts on the tensor product space $\mathcal{H} \otimes \Gamma(L^2(0,t))$ and $I_{t]}$ is the identity operator acting on the tensor product space $\Gamma(L^2(t,\infty))$. The operator $A_t : \Gamma_n(L^2(\mathcal{R}^+)) \to \Gamma_{n-1}(L^2(\mathcal{R}^+))$ is called the *annihilation operator,* A_t^+ the *creator operator*, and Λ_t the *particle number* or *conservation* operator. These particle operators are also martingale operators, i.e., $E_t(A_s) = A_t$ for $s > t$ (Holevo, 2001, pp. 119-127). These operators are defined formally on $\Gamma(L^2(\mathcal{R}^+, C))$ as:

$$A_t(\psi(g)) = \int_0^t g(s)ds\psi(g)$$
$$A_t^+(\psi(g)) = \frac{\partial}{\partial\varepsilon}|_{\varepsilon=0}\ \psi(g+\varepsilon\chi_{[0,t]}) \tag{5.12}$$
$$\Lambda_t\psi(g) = \frac{\partial}{\partial\varepsilon}|_{\varepsilon=0}\ \psi(ge^{\varepsilon\chi_{[0,t]}})$$

The Heisenberg relationship $[A_t A_t^+] = tI$ exists, that is, the commutation relations follow:

$\left[A_i(t), A_j(s)\right] = \delta_{ij}\min(t,s)$. The incremental time evolution can be given as:

$$V_t\left[A_j(s)\right] = A_j(t+s) - A_j(t) \tag{5.13}$$

The differential forms are defined as:

$$\begin{aligned} dA_t &= A_{t+dt} - A_t \\ dA_t^+ &= A_{t+dt}^+ - A_t^+ \\ d\Lambda_t &= \Lambda_{t+dt} - \Lambda_t \end{aligned} \tag{5.14}$$

and the multiplication rules for the differentials are defined by the table:

Table 2

Itô Multiplication Rules for Quantum Differential Operators

Adapted from Hudson and Parthasarathy,1984 and Boukas, 2004.

Operators	dA_t^+	$d\Lambda_t$	dA_t	dt
dA_t^+	0	0	0	0
$d\Lambda_t$	dA_t^+	$d\Lambda_t$	0	0
dA_t	dt	dA_t	0	0
dt	0	0	0	0

The processes depicted by:

$$\begin{aligned} B_t &= A_t - A_t^+ \\ P_t &= \Lambda_t + \sqrt{\lambda}(A_t + A_t^+) + \lambda t \end{aligned} \tag{5.15}$$

are the quantum analogies to the Brownian process and Poisson process of intensity λ, respectively, via the vacuum characteristic functionals (Parthasarathy, 1992),

$$\begin{aligned} \left\langle \psi(0), e^{isB_t}\psi(0) \right\rangle &= e^{-\frac{ts^2}{2}} \\ \left\langle \psi(0), e^{isP_t}\psi(0) \right\rangle &= e^{\lambda(e^{is}-1)t} \end{aligned} \tag{5.16}$$

This model assumes that an entangled informaton is the particle placeholder of interest.

Next, the noise field in the above quantum stochastic model will be transformed using the group of Poincaré transformations, $P_+^* = \mathbb{R}^2 \times L_+^*$ where L_+^* is the proper orthochronous Lorentz group, as a prerequisite for GR consideration (Frigerio & Ruzzier, 1989). In the operator L_3 of the evolution equation (5.11) is embedded the operator H considered an inertial system (IS). Now apply a Poincaré transformation, $p \in P_+^*$ to H resulting in the transformed IS, $H^p = pH$. Let $L_3^p = (iH^p + \frac{1}{2}L^*L)$ be the newly transformed evolution operator in (5.11). We write the new linear relativistic quantum stochastic differential equation with Poincaré transformation p as:

$$dU_t = \left[L_3^p dt + L_2 dA_t^* + L_1 dA_t + L_0 \Lambda_t \right] U_t = G_t^p U_t \tag{5.17}$$

Then applying the above conditions as in (5.11), (5.17) has a unique solution, U_t^p which is an *adapted process* and strongly continuous.

We now consider the time evolution of general informatons, that is, of an *N* multipartite systems with entanglement. Busse (2006) considered multipartite Markovian open systems that are capable of being entangled and the informaton model will be adapted, in part to this formalism. Consider the case of a d_i-dimensional state space for the i^{th} particle in an open quantum system (Breuer & Petruccione, 2006, pp. 105-110). A Markovian open quantum system follows the time evolution of the stochastic differential equation:

$$\begin{aligned} U_t &= \mathcal{L}U_t = \frac{i}{\hbar}\left[H, U_t\right] + \sum_{k=1}^{N} \mathcal{L}_k U_t \\ \mathcal{L}_k U_t &= (J_k U_t J_k^\dagger) - \frac{1}{2} J_k^\dagger J_k U_t - -\frac{1}{2} U_t J_k^\dagger J_k \end{aligned} \tag{5.18}$$

where J_k is the (generally nonHermitian) jump process defined for the k^{th} informaton. The first term on the RHS of (5.18) defines the unitary evolution, while the second part defines the coupling between the system and its surrounding environment. Here, $N \le \max_{1 \le i \le N} d_i$. Entanglement dynamics will be described for this open system evolution.

Now consider a general entanglement measure, a monotone map M such that:

$$M : U_t \to M(U_t) \in \mathbb{R}, \text{ where } U_t \in L(\mathcal{H}_N) \tag{5.19}$$

A direct propagation of entanglement in which M is directly propagated under the action of a system Hamiltonian coupled with environment will be done. This avoids some computation problems. *M* contains only reduced information on the system and as such will depend on function, $f_i(U_0)$ of the initial state, U_0. In other words, the evolution is governed by a mapping in which

$$M(0) \underset{f_i(U_0)}{\overset{H,\mathcal{L}}{\rightarrow}} M(t) \tag{5.20}$$

In the spirit of solving the master equation (5.18) with entanglement measure *M*, and the above suggestive form, consider the differential equation:

$$\dot{M}(t) = f(M(t), H, J_k, U_0, p(U_0)_i) \tag{5.21}$$

A computational approach will be taken involving Monte-Carlo simulation of quantum trajectories $\{|\psi_i(t)\rangle\}$. First consider a multipartite system *S* that is described by a mixed state $U_S(t)$ which is weakly coupled to an environment (bath dependent), consisting of jump operators, $\{J_k\}$. Its master equation is:

$$\dot{U}_S(t) = \mathcal{L}(H_S, J_k)U_S(t) \tag{5.22}$$

Apply a quantum Monte-Carlo simulation to (5.22) which will produce a series of runs $\{|\psi_i(t)\rangle\}$ such that the average trajectory is:

$$U(t) \simeq \frac{1}{N}\sum_{i=1}^{N}|\psi_i(t)\rangle\langle\psi_i(t)| \tag{5.23}$$

and which solves (5.22). Utilizing this Monte-Carlo series, one can average over the pure state expectation values in this ensemble to produce:

$$\overline{A}(t) \simeq \frac{1}{N}\sum_{i=1}^{N}\langle\psi_i(t)|A|\psi_i(t)\rangle \tag{5.24}$$

Using the technique of Lindbladian transformations, the entanglement operator can be written as:

$$M(U) \simeq \inf_{\{\mu,U\}}\frac{1}{N}\sum_{i=1}^{N}M(|\psi_i(t)\rangle) \tag{5.25}$$

where $\{\mu, U^*\}$ is a parameterization in the optimization of (5.25). Taking the continuous limit,

$$M(U) \simeq \inf_{\substack{\text{continuous} \\ \text{measurement} \\ \text{setups}}} \frac{1}{N} \sum_{i=1}^{N} M(|\psi_i(t)\rangle) \tag{5.26}$$

and the computation of the entanglement measure is completed. See von Marc Busse (2006) for details on the computational scheme for its final calculation. Informatons under the master evolution equation (5.22) can then be applied to Lerner's controlled constrained optimization formalism for microlevels.

Zadeh (2005) general uncertainty (GU) constraints may now be applied to the underlying process in (5.17). To do this we appeal not only to Zadeh's GTU, but to notions for uncertainty in processes so as to build a legitimate uncertain process. This work was originated by Liu in which a series of notions on fuzziness and uncertainty in variables was extended to processes and subsequently to a concept for calculus of uncertain process (Liu, 2008).

Recall that fuzzy logics using the Lukasiewicz operators completely generalize Quantum logics when a series of fuzzy subsets of the underlying measure space are used. To this end, we assume this decomposition. Next, a credibility measure, denoted by Cr extends the idea of a measure space for fuzzy variables when as a set function it satisfies:

(1) normality, $Cr(\Theta) = 1$,

(2) monotonicity, $Cr(A) \le Cr(B)$ whenever $A \subset B$,

(3) self-duality, $Cr(A)+Cr(A^c)=1,$ for any $A\in\mathcal{P}$ and

(4) maximality, $Cr(\bigcup_i A_i)=\sup_i Cr(A_i)$ for any events $\{A_i\}$ with $\sup Cr\left(A_i\right)$ <0.5

where $\mathcal{P}$ is the power set of a nonempty set, Θ. Normally, Θ will depict the sample space. A *fuzzy variable* is then a function, $f:(\Theta,\mathcal{P},Cr)\rightarrow\mathbb{R}$. A *fuzzy process* is a function, $X_t:T\times(\Theta,\mathcal{P},Cr)\rightarrow\mathbb{R}$ where *T* is an index set, so that for each index *t*, X_t is a fuzzy variable.

A fuzzy process X_t is said to have independent increments if the increments $\left\{X_{t_i}-X_{t_{i-1}}\right\}_{i=0,\ldots,k}$ are independent fuzzy variables for all $t_0<t_1<\ldots<t_k$ and it is said to have stationary increments if for any given $t>0$, $\{X_{s+t}-X_s\}$ are identically distriburted fuzzy variables $\forall s>0$. A fuzzy process C_t is a *C process* if:

(1) $C_0=1$,

(2) C_t has stationary and independent increments,

(3) Every increment $C_{s+t}-C_s$ is a normally distributed fuzzy variable, with mean, *et* (drift) and variance, σ^2t^2 (σ diffusion) whose membership function is:

$$\mu(x)=2\left(1+\exp\left(\frac{\pi\left|x-et\right|}{\sigma t\sqrt{6}}\right)\right),\ x\in\mathbb{R} \qquad \underline{(5.27)}$$

When $\left(et,\sigma^2t^2\right)=(0,1)$ then the C process is a *standard C process*. A fuzzy calculus may now be constructed and defined based on a fuzzy integral, with a chain rule and integration by parts using standard C processes (Liu, 2008). Moreover, if C_t is a standard C process and *f* and *g* are functions then,

$$dX_t = f(t, X_t)dt + g(t, X_t)dC_t \tag{5.28}$$

is called a *fuzzy differential equation* whose solution is a fuzzy process X_t that satisfies (5.28) identically in *t*. Now define a chance space to be the product space of a classical probability space and credivility space, $(\Theta, \mathcal{A}, P)\times(\Theta, \mathcal{P}, Cr)$. A hybrid process is a function $X_t : T\times(\Theta, \mathcal{A}, P)\times(\Theta, \mathcal{P}, Cr) \to \mathbb{R}$, *T* an index set. A hybrid process, X_t has independent hybrid variables and stationary increments in an analogous way to fuzzy processes. Define a (standard) *D process* as $D_t = (B_t, C_t)$ where B_t is a (standard) Brownian process and C_t is a (standard) C process. When B_t is a standard Brownian process and C_t a standard C process, then the hybrid process defined as:

$$X_t = et + \sigma_B B_t + \sigma_C C_t \tag{5.29}$$

is called a *scalar D process* with drift *e*, random diffusion, σ_B, and fuzzy diffusion, σ_C. The hybrid process,

$$X_t = e^{et+\sigma_B B_t+\sigma_C C_t} \tag{5.30}$$

is called a *geometric D process*. If B_t is a standard Brownian process, C_t a standard C process, and *f, g*, and *h* are functions then,

$$dX_t = f(t, X_t)dt + g(t, X_t)dB_t + g(t, X_t)dC_t \tag{5.31}$$

is called a *hybrid differential equation* whose solution is a hybrid process X_t that satisfies (5.31) identically in *t*. Finally, we define an uncertain variable, process, and calculus. Let Γ be a nonempty set, and $\mathcal{L}$ an algebra over Γ. An element $\Lambda \in \Gamma$ is called an event analogous to our other process type events. Let $M(\Lambda) \to \mathbb{R}$ assign a level for Λ such that the following are satisfied:

(1) (normality), $\mathcal{M}(\Gamma) = 1$,

(2) (monotonicity), $\mathcal{M}(\Lambda_1) \le \mathcal{M}(\Lambda_2)$ whenever $\Lambda_1 \subset \Lambda_2$,

(3) (self-duality), $\mathcal{M}(\Lambda) + \mathcal{M}(\Lambda^c) = 1$, for any event Λ and

(4) (countable subadditivity), $\mathcal{M}(\bigcup_{i=1}^{\infty} \Lambda_i) \le \sum_{i=1}^{\infty} \mathcal{M}(\Lambda_i)$ for any events $\{\Lambda_i\}$

The set function, $\mathcal{M}$ is called an *uncertain measure* if it satisfies the above four conditions (axioms). An uncertain variable is a measureable function, $X : (\Gamma, \mathcal{L}, \mathcal{M}) \to \mathbb{R}$, where $(\Gamma, \mathcal{L}, \mathcal{M})$ is an uncertainty space and for any Borel set *B* of real numbers, $\{\gamma \in \Gamma \mid X(\gamma) \in B\} \in \mathcal{L}$. An *uncertain process* is an uncertain function, $X_t : T \times (\Gamma, \mathcal{L}, \mathcal{M}) \to \mathbb{R}$, *T* an index space, so that $\{\gamma \in \Gamma \mid X_t(\gamma) \in B\} \in \mathcal{L}$ for each *t*.

An uncertain process has independent uncertain variables for all times and stationary increments for a given time in an analogous manner to the above definitions of processes. A *canonical process* is an uncertain process W_t that satisfies the following:

1. $W_0 = 0$ and W_t is sample continuous,
2. W_t has stationary and independent increments, and
3. $E[W_1]$=0 and $Var[W_1] = 1$

It follows that $E[W_t]$=0. If *dt* is an infinitesimal time interval and $dW_t = W_{t+dt} + W_t$ then $E[dW_t] = 0$ and $dt^2 \le E\left[dW_t^2\right] \le dt$. A chain rule and integration by parts can be suitably defined for canonical process. Moreover, is W_t is a canonical process and *f* and *g* are functions, then:

$$dX_t = f(t, X_t)dt + g(t, X_t)dW_t \tag{5.32}$$

is called an *uncertain differential equation* whose solution is an uncertain process X_t that satisfies (5.32) identically in *t*.

The Zadeh GTU process generalization to these processes follow from these definitions by replacing the uncertain process by the appropriate constraint in the definition of the GC $G = (X, r, R)$ and applying the suitable calculus framework for the granularity defined, i.e., the fuzzy and uncertain component in G. Again, the quantum stochastic is fully generalized in this formalism. It follows from the development for a relativistic-quantum Itô stochastic model that one can naturally extend to a relativistic-GTU Itô stochastic model:

$$dU_t = f(t, X_t)dt + g(t, X_t)dW_t \tag{5.33}$$

where *f* and *g* are general functions, and W_t is a $G = (X, r, R)$ process. The process defined in (5.33) will then be referred to as the nonevolutional version of the GTU-process. The general evolutional version will be developed below. Prior to that development, a fractal generalization to the differential system of (5.33) will be constructed.

In the development of all the extensions to Itô processes above, differentiability (stochastic differentiability) was assumed. Consider now a generalization to these process systems where fractility and continuity replace differentiability. Recall that all these differentiable systems are continous approximations to the discrete systems of spinfoam LQG and Planck-scale processes. Quantum mechanical pathways contributing to the path integral of QM have been shown to be more akin to nondifferentiable fractal of dimension 2 (Feynman & Hibbs, 1965). This study continues to hold that informatons behave discretely within such environments, which are not plagued by singularities and of which no experimental evidence of continuity has been produced. Of course, continuity is a mathematical concept that is philosophically not falsifiable. Fractal systems replace differentiable ones by using a scale-resolution dependent surrogate function, $f_\varepsilon, \varepsilon > 0$ for a differentiable one, f and a fractal Lagrange system, $\mathcal{L}(x,\varepsilon)$ for a differentiable Riemannian Lagrange system, $\mathcal{L}(x)$ (Nottale, 2007). The new fractal displacement, $d_f X$ can be written as the composition of a differentiable displacement dx and a pure fractal one, ξ:

$$d_f X = dx + d\xi \tag{5.34}$$

In Nottale (2007), the fractal presented was of dimension 2, as presented by Feynman. Because of the nondifferentiability of ξ, stochastic differentials are used. Nonetheless, this definition propagates through the Itô stochastic differentiation process as well. The scale-resolution, ε, remains the parameter of interest in the final Itô differential. The GTU-process differential for a fractal system can then be generalized to a synonymous system:

$$d_f U_t^\varepsilon = f_\varepsilon(t, X_t) d_f t + g_\varepsilon(t, X_t) d_f W_t^\varepsilon \tag{5.35}$$

where U_t^{ε}, f_{ε}, g_{ε}, and W_t^{ε} are the corresponding fractal operators. One important effect of the scale-resolution method is that these fractal systems become scale-relative, i.e., dependent on the scale resolution of the level in a fractal system. This is then relevant for different scales and holonic levels in an info-holarchy.

In the absence of a fully verifiable unified theory of everything (TOE) and hence of information, it has been posited that the universe's laws are evolutionary, that is, physical laws change with conditions based on thermodynamic entropy or other measures of disorder or nonergodicity. This is championed most forcefully by the biologist Kaufmann and the physicist, Wheeler, along with skeptics of static grand unification theories such as Smolin (Wheeler, 1983; Kaufmann, 2000, pp. 141-265; Smolin, 2007; Frank, 2010). Kaufmann, more forcefully posits that natural selection is manifested by local interactions. Evolution is the result of an auto-catalysis brought on by the microcoopetition between agents in an organism. Auto-catalysis is simply a chain of causal catalyses that loops back onto itself and each catalysis is an exchange of information between agents. This exchange, as posited in this study, is an interaction of informatons in an info-holarchy. Hence, evolutional operators acting on an organism are auto-catalytic chains of informaton microinteractions, the stuff of informaton connectivity.

Evolutional processes will now be introduced into the adaptive informaton model. They will be based on a general category of four evolutionary trait operators - (1) genetic, (2) epigenetic, (3) behaviorial, and (4) symbolic variation (Jablonka & Lamb, 2005). Genetic operators are microscale functors acting on the genotype or microagents of a system. Epigenetic operators act on the phenotype or macroscale of a system. Behavior operators act on certain substructures of the system that designate the major behavorial processes of that system. Finally, the symbolic variation operator acts on the equivalent communicative apparatus, some would say language substructure of the system. Cross causality is evident within these operators.

These inheritance operators generalize Darwinian evolution that is solely dependent on genetic information transmission and communication. These operators, in no small way, also generalize the ideas of Hamilton and Dawkins - the selfish (spiteful) gene paradigm (Hamilton, 1970). In this broad spectrum of ideas, replicators are general entities that are capable of being copied. Vehicles are entities that contain groups of replicators with the intent of alleviating their survivability and consistency. Furthermore, gene processes may be described by the transgressions in a game theoretic setting, possessing strategies, payoffs, and competitors (other genes or players). These are the tenets of evolutionary game theory further progressed by Hamilton and Smith (Hamilton, 1967; Smith, 1982). Dawkins further generalized this structure of inheritance by positing that genes "act" to increase the probability of their trans-generational propagation, sometimes to the detriment of the vehicle(s) containing them (Dawkins, 1989). Hence, the word "selfish" in their description.

Pre-dating the development of the gene theory of reproduction was the ingenious work of the polymath mathematician von Neumann. Von Neumann developed the concept of self-replicating machines (SRMs) (he termed it "universal replicator"), abstract computational entities that are capable of self-replicating or copying their architectures for progeny (von Neumann, 1966). In this paradigm of computation, the SRM consists of a triplet (R, M, P), the resources for construction, *R*, a manipulator to build such a construction, *M*, and a program, *P*, for its construction that is self-referential in the sense that it copies these instructions into the progeny.

This first model led to the refined cellular automaton. This is the precise nature of gene replication. In a sense, SRMs may be made general to self-replication. Furthermore, self-assembly can be built into the instructions as well so that the fathering SRM may further enhance or change its architecture and update its instructions for such. This ability to do further self-assembly and change is an extension of the original von Neumann SRM and gives a means to achieve the other types of evolutional operators mentioned here. A contemporary prototype of a SRM is the open source GNU GPL project, Replicating-Rapid (RepRap 2.0) in which a robot was able to successfully replicate all of its plastic parts autonomously, forming its child machine (Bowyer, 2006). See Figure 24 below.

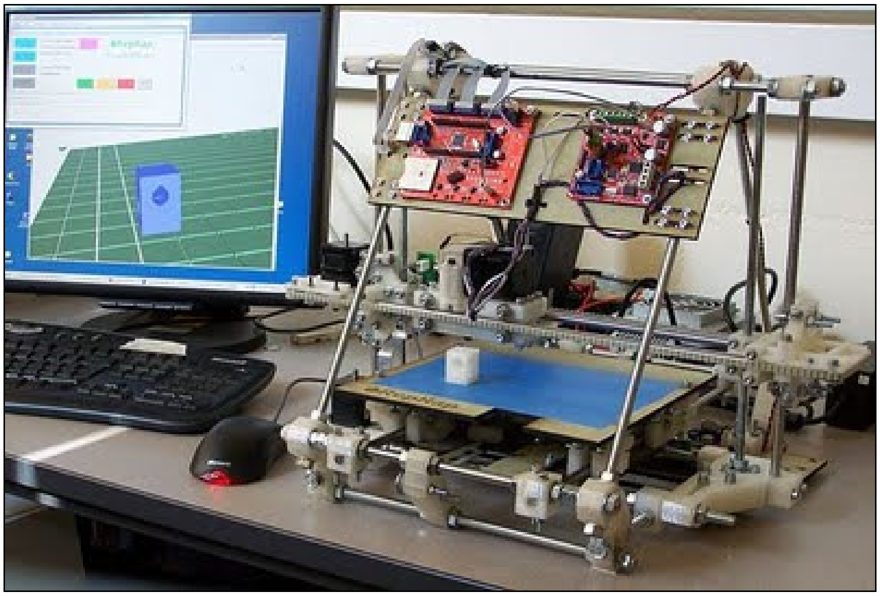


Figure 15. RepRap rapid self-replicating machine
Adapted from “RepRap v2 Mendel: Self-replicating fused deposition modeling (FDM) machine” In A. E. W. Rennie, C. Bocking & D. M. Jacobson (Eds.), *Proceedings of the 8th National Conference on Rapid Design, Prototyping & Manufacturing*, 1-8. By Bowyer, 2010. Copyright 2010 Bowyer. Reprinted with permission under the GNU Free documentation license – Creative Commons Attribution-ShareAlike 3.0.

Dawkins extends this idea to information transmission by analogizing to an entity of information he referred to as a *meme*. The meme was then the equivalent gene for information. Memes can then be viewed as units of information lacking a particular physical description. Informatons endeavor to fulfill this gap for information subcomponents that build observed information chunks in the form of, among other entities, memes. In this sense, informatons will be envisioned as subquanta of memes and other forms of information transmission. Each of the four evolutional operators that are considered, are constructed via informaton structures or info-holarchies. Epigenetic operators center on the processes of meme transmission via nonDNA media, the epigenetic inheritance systems (EISs) that form the ultimate phenotype of the organization.

Behavorial operators focus on meme transmission by acquired behavioral patternization through experiential exposure, socially mediated transmission and cultural meme changes. Lastly, symbolic variation operators concentrate on the processes surrounding language or sign transmission between entites, the semiotics of information. Semiotics will be discussed in detail and generalized for the informaton model in a subsequent section. The ongoing arguments between selfish gene proponents and opponents center on the relative importance of the vehicle-replicator relationship, i.e., which is the more important entity for inheritance. In this study, this distinction is vaporized because the various holonic levels of the info-holarchy - the vehicle in one and replicators and their subcomponents in others, equally collaborate for the whole organizational prowess and survivability.

Corresponding to each general evolutional operator will an information pipe, an operator on input parent information producing prodigal information corresponding to that evolutional concept. The four evolutional information pipes corresponding to the major evolutional operators of Jablonka & Lamb will be treated as separate information components within each physical component that will share information particles or informatons, although cross causality is valid between these evolutional pipes (Jablonka & Lamb, 2005. pp. 245-317). This genetic leakage between operators is accomodaed for by utilizing cross composition, that is, components that consist of cross products between operators. Each of those information pipes exhibit Markovian behavior in the following fashion: the genetic information pipe is governed by a complex process in which information flows from one generation to another where various history data may be passed based on the evolutional operators of mutation, crossover, and reproduction. These operators are Markovian because they possess memory of prior information. The microlevel equations discussed must then accommodate Markovian dynamics.

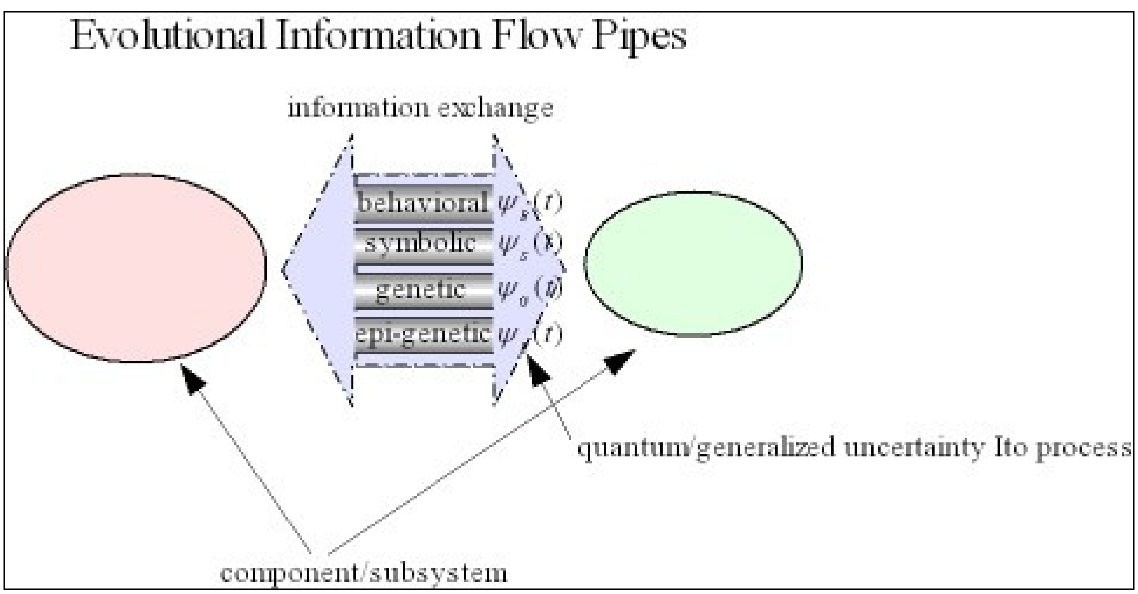


Figure 16. Evolutional information flow pipes as state operators

Macrodynamics are generated based on utilizing the variational principle (VP) applied to a path integral of the expectation of an information divergence between the microstate solutions, U_t of the before mentioned GTU micro model and the Lagrangian, L of the diffusion process, $b_t = \frac{1}{2}\sigma_t \sigma_t^*$ of a Weiner-Levy stochastic process. This divergence measure is given by:

$$D(|U_t\rangle, L(b(t))) = Tr[|U_t\rangle \log |U_t\rangle - \log L(b_t)] \qquad (5.36)$$

By optimizing this measure, a sort of "regularization to macro order" is established for the random observations, $x(t)$. This divergence can be viewed as a functional, $S_{b(t)}$ on the space of Itô processes in the microdynamic conditions: $S_{b_t}(|U_t\rangle) = D(|U_t\rangle, L(b_t))$. Now utilize the quantum variation minimax principle on S_{b_t}. Denote the variational minimax solution by $|U_t^{vp}\rangle$. The extremal trajectories of $|U_t^{vp}\rangle$ dictate the macrolevel process, resulting in a proper probability for the macro entropy functional, S_{PF}. Using the phase space coordinates (U_t, Ψ_t), where $\Psi_t = \frac{\partial S_b}{\partial U_t}$, the quantum Hamilton equations are:

$$\begin{aligned} \frac{dU_t}{dt} &= \frac{\partial \langle H \rangle}{\partial \Psi_t}, \\ \frac{d\Psi_t}{dt} &= -\frac{\partial \langle H \rangle}{\partial U_t} \end{aligned} \qquad \underline{(5.37)}$$

Together with the differential constraint condition, $C(U_t, \Psi_t) = \dfrac{\partial \Psi_t}{\partial U_t} + 2\Psi_t \Psi_t^* \geq 0$ the macro dynamic model is regularized. Here $\langle H \rangle$ is the ensemble average value of H over the quantum trajectories. The solution to this Hamiltonian system is:

$$\langle H(t) \rangle = \Psi_t^* \frac{dU_t}{dt} = -\frac{1}{2} b_t \frac{\partial \Psi_t}{\partial U_t} \tag{5.38}$$

Information transfer from microlevels to macrolevels is optimized when information loss in H is minimized (Lerner, 1998). In the classical model, Lerner uses a double-coping feed-back control mechanism (vector), v and conjugate vectors possessing a macro model process, $A(t)$ defined on the discretized points $t \in [t_1,,,t_n]$ as:

$$\begin{aligned}
&\Psi_t = \frac{A(U_t + v(t))}{2b_t} \\
&A(t)v(t) = u(t) \\
&A(t) = (\lambda_i(t)), \text{ where } \lambda_i(t) = \alpha_i - j\beta_i,\ i = 1,\ldots,n \\
&v(t) = -2U_t
\end{aligned} \tag{5.39}$$

This leads to the formulation: $A(t) = \dfrac{b_t}{2\int_0^t b_t dt}$, leading to the macromodel differential equation:

$$\frac{dU_t}{dt} = A(U_t + v(t)) \tag{5.40}$$

In general, using a quantum variational principle, if $\phi = \sum_n a_n \psi_n$ is a normalized wavefunction (mixed quantum state) and H the Hamiltonian, then $\langle \phi \mid H \mid \phi \rangle \geq E_g$ where E_g is the ground state (lowest energy state). Hence, if

$\varepsilon(\Psi) = \dfrac{\langle \Psi | H | \Psi \rangle}{\langle \Psi | \Psi \rangle}$, then $\varepsilon \geq E_0$. This is the target for an approximation to a ground state and the variational minimax problem of finding a greatest lower bound ground energy.

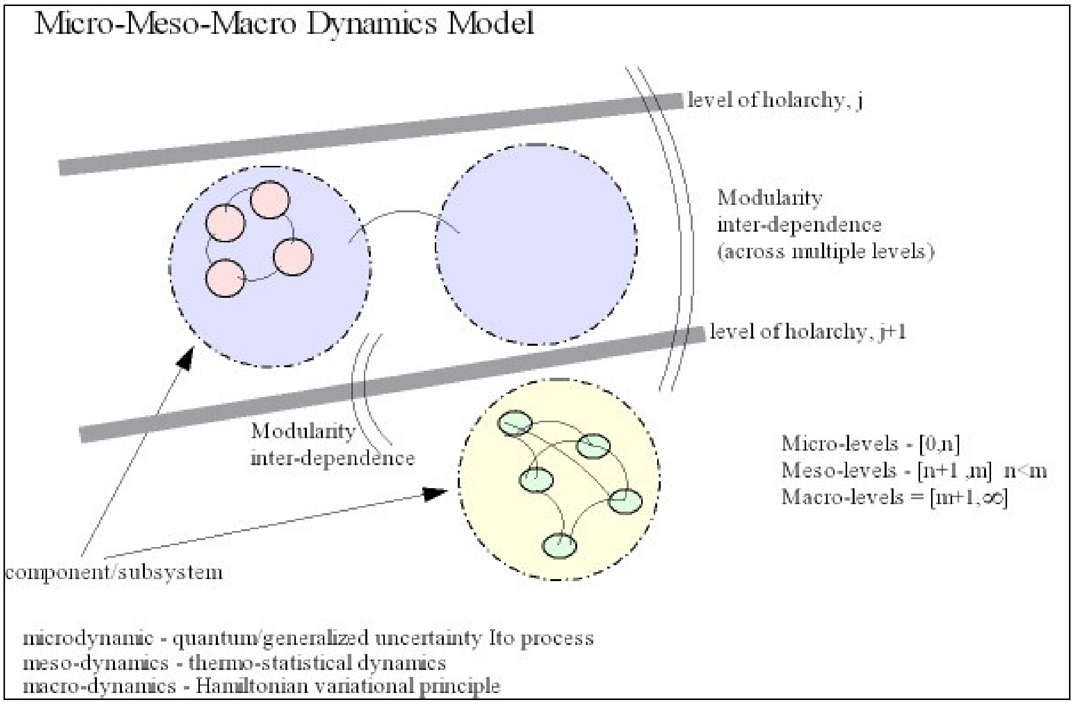


Figure 17. Micro/meso/macro multiscale model

Corresponding to the four evolutional information pipes will be four state densities, ψ_B, ψ_S, ψ_E, and ψ_G manifested by four distinct GTU Itô processes. This four-way information pipe between components of a system is shown in Figure 30 above. While these information pipes may share a certain amount of information and cross-pollinate, the microdynamic system for each will be distinct. The evolutional operators will be in the form of genetic functionals utilizing a fitness function to select quantum states in the stream and hence influence the new state. Let $\left|\psi_t^k\right\rangle$ denote the state of the k^{th} component at time t. Form the fitness operator, U_f based on a fitness function, f applied to comparing the pair $\left(\left|\psi_t^k\right\rangle, \left|\psi_t^j\right\rangle\right)$ as:

$$U_f(\left|\psi_t^k\right\rangle \otimes \left|\psi_t^j\right\rangle) = \left|\psi_t^k\right\rangle \otimes \left|\psi_t^j + f\left(\left|\psi_t^k\right\rangle\right)\right\rangle \tag{5.41}$$

Applying this to the case of comparing a general component to the ground-state $\left|0\right\rangle_i$, one obtains a "normalized" value of fitness of a component,

$$U_f(\left|\psi_t^k\right\rangle \otimes \left|0\right\rangle) = \left|\psi_t^k\right\rangle \otimes \left|f\left(\left|\psi_t^k\right\rangle\right)\right\rangle \tag{5.42}$$

Consider a general mixed quantum state of the i^{th} component, $\left|\xi_t\right\rangle_i = \sum_{n=1}^{N} (a_n)_i \left|n\right\rangle_i$.

Applying the fitness operator,

$$\begin{aligned} U_f(\left|\xi_t\right\rangle_i) &= \sum_{n=0}^{N} U_f(a_n \left|n\right\rangle_i \otimes \left|0\right\rangle_i) \\ &= \sum_{n=0}^{N} (a_n)_i(\left|n\right\rangle_i \otimes \left|f(n)\right\rangle_i) \end{aligned} \tag{5.43}$$

Selection operators, U_{S_λ} can be defined based on various schemes such as (i) proportional selection (roulette wheel), (ii) pair-wise tournament, and (iii) rank selection. In proportional selection, a component, i is selected with probability $p_i = \dfrac{f_i}{\sum_n f_n}$ (a roulette wheel is turned with the component taking up $100\,p_i\%$ of the wheel slots) where its individual fitness value is f_i. In pair-wise tournaments, two components are chosen at random and the one with the largest fitness value is chosen. In rank selection, all components will be ranked based on their respective fitness values and the larger ones are then selected. In all three cases, more than one component is compared. If the comparison is made against a ground-state energy, a selection operator, U_{S_λ} can be based on a fitness operator, f and threshold value, λ.

$$U_{S_\lambda}(|\xi_t\rangle_i) = \begin{cases} |\xi_t\rangle_i & \text{if } U_f(|\xi_t\rangle_i) > \lambda, \\ |0\rangle_i & \text{otherwise} \end{cases} \qquad \underline{(5.44)}$$

i.,e., $U_{S_\lambda}(|\xi_t\rangle_i) = \chi_{A_\lambda^c}(|\xi_t\rangle_i)|\xi_t\rangle_i$ where $A_\lambda = \{|\xi_t\rangle_i : U_f(|\xi_t\rangle_i) < \lambda\}$. In a proportional selection operator, if R_i is a region in the interval $[0,1]$ with length p_i, representing the i^{th} component, and $Rand_U([0,1])$ is a uniformly random selection in $[0,1]$ then

$$U_{S_\lambda}(|\xi_t\rangle_i) = \begin{cases} |\xi_t\rangle_i & \text{if } \mathrm{Rand}_U([0,1]) \in \mathrm{R}_i, \\ |0\rangle_i & \text{otherwise} \end{cases} \qquad \underline{(5.45)}$$

In a rank selection, if $r_{i_1} \le r_{i_2} \le ... \le r_{i_n}$ is the ranking of fitness (ranked-ordered permutation) of all indexed components $(1,2,...,n)$ in the system, and when $j = i_k$ for some $1 \le k \le n$, then

$$U_{S_\lambda}(|\xi_t\rangle_j) = \begin{cases} |\xi_t\rangle_i & \text{if } i_k \ge i_m, \\ |0\rangle_i & \text{otherwise} \end{cases} \qquad \underline{(5.46)}$$

for some threshold minimum ranking m, $1 \le m \le n$. Crossover operators, U_C will be defined using a convex combination of a process, u_t dependent on a probability, $p(\theta)$ with parameter θ,

$$U_C(|\psi_t^k\rangle \otimes |\psi_t^j\rangle) = u_t|\psi_t^k\rangle + (1-u_t)|\psi_t^j\rangle,\ u_t \sim p(\theta) \qquad \underline{(5.47)}$$

Mutation operators, U_M will be defined as,

$$U_M(|\psi_t^k\rangle) = |\psi_t^k\rangle(Cw_t + 1) \quad \underline{(5.48)}$$

where C is the amplitude of a *mutation perturbation process*, w_t, with $w_t \sim q(\tau)$, distributed as the pdf $q(\tau)$ with parameter τ. An evolutional operator on GTU states can then be generalized as the composition operator, $U_M \circ U_C \circ U_{S,f}(|\psi_t^k\rangle)$. Denote the four evolutional operators by U^G, U^E, U^B, U^{SV}. Each can subsequently be written in the above form–a specification of mutation perturbation, crossover mixture, selection criteria, and fitness function *f*, i.e., $U^h(|\psi_t^k\rangle = U_M^h \circ U_C^h \circ U_{S,f}^h(|\psi_t^k\rangle$, $h = G, E, B, SV$. The grand evolutional operator, U_E, taking into account cross-causality, can be constructed by a multidimensional cross component functional:

$$U_E\left(\left(|\psi_t^k\rangle_{k=G,E,B,SV}\right)\right) = \left(U_M^h \circ U_C^h \circ U_{S,f}^h\left(|\psi_t^h\rangle\right)\right)_{h=G,E,B,SV} \quad \underline{(5.49)}$$

which produces a 4-D evolutional output that manifests the new state of the four evolutional pipes of the system. These evolutional microprocesses will dictate four macrodynamic phenomena. These macrodynamics will, in turn, dictate evolutional processes, such as reproduction, death, modularity, and holonic behavior homogenized into a master macrodynamic. The mesodynamics are to be defined by the master information transfers that form intermediate or mesoscopic levels in the holarchy via the macrodynamic variational solution. This variational solution takes the form of a statistical thermodynamic process in this intermediate stage and so, mesoscopic levels are dictated by thermal flow. Mesoscopic levels will be defined as modules that exhibit weaker inter-dependency than other inter-level dependencies. This set of *weakly inter-dependent* modules may consist of more than one module or level and hence, mesoscopic regimes may consist of multiple levels in the holarchy. Mesoscopic levels are inherently observer-dependent because they are mesoscopic in scale. An observer component possesses a *sensorial boundary* of which a *level-lens* is fundamental. This level-lens dictates the unaided spectra of observation for the observer. Mesoscopic levels are defined by this level-lens as those which as observable without aid, i.e., within the natural level-lens of the observer. The optical spectrum of humans dictate what is visual (rhopsidan molecules). This optical spectrum is thus the natural optical mesoscopic level of observation for humans.

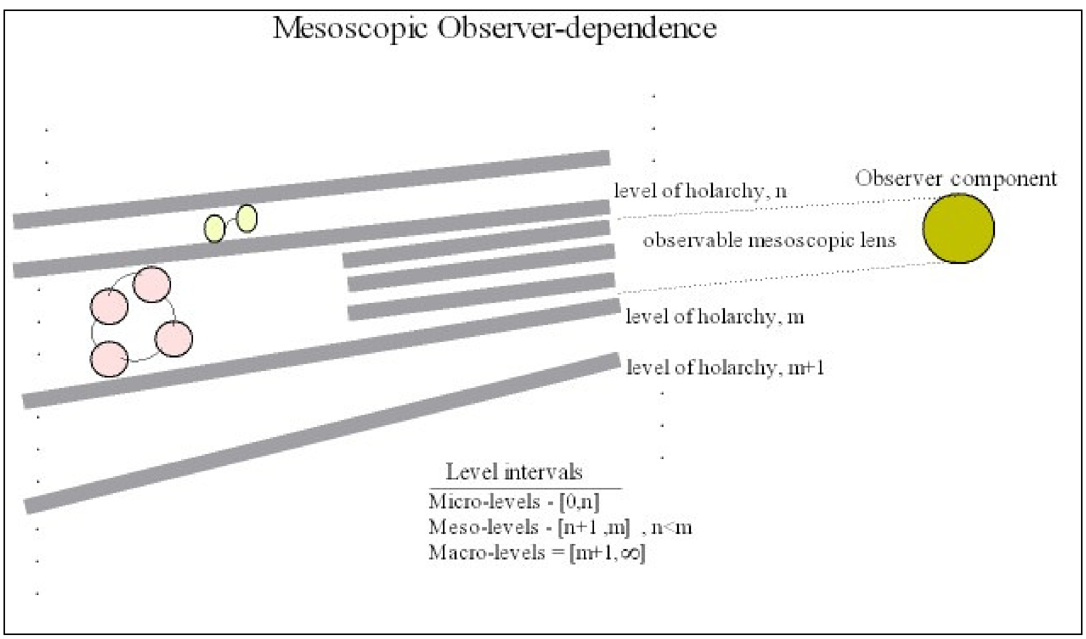


Figure 18. Mesoscopic observer-dependence lens

A general systems-theoretic approach to building a state system that exhibits dynamic behavior manifested in chaotic and quantum microprocesses and fluid-dynamic macroprocesses was presented by Selvam (2007). In this model, each component was simulated as rotating fluids emulating small eddies. This compares similarly with the quantum-general uncertainty stochastic Itô processes in the model presented.

Building Holarchies

Quantum entangled informaton games and holarchies, having now been described, will be utilized to construct the proposed information framework, the info-holarchy. This dynamic structure will also serve as a calculus for forming CAMS organization. The dynamics described by the GTU information macrodynamics and LQG-spinfoam models for informatons can be utilized to construct informaton states. First, I describe the setup for establishing a quantum game of entangled particles (informatons) using the general rules of holons as a backdrop.

The mainstay of holarchy development is the procedure of adding and deleting members from a given holonic level. The morphology of the holarchy is determined by the width at each holonic level and the depth of the holarchy. We denote the width of the l^{th} holonic level of a holarchy $\mathcal{H}_n$ of depth *n* at time *t* by $w_l(t)$. This width is determined by the dynamics of holonic activity at neighboring and resident holonic levels. Note that the depth may be time dependent as well since a new holonic level can dynamically appear or disappear. Let $G = (n, C, A, J, \omega)$ be the game for the holarchy $\mathcal{H}_n = \{H_{n-1}, O, \pi\}$. We will convert the satisfaction and resource rule set of *H* into a strategy S for coalition building based on utilities and a GTU structure. Consider the holonic state transition matrix in Table 1. A holon (agent) has a role within its present holon level. So, the state of a holon can be described by the pair, (l, R). These roles and their interactions are uniquely determine by the organization $O = \left(R^O, I^O, P^O\right)$. Within this organization lay the rules and patterns of behavior and interaction. These will then be adapted from the game structure $G = \left(n, C, A, J, \omega\right)$. In this setup, the holarchy $\mathcal{H}_n$ will be overlaid with the lattice of informatons (entangled pairs). Each holon agent in $\mathcal{H}_n$ will then share subsets of informatons with other holons. For informatons, entanglement is fundamental. Entanglement between informatons means that an event, e or observer o is shared by informatons. In this way a cluster informaton is constructed. Two bi-partite systems (such as informatons) can be partially entangled quantum mechanically if their combined state is expressible in a nonseparable way.

When a holonic level is formed, coalitions of holons are built where affinities, as specific divergence measures between any two holons in the coalition holonic level, (h_i, h_j), $D(h_i \| h_j)$, are small, i.e., are within thresholds, $D\left(h_i ; h_j\right) < \varepsilon$ where $\varepsilon > 0, \varepsilon \approx 0$. Normally, the representative holon, the HEAD, is compared to the requesting holon for membership. As mentioned before, this process can be weighted across the family of holon members producing different types of control in a level, i.e., oligarchy, etc. Now, in the game structure, G, a coalition is formed based on shared and individual utilities of the players (holon agents). The transition matrix in Table 1 will dictate the rules for a player joining a holonic level or coalition. The action rule will then take the form

(3.180) with affinity divergence (3.178). Recall that statically, $\mathcal{H}_n$ can be affiliated with a higraph. One now endows $\mathcal{H}_n$ with a game G and a GTU higraph, $\{B, E, \rho, \pi\}$ creating a GTU type holarchy. Next, an evolutional GTU-process information macrodynamics (5.49) will be implemented to describe the dynamics of $\mathcal{H}_n$. The resulting dynamic model of information, coupled with an LQG-spinfoam lattice defined by the Hamiltonian operator of type (3.43) completely describes the evolution of the info-holarchy, $\Im = \{\mathcal{H}_n, G, B, E, \rho, \pi\}$.

Info-holarchies $\Im$ endowed with these LQG-spinfoam spatio-temporal structures, general uncertainty GTU macrodynamic processes, and their accompanying information fields as described earlier by the hamiltonian field (5.9), serve as metamodels for an information generated universe of objects and their organization. Consider a holarchy view of an info-holarchy and a different, but simultaneous view of the structure through a spatio-temporal lattice, i.e., a geographic view. The holarchic view could be in relation to some functional process that informatons share, i.e., energy-mass transfer, socio-economic goals, community and familial bondage, organization tasks. The accompanying spatio-temporal view relates the spacetime relationship (divergence) between informatons, $S_{i+1,j}$ and $P_{i,k}$ located simultaneously at different functional holarchic levels. See Figure 28 below as an example of these dualistic views. They will be revisited when novel displays of organization evolution will be attempted in chapter 4.

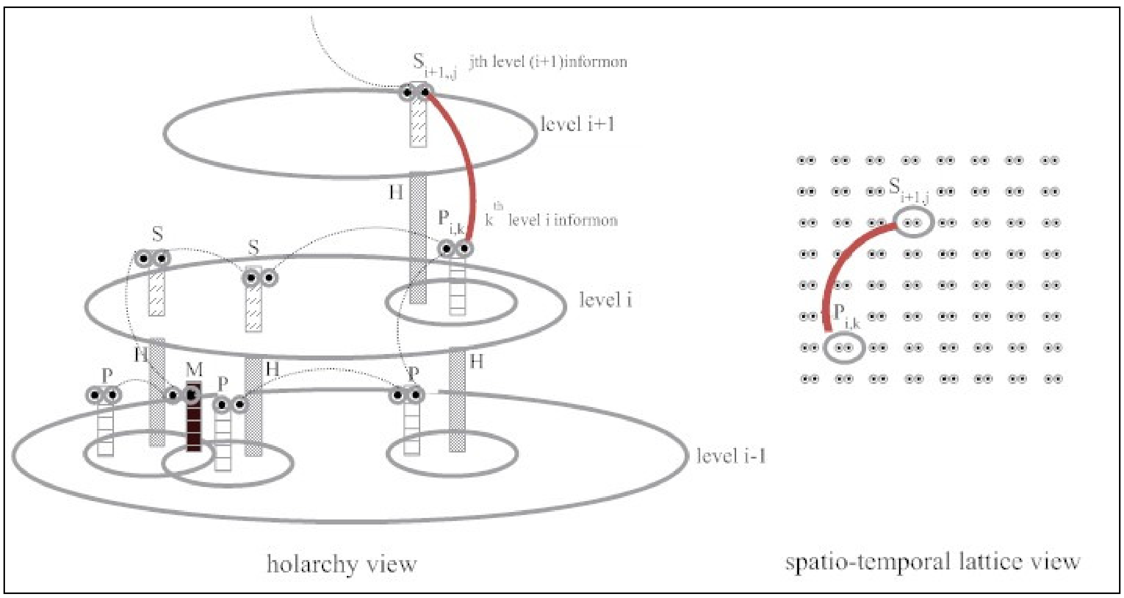


Figure 19 - Info-holarchy dualisms

Topoi of Holarchies

Previously, in chapter 2, we reviewed the power and generalizability possible using Topos Theory as a means to represent the mathematical structure of logical systems, such as natural processes in physical sciences. Here, we attempt to represent a GUT holarchy or super-holarchy by an appropriate topoi in order to predict a generalization of structure and dynamic. We have labeled a holarchy $\mathcal{H}_n$ as the GUT-higraph tuple, $\{B, E, \rho, \pi\}$ together with a GTU game, G with an informaton lattice, $\mathcal{I}$ and strategies S, developed from the choice of operators, J and starting state, ω and an underlying organization, $\{H_{n-1}, O, \pi\}$ and rules. To develop a topoi for this tuple, we start with each representation separately. First, a GTU game, G is one which is endowed with a quantum game structure in which the probabilistic rules are replaced by GTU rules. Topoi representing LQG spinfoams were presented in chapter 2. The topoi of random processes and underlying calculus, such as in the development of the micro and macro models of the information macrodynamics model can be constructed based on the structure of the time series of processes in the random process.

First, we consider the system topos, **Sys**, of subsystems of the universe. If $S \in Sys$ and $S_1 \subset S$ is a subsystem of S, then $S_1 \in Sys$. Holarchies fall into the topos of **Sys** loosely. However, they exhibit special behavior, that of self-similar embededness. Now attach the propositional and higher order type language that are associated with GTU, that is, with the GCL. We shall call such languages for a GTU-holarchy S, $\mathcal{PL}_{GCL}(S)$ and $\mathcal{L}_{GCL}(S)$ respectively. The set of function symbols (physical quantities) in such a topos will be the tuplet of S, namely, $F_{\mathcal{L}_{GCL}(S)}(\Sigma, \mathcal{R}) = \{G, O, \mathcal{I}, \mathcal{H}_n\}$. Now let $S_1, S_2 \in Sys$. We want to define a relationship between a generalized topos disjoint sum, $S_1 \Diamond S_2$ and its parts, S_1 and S_2 and between S_1 and S_2 if one is a subsystem of the other. This can be done through the relationships between their languages, $\mathcal{L}_{GCL}(S_1 \Diamond S_2)$, $\mathcal{L}_{GCL}(S_1)$ and $\mathcal{L}_{GCL}(S_2)$. It may be reasonable to define an isomorphism:

$$F_{\mathcal{L}_{GCL}(S_1 \lozenge S_2)}(\Sigma, \mathcal{R}) \approx F_{\mathcal{L}_{GCL}(S_1)}(\Sigma, \mathcal{R}) \times F_{\mathcal{L}_{GCL}(S_2)}(\Sigma, \mathcal{R}) \qquad \underline{(5.50)}$$

between their respective languages. First, an arrow $j : S_1 \to S$ in **Sys** is induced and defined by a corresponding translation from the languages, i.e., $\mathcal{L}(j) : \mathcal{L}(S_1) \to \mathcal{L}(S)$ which means that physical quantities in S are pulled-back to give physical quantities in S_1. Arrows that define subsystems, $\subset$ and compositions, $\lozenge$ are the main builders of this topoi. We add to this the list of operations on **Sys**: if $S_1, S_2, S_3 \in Sys$

1. $(S_1 \lozenge S_2) \lozenge S_3 \simeq S_1 \lozenge (S_2 \lozenge S_3)$
2. $S_1 \lozenge S_2 \simeq S_2 \lozenge S_1$
3. $\exists$ arrows i_1, i_2 in Sys such that $\mathrm{i}_1 : S_1 \to \mathrm{S}_1 \lozenge S_2,\ \mathrm{i}_2 : S_2 \to \mathrm{S}_1 \lozenge S_2$
4. $S_1 \lozenge S_2 \in Sys$
5. $\exists$ arrows p_1, p_2 in Sys such that $\mathrm{p}_1 : \mathrm{S}_1 \lozenge S_2 \to S_1, \mathrm{p}_2 : \mathrm{S}_1 \lozenge S_2 \to S_2$
6. The trivial system 1 exists such that, $S_1 \lozenge 1 \simeq S_1 \simeq 1 \lozenge S_1$
7. An empty system (terminal object), 0 exists such that, $S_1 \lozenge 0 \simeq 0 \simeq 0 \lozenge S_1$

General Semiotics of Info-holarchies

We close out the description of info-holarchies with a semiotic representation. Recall in our discussion on general semiotics of quantum systems in chapter 2, that a representatum was mapped to the evolution equations of QM. In the case of info-holarchies, the GTU Itô differential equation of (4.3.3) is the representatum of the semiotic triad for info-holarchies. Once a measurement is taken of the state of an info-holarchy, by means of a GTU precisiation operator, $\Gamma_{PNL} : p \to g$ (substitute a POVM operator for quantum probability), it corresponds to an instanciation of that info-holarchy. Therefore, the precisiation operators of the GTU constraints in the info-holarchy are the object of the semiotic triad of the info-holarchy. Finally, the interpretant is the set of game rules and strategies employed in the info-holarchy. This is given by the game G and the organization O. The pair (G, O) describes a shorthand pattern for the instanciation of the info-holarchy, the means of generating that instanciation. In this manner, a complete semiotic triad $(F(U_t), \Gamma_{PNL}, (G, O))$ is constructed for an info-holarchy where $F(U_t) = 0$ defines the GTU Itô differential equation describing the info-holarchy infodynamics.

Using semiotic chaining, info-holarchies can be represented by a semiotically interconnected series of info- holarchies, to be utilized in a system of semantical information structures. This mapping process constructs information structures holistically, where semantics, pragmatics, and contextual issues are components of the same mathematical structure – the info-holarchy semiotic triad. The semiotic chain representing an info-holarchy is laid out such that the macroscopic and mesoscopic levels depicted respectively by the interpretant, $I_{(i,j-1)}$ on the *(i.j-1)*th triad (*i*th triad on the *(j-1)*th holonic level) and representamen, $R_{(i-1.j-1)}$ on the *(i-1.j-1)*th triad is isomorphic to the microscopic level depicted by the object, $O_{(i,j)}$ on the next higher *(i,j)*th triad. Then the macroscopic level interpretant $I_{(i-1,j-1)}$ and the microscopic level object $O_{(i-1,j)}$ map into $R_{(i,j)}$, the mesoscopic level of the *(i,j)*th triad. In the info-holarchy structure, this chain is glued by the GTU Ito differential equation for microscopic processes, the macroscopic entropic function and the mesoscopic control processes—GTU infodynamics defined before. Each holon and holonic level in an info-holarchy are therefore of a certain scale level based on their representative semiotic triad designation. The triad in an info-holarchy is the generator of the scale relevance in that holonic level.

Applications

Various themes were discussed with respect to leading models of organism dynamics. Dynamic and emergent complex adaptive systems (CAMSs) were used as a background for the development of a super-organism metamodel consisting of GTU process microdynamics, leading to intermediate thermodynamic mesodynamics and Hamiltonian macrodynamics. This generalizes a quantum mechanical and relativistic dynamic for organization. By the use of causaloids, a time causal model can be generalized to a time-less structure, thereby generalizing our concept of evolution in organizations, large and small. Using Topos Theory, an abstract, high level description of these systems can be made, leading to further development in the abstract dynamics and structures of organizations in a universe. The next discussion will reveal applications for this abstract model, pointing to the specialization to certain natural and human organisms. Two such organisms will be expanded on and specialized from the info-holarchy metamodel: (a) inference machines, specifically a brain-neural organism, and (b) socio-economic organisms, most ostensibly for this study, the business dynamic as seen through a specialized holographic performance virtual reality dashboard-cave.

The informaton model depicting a generalized uncertainty scaffold serves as a calculus for constructing organizations of physical presence. In particular holarchies and more specifically, HMASs, were targeted as the general structures for organisms and prototypical adaptive organizations with many interacting agents. In this chapter we investigate applications of the informaton inspired info-holarchy model. Two applications of info-holarchies will be reframed based on the application of this calculus. They are; (a) the organization of brain cell structures, including the new found importance of brain and spinal cord glial cells known as astrocytes, the so-called other 90% of our brain, dendritic systems, and the dominantly studied neuronal structures, and (b) a holographic representation of a performance dashboard for a general business organization. Passing reference will be made to two other large scale examples of info-holarchies; (a) financial monetary networks, and (b) cosmological substrates such as galactic structures and other organized celestial clusters.

All these systems have one very broad pattern in common in their respective morphological evolution-their network development via holarchical structures, relationships, and informational processes. More generally, it can be seen that almost any organization in nature or of human-made origin is a holarchy. Here, we take a further step in positing that these holarchies are, in fact, informational structures subsuming physical presence, that is, in our language, they are info-holarchies. The philosopher-neuroscientist Chalmers makes the bold claim that all things are conscious in the universe, including the collective universe itself, and hence, so are information structures through his doctrinal version of panprotopsychism (Chalmers, 2002). This idea fits into our framework for info-holarchies, at least metaphysically.

Taxonomies for natural holarchies have been developed by various social and biological systems theorists including von Bertalanffy, Laszlo, and Wilber (von Bertalanffy, 1950; Laszlo, 2004; Wilber, 1996). In particular Laszlo develops a holarchy of natural organisms and organizations in the universe of discourse (Laszlo, 2004):

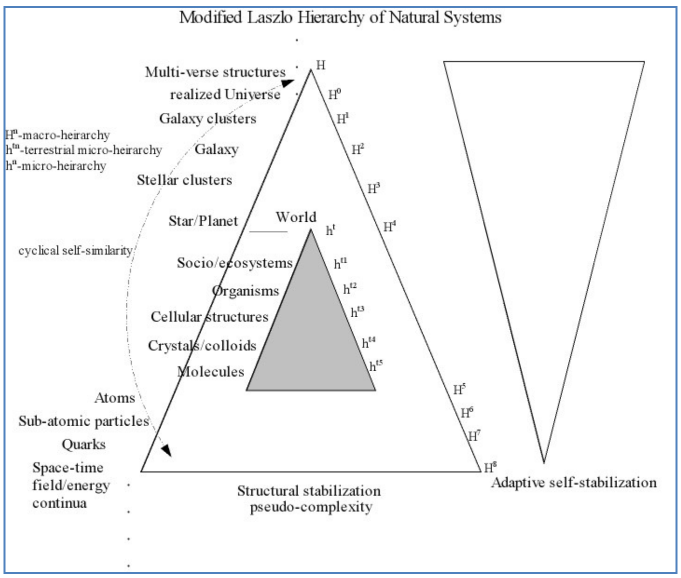


Figure 20. Metaverse holarchy

Here, we have modified this taxonomy to include a higher-order structure that represents the mathematical multiverses that are possible in describing any realized personal universe. Personal universes are generalized metaphors for consciousnesses. So, a multiverse contains all possible realizations of thoughts and physical systems based on information languages of which logics are a kind of. Transpersonal theorists posit higher-order consciousness in vague terms that resemble unfalsifiable theories of existence (Wilber, 1996). This is done in the guise of a holarchical existence.

Here, instead the concept of multiverses based on a noncollapsable wave equation which itself will be based on generalized uncertainty principles discussed in this dissertation is a sub model for our universal holarchy. This is falsifiable based on the application of tests of uncertainty to the structures that arise from such calculus. In the above taxonomy, we adapt a generalization of the concept of Everett's many-world's interpretation (MWI) of quantum mechanics in which the Schrodinger wave equation for the universe does not collapse. Instead, an Ultimate Ensemble exists in which all possible worlds with all possible mathematical descriptions exist as separate realizations of separate governing equations (Tegmark, 2007). One such subset of ensembles comes from the usual quantum mechanical wave equation. The transcendence from one multiverse to another and the decision logic that points to the version of the multiuniverse one experiences are the points of contention in a comprehensive physical law of multiverses and ultimate ensembles. The commonality between each universe ensemble is the concept of information, albeit, with different transmission and interpretational logic systems. A parameterization for such information logics would then be uncertainty via GTU and the generated propositional language and logic systems.

We return to the ultimate constructs of informatons, HMASs. Adaptive multiagent systems that are self-aware in nature are holarchies. This is so because of the fractal nature of holons and how its information flows in a dynamic dance with its adaptively formed organization shell. Holons, by the nature of their existence, learn from within and outside of their artificial physical membranic boundaries. In their holonic organizations, the chain of holons connecting them to hierarchically lower and higher holons is itself a subholarchy of the above taxonomy. In the highest level holon are included logical systems, including any human-made Gödelian mathematical system. This ultimate holon as well as the lowest holon are labeled as self-infinite, that is, they possess self-contained infinite subholarchies. These boundary holons hide the detail of unknown ends but nonetheless act as bookmarks for exploratory structure. In the gist of this taxonomy, we put into action info-holarchies which incorporate GTU processes in order to add higher informational structure and dynamics to natural and anthropomorphic holarchies.

Inference Machines and Brain-Neural Structures

We now consider in overview brain cell structures which include the microbiology and physiology of neural systems consisting of the main cell neuron or soma, the input signal conduits supplied by the dendritic subsystems, the axons, supplying the output signal conduits, the synaptic junctures joining these subsystems for connectivity, and the novel findings that have established the synergism between the abundant neural supporting and dependent glial cells in the brain – the astrocyctes, and these neural systems (Koob, 2009, pp. 29-41; Gourine, et al., 2010). These microsystems comprise a vastly complex organization of interacting, adaptive, and independent subsystems (split-brain corpus callosum separation) that are the foundation of human thought, consciousness, and motor action. Indeed, their functional and morphological characteristics manifest a distinguished and magnificent intelligent HMAS. To see this we first consider the organization of brain cell types, how information is transmitted between them, and finally state a hypothesis about their high-level adaptive structure in the language of an info-holarchy.

The human brain consists of approximately 2% of the total body weight, yet consumes 20% of its total energy. Of that energy consumption over 80% is devoted to the maintenance of neuronal connectivity (Raichle, 2006). There are approximately 10^{10} neurons in the human brain upon optimal maturation after birth. On average, 20% of these neurons die during the human lifespan. This turnover is as of a result of external stimuli, such as environmental pollutants, ingestion of carcinogentic elements, injury, and of internal stimuli, such as genetic dispositions to particular structural changes, diseases, and maturation of evolutional patterns in the structural formation of these cells and their long-term connectivity. This neuronal-based cellular structure of human brains was discovered and developed by Santiago Ramón y Cajal in the early twentieth century through animal experimentation and human autopsy (y Cajal, 1899).

Each neuron has on the order of 10^3 connections with other neurons through gap junctures situated in the dendrites and axons of the neuronal structure. The average speed of an electrical signal in neurons is 100m/s and the average voltage gradient in a neuron is 70 mV. More incredible is the effective bandwidth of human consciousness. This was posited to be approximately 16 b/s even though the stimuli received by the brain from the senses is well over 11Gb/s (Helmuth, 1969, p. 69). The perception-to-sense ratio in humans is thus less than 10^{-7}. It is a form of exformation, the notion that conscious information has been compacted into an efficient package through the discarding of redundant or unnecessarily lengthy strings (Nørretranders, 1999, p.124-156). During the experiental and genetic lifespan of a human, these connections are either strengthened or weakened, many times to the point of synaptic death or nearly synaptic immortality. This morphogenesis of the topology of the neural space of the human brain via the changing set of neurons and their connectivity lattice is called brain plasticity. These strength values are normalized and then compared to universal threshold values in order to categorize synaptic mortality. The Hebbian Theory of connectivity establishes the well known dictum that synaptic strength between two neurons is a result of nearly simultaneous electrical firing and through a reinforcement law relating presynaptic and postsynaptic signals (Hebb, 1949). In simplified terms, the Hebbian rule is:

$$\Delta w_i = \eta x_i y \tag{6.1}$$

which expresses the connection weight gradient or change, Δw_i of the *i-th* dendritic presynaptic input to a neuron with its postsynaptic output *y*, and a learning rate given by η. In terms of a dynamic differential system, this can be restated in the form:

$$\frac{dw_i(t)}{dt} = \eta x_i(t)\, y(t) \tag{6.2}$$

Hebbian learning rules are obviously unstable for a neural network because it produces unbounded learning gradients and does not provide for possible diverging weak connections leading to potential synaptic turnover. Two modifications to Hebb's rule - the Generalized Hebbian Algorithm (GHA) and the BCM Synaptic Modification, normalize synaptic values and account for weak and/or bounded learning gradients, hence resulting in stable neural networks. Moreover, these rules have successfully approximated in-vivo experimental results in some brain subsystems such as the hippocampus and neocortex. Further enhancements to these rules in the unsupervised learning neural network field of study have been developed to further refine learning rates and computational efficiency towards the optimal estimation of system parameters in training and generalization problems. Of these the two most ostensible belong to the class of self-organizing algorithms or maps (SOM) referred to as Kohonen maps and unsupervised versions of Kernel machines (Baudat &Anour, 2001) (Hofman, Schölkopf, & Smola, 2008). Kohonen's SOMs endeavor to conserve the topological relationships between neurons, i.e., topologically close neurons tend to correlate better with each other. For completeness we give these modified Hebbian rules, which are generalized to handle multiple outputs. The GHA is given by:

$$\Delta w_{ij} = \eta y_j \left(x_i - \sum_{k=1}^{j} w_{ik} y_k \right) \tag{6.3}$$

where w_{ij} is the connection weight between the *i-th* presynaptic input and the *j-th* postsynaptic output, x_i is the *i-th* input neuron signal and y_j is the *j-th* output signal (Sanger, 1989). The BCM rule is given by:

$$\frac{dw_j(t)}{dt} = \varphi\left(\overline{c}(t)\right) c_j(t) - \varepsilon w_j(t) \tag{6.4}$$

where for time *t*, for the *j-th* synaptic connection, $w_j(t)$ is the weight and $c_j(t)$ is the current, and $\overline{c}(t)$ is the weighted presynaptic vector signal, ε is the time constant of uniform decay for all synapses and φ is a postsynaptic activation function which has the form that it changes sign at a given threshold, θ_w (Bienenstock, Cooper, and Munro, 1982). The BCM learning rule is a dynamic analogue rule. The Kohonen learning rule is given by:

$$\Delta w_i = \eta_i N_{ij} D\left(x \,\|\, w_i\right) \tag{6.5}$$

where w_i is a vector of weights that connect the inputs to the *i-th* output, N_{ij} is a node neighborhood (similarity) function such that $N_{ii} = 1$ and $N_{ij} \to 0$, as $D(r_i \,\|\, r_j) \to \infty$, r_i is the *i-th* output node, *D* is a suitable metric or more generally, a divergence measure in the space of node vector values, and x_j has been chosen as the winning node, i.e., $w_j \cdot x \le w_i \cdot x,\ \forall i$ for a randomly chosen beginning input vector *x*. The learning rate, η_i is now dynamic.

The most interesting form of learning because of its generalizability and mathematical power is the kernel machine. The kernel machine employs a kernel method which is a device that separates the output features (output node values) of a neural network in a different space, the kernel space, in such as way that more distinguishability is accomplished, hence providing better separability. This learning algorithm is a higher level generalization of the other methods and can represent other types of network or input-output systems. The details of such method follow. The kernel machine approximates a function $y_d = f(u)$, where $u \in \mathbb{R}^N$, $y \in \mathbb{R}$ using a training dataset of dimension *P*, $\left\{u_k, y_d^k\right\}_{k=1}^{P}$. The input vectors are then projected into a higher dimensional feature (node value) space, by a set of nonlinear functions, $\varphi(u): \mathbb{R}^N \to \mathbb{R}^M$. Finally, the output is obtained as the linear combination:

$$y(u) = \sum_{i=1}^{M} w_i \varphi_i(u) + b = w^T \varphi(u) + b \tag{6.6}$$

where w is the usual weight vector relating input to output nodes and b is a bias term selected for the purposes of suitable robust approximation and error correction in a statistical manifold. Now form a kernel operator, K as the inner product:

$$K(u_k, u_i) = \varphi^T(u_k)\varphi(u_i) \tag{6.7}$$

Map expression (6.6) in terms of this operator:

$$y(u) = \sum_{k=1}^{P} \alpha_k K(u, u_k) + b = \alpha^T \overline{K} + b \tag{6.8}$$

where $\overline{K}$ is the appropriate matrix representation of the kernel operator, K (Horváth, 2006). The new weights are designed by the vector α. The new space where the output feature nodes are mapped has dimension P and is called the kernel space. The modus operandi for these consecutive mappings of the output nodes is that the sepability of the features of the output nodes becomes easier because a linear line can now distinguish or separate them. Additionally, a device called the kernel trick can now be applied to the solutions of (6.8) in the kernel space in which the solutions are built without direct knowledge of the original feature output node space, in which in turn, it may be difficult to separate features of the output nodes. Kernels can be used that generalize all the aforementioned learning rules and extend them to other more powerful mathematical and computational ones.

When a neural network contains many layers of neuronal subsystems, a learning rule applied to it is considered "deep learning". Otherwise, for relatively small numbers of layers, usually less than three, a learning rule applied is considered “shallow learning”. Most practically applied neural network algorithms utilize three or fewer layers and hence are considered shallow learning. Learning rules are essentially rules for neuronal evolution. They dictate the pattern of neuron lifecycles. Important to this is the dimension of the neural network since a vast array of such can overwhelm computational resources, including the wetware of the human brain. This is called the “curse of dimensionality” and is no new problem for estimation algorithms. Related to this is the effective dimension of such algorithms. This can be generalized to the wetware of the human brain and other biologically inspired computational machines. The effective dimension of an algorithm is a measure of the computational complexity of the algorithm in terms of the minimum dimension of the input data needed to effectively accomplish the task of learning such as in estimation and classification. Recently, the effective dimension of kernel machines was shown to be an indicator of their convergence rate in solving problems of identification (Zhang, 2003). Projecting this to human brains, a mathematical construct can be made to reduce the effective resources needed, i.e., an efficient resource management of wetware.

Looking more closely at the synaptic junctures, the synaptic strength is more akin to a function (usually a sigmoidal) of the mathematical summation of contributing electrical charges from denritic inputs to that neuron. A threshold function is then applied to this function resulting in an output signal strength value. These electrical pulses are binary signals. These signals are in turn created by ionic gradients normally manifested through wave transmissions of calcium or potassium ions. These waves also cause the transmission of neurotransmitters such as serotinon, dopamine, and epinephrine which facilitate these gradients. Analogue signals are sent from mechanical stimuli such as muscle manipulations and are in turn converted to digital action potentials by neuronal subsystems. These action potentials are summed according to the aggregate of similar action potentials.

Brain cells, as connectivist microsystems of the central nervous system (CNS) are sensorial in that they receive coded information via electrical pulses from ionic gradients. Shannonian doctrine shows that these analogue information potentials are reliably represented by their binary coded representations given adequate channel width and strength. This is the computational basis for most artificial neural network dynamics.

Connections may be inhibited or excited (strengthen) depending on the resultant potential. Additionally each contributing input to a neuron may be weighted before summed. As an example consider the following single neuron environment. Neuron n_i^j in a brain subsystem, B_i receives input signals from m other neurons, labeled as the neuron subset $\left\{n_i^k\right\}_{k=1,\ldots,j-1,j+1,\ldots m}$. The action contribution from neuron n_i^k is given by α_i^k and its respective weight is w_i^k. Then the output action of neuron n_i^j is given by $g_i\left(\sum_{\substack{k=1\\k\neq j}}^{m} w_i^k \alpha_i^k\right)$ where g_i is the transfer or threshold function. More generally, this output action may be subdivided into all the output connections emanating from neuron n_i^j, as an assignment to its *l-th* output, $f_l\left[g_i\left(\sum_{\substack{k=1\\k\neq j}}^{m} w_i^k \alpha_i^k\right)\right]$. Artificial neural networks are usually simplified to consist of distinct layers of neurons that successively receive inputs from the preceeding layer and produce output to the following one. The human brain has vastly more complicated connectionist subsystems with various nonlinear and topologically foliated loops possible between subsystems of neurons. Neural subsystems may also involve feedback mechanisms in which a subset of output signals are feed back into the source neuron or to preceeding contributor neurons, emulating a control system.

Human brains are a subset of the category of learning machines in which recurrent systems, such as neural networks are statistical operators that produce iterated estimators of a target parameter(s) given training data as perceived through experiental observations and sensory inputs. In this way, artificial neural networks are caricatures of the human brain dynamic. Connections in the human brain are more highly complicated, signals involve the movement of chemical molecular systems, i.e., neurotransmitters and ionic charges, and analogue information is more diverse and exquisitely partitioned to specialized subsystems. Nonetheless, these dynamics are approximated using differential systems that involve transmembrane potentials. As an example of such models, neuronal dynamics are often approximated as integrate-and-fire mechanisms. These membrane potentials are modeled by the differential equation:

$$\tau_n \frac{dV(t)}{dt} = -V(t) + V_0 + R_n I_{syn}(t) \qquad (6.9)$$

where τ_n is a membrane time constant, R_n the membrane resistance, I_{syn} the total synaptic current, and $V(t)$ the potential at time t (Tuckwell, 1988). $C_n = \frac{\tau_n}{R_n}$ denotes the capacitance. The potential, V_m is reset to the resting value of μ_d for a refractory period of t_r and a spike is sent when the potential exceeds some threshold value, V_α .

Dendrites, on the other hand, while supplying the conduits for input signals to a neuron from other neurons, resemble cylindrically-shaped pipes with numerous dendrite spines protruding as dendrite necks from the sides, each culminating in dendrite spine heads. The spine head makes junction with the presynaptic area of the axon of an input neuron. The synaptic gap is approximately 1/100 micron wide. The surface of the spine head contains a dense region of electrons, called a postsynaptic density. The main cargo at this junction is glutamate receptors (GluRs). When the input neuron's axon releases a neurontransmitter, it binds to the postsynaptic density of the spine head that expresses GluRs. The GluRs are held to the spine head membranes by cytoskeleton elements. Cadherin, a calcium synergism partner, is responsible for the inter-neural juncture "glue". GluRs pass signals to the cytosol of the dendrite for further propagation to the neuron cell. This is further mediated by protein kinases. The shapes and sizes of dendrites are posited to be correlated with learning strength, i.e., large, wide spine heads mean stronger connections.

Cabling or the Neuronal cable theory model is an accurate method to model the flow of current in consecutive portions of dendrites. The model obtains its name from the cabling mechanism that it emulates connecting one section of a dendrite to another. It utilizes a differential system to describe the flow of current in passive uniform cylindrical neural pathways. Linear neuronal cable theory starts with the following differential system relating the flow of current in dendritic cylinders with perimeter *P*, cross-sectional area *A*, constant specific capacitance , C_m constant membrane conductance , g_m constant axial conductance g_a, and $V(X,T)$ the difference in potential at (X,T) from its equilibrium value:

$$\frac{\partial V}{\partial T}+V=\frac{\partial^2 V}{\partial X^2}-\frac{1}{\tau}\frac{J}{g},\ X\in(0,L) \tag{6.10}$$

where $X=\dfrac{x}{\lambda}$ is the electrotonic distance, *x* is regular distance, $T=\dfrac{t}{\tau}$ is nondimensional time, and *J* is the nondimensional density of exogenous current. Additionally,

$$\lambda=\sqrt{\frac{Ag_a}{Pg_m}},\qquad \tau=\frac{C_m}{g_m},\qquad g=\sqrt{g_m g_a AP} \tag{6.11}$$

where *g* is the conductance. These circuits are myeliated which means that they are insulated by membranes. Branching of these cable segments occurs in dendritic development. An extension of this cable differential system for branching and hence a way of defining subunits of dendrites for the purposes of accurately modeling current in nerve cells has been given by Lindsey, Ogden, and Rosenberg (2001). Nonlinearities in the model can be expressed approximately as:

$$\begin{aligned}\frac{\partial^2 V}{\partial X^2} &= \frac{\partial V}{\partial T} + I_{ionic} + D \\ &= \frac{\partial V}{\partial T} + I_{Na} + I_K + I_m + D \\ &= \frac{\partial V}{\partial T} + G_{Na} m^3 h \left(V - E_{Na}\right) + G_K n^4 \left(V - E_K\right) + G_m \left(V - E_{rest}\right) + D\end{aligned} \tag{6.12}$$

where D is a differential system that describes the evolution of the triplet (m, h, n).

The transmission of glutamate receptors, in the case of excitatory neuro transmission involves the opening of transmembrane ion channels through glutamate binding. It has been shown that these receptors are crystalline in organization and symmetry with respect to two well known receptors crucial to all transmission of chemical information between membranes and hence to all brain cell communication (Sobolevsky, Rosconi, & Gouaux, 2009). This is where the diffusion of neurotransmitters occurs and hence the potential for charge gradients and of signal signature into the neuron cell.

The neuronal system is part of a holarchical system of cellular, subcellular, suborgan, and super-organ structures based on these transmembrane dynamics, the resulting evolutionarily formed subsystems in the brain, such as the hippocampus and neocortex, and proceeding upward in the dependency chain, the noosphere of collective human and extra-human thought and activity. Quantum systems have been hypothesized to be the backbone of consciousness and human activity through the actions of subcellular organizations named neuro microtubules and known as the Orch OR (Orchestrated Objective Reduction) theory of consciousness (Hameroff & Penrose, 1996). Microtubules are components of two-sided (tubulin) protein polymer protofilaments that form as an asymmetric helix that are in turn, components in the space-filling cytoskeleton for all cells. These proteins generally have hydrophobic pockets where π electrons reside. These electrons do not bond with other electrons in their outer shell.

The tubulin proteins have hydrophobic pockets that are approximately within two nanometers of each other. Because of this proximity Hameroff and Penrose have postulated that quantum-level manipulations, specifically quantum entanglement, are manifested in these microtubules in brain cells and hence propagate information faster than would be possible from chemically induced information transmission. More specifically, Einstein-Bose like condensates develop from the chaining of these entanglements across synaptic and gap junctures. This produces a macroscopic quantum behavior across regions of the brain. Hence, they posit that consciousness, at a higher level than physical stimuli is the result of the microtubule collective activity. Penrose cites Godelian incompleteness of a mathematical system that can be represented purely within a Planck-scale region (Penrose, 1989, p. 457). Outside this region the quanta collapse and classical commnunication ensues. So, within microtubule structures, Godelian incompleteness manifests the subjectivity of consciousness within the Planck region and is not dependent on the topo-geometric constraints of general relativity. Nonetheless human anticipatory stimuli occurs that proceeds actual physical stimuli, as in the case of the human brain anticipating one tickling oneself and hence not losing control of laughter or the recorded pre-physical anticipation of stimuli before proceding with the thought of carrying through with an action.

Orch OR theory is not without its strong critics. Quantum decoherence occurring in the brain would spoil any such quantum entanglement schema. The distances for neuronal microtubule entanglement and coherence times speculated by Hameroff do not agree with experimental measurements. Hameroff and Penrose cite the experimental results from the detection of gamma synchronization in the brain as the product of Orch OR. Moreover, Hameroff claims that these microtubules have an A-lattice structure that facilitates quantum manipulation, specifically topological quantum error correction. These results have not been shown and further have been disproved by other results that show different structures (B-lattice and a seam) (Kikkawa & Metlagel, 2006). Lastly, the estimate of number of tubulin (dimers) in a neuron given by Hameroff was incorrectly calculated, leading to an incorrect number of neurons that would be entangled for conscious thought. Instead, on the order of 15 neurons was calculated for a time period of 25ms as required by Orch OR (Georgiev, 2009).

These refutations of the Orch OR theory do not rule out any quantum phenomena of neuronal and consciousness activity. In fact, the most stringent criticisms point to respecifications of the theory and not refutations. By specifying different parameterizations for the microtubules and quantum coherence limitations, the potential credibility of the theory endures (Georgiev, Papaioanou, & Glazebrook, 2004). It remains to construct a more plausible metamodel of microneural systems before attempting to link consciousness to such constructs.

Nonetheless, it appears that mesoscopic neural subsystems act in a thermodynamic way. Microscopic quantum behavior is trivially true since ionic activity, which carries nonneglibile quantum effects, is at the heart of neural communications. However, quantum entanglement schema will be more difficult to experimentally display. Large objects lose quantum coherence and hence their quantum entanglement properties. The size boundary of where this decoherence happens in an object is not known.

Notwithstanding the unknown size limit of quantum systems in nature, recent experiments have shown that mesoscopic systems consisting of larger ionic structures that mimick mechanical oscillators exhibit quantum entanglement behavior (Jost, 2009). By constructing holarchies of cellular and subcellular holonic structures from these distinguished neuronal subsystems with their accompanying quantum ionic communciation, a brain info-holarchy is proposed. Information transfer within and between these structures has been shown to be by transmembrane activity, ultimately mandated by ionic flow channels. Notwithstanding specific criticisms of Orch OR theory and its quantum entanglement schema for microtubule activity, the info-holarchy generalizes the brain structure. Partial quantum entanglement as previously reviewed and discussed in an info-holarchy serve as general measures of information complexity in the specific brain info-holarchy.

In a curious application of Zeno's paradox to quantum decoherence, continuous measurements of coupled quantum systems theoretically lead to perpetual coherence. Brains as quantum systems can be in a trapped coherent brain state leading to targeted actions and thus escaping the ambiguous smear of clouds of probabilistic quantum states and nonaction (Stapp, 2009, pp. 225-226) . This quantum version of the Zeno paradox is known as the quantum Zeno effect (QZE) (Misra & Sudarshan, 1977). Discrete Turing machines processing in discrete, finite steps are immune from the QZE because their measurements in the shape of computations are not continuous. Generalized brains that approach continuous measurement capability are therefore asymptotically quantum mechanical and can terminate computations leading to an eventual system action or reaction. Info-holarchies with this asymptotic measurement apparatus can therefore perform actionable computations, leading to the simulation of actionable brain states, i.e., decisiveness. Even in discrete Turing machines, for large numbers of computations, an asymptotic QZE may approximate a large enough period of coherence, on the scale of the experimentally validated nanosecond of "thought-fix", that an action is manifested.

This may be the evolutional mechanism for thought and information processing in brains simulated directly in quantum and super-quantum info-holarchies. Suppose that particular stimuli have conditioned a group of neural subsystems so that the asymptotic QZE has manifested a fixed thought-state in that federatation of subsystems for a period of quantum-level time. This thought-state is partial for the whole brain because it is manifested only in a subsystem of the brain. That thought-state is in direct competition with other thought-states resident in other subsystem federations of the brain. The neo-Darwinian process is then one in which these thought-states are compared by a metric that measures the effectiveness and size of these subsystems, i.e., the complexity of the subsystems possessing the rival thought-states.

Federations of thought-states are formed by cooperation based on similarity of thought-states. Let $\{n_i\}_{i=1}^{N}$ be the set of neural subsystems of a brain info-holarchy. These neural subsystems are holons in the info-holarchy, not particularly hierarchically ordered.

Corresponding to a neural subsystem, n_k is a thought-state, $s(n_k)$. Federated super-systems are then formed by combining several neural subsystems, $\{n_{k_i}\}_{i=1}^{M_j}$, $j = 1, 2, \ldots, M$ where M is the number of federated systems formed and M_j is the number of neural subsystems in the j^{th} federated system based on the proximity of their respective thought-states, $\{s(n_{k_i})\}$, that is, $D(s(n_{k_i}), s(n_{k_j})) < \varepsilon_k$ for some threshold, $\varepsilon_k > 0$ and similarity measure or divergence D acting on the 2-product space of thought-states.

These groups of thought-states form equivalence classes of thought-states which then represent the template for the thought-state of that federated subsystem. Hence, a federated thought-state is the development of similar thought-states. The federated thought-state with the largest evolutional fitness measure is one which possesses the largest information complexity measure (see Figure 27). The prior reviewed complexity measures for info-holarchies are considered as candidates for this thought-state metric. This is the evolutional mechanism of the brain info-holarchy. Note that these neural subsystems most assuredly share overlapping physical regions, but the thought-states correspond to unique neural subsystems.

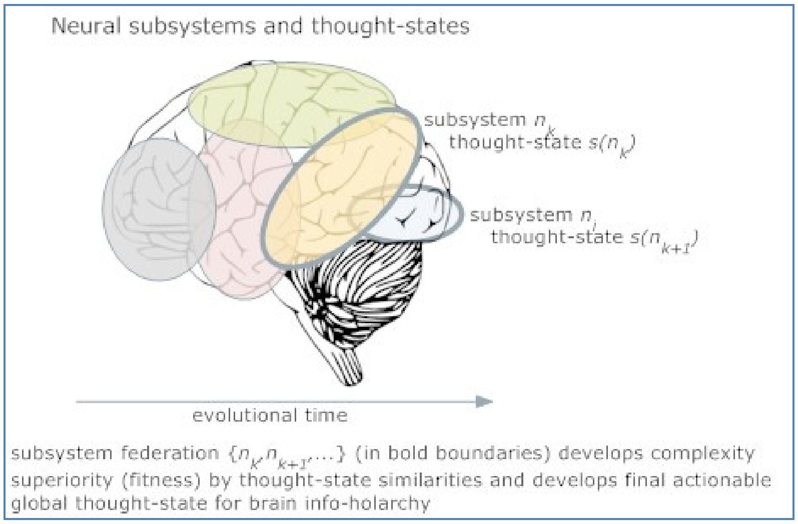


Figure 21. Brain info-holarchy and thought-states

Causal rules such as Bayesian quantum probabilities may be superimposed on the info-holarchy network structure to produce a quantum causal network over the holon infrastructure (Laskey, 2006). In this way any previous inference based on the formation of evolutionary prior distributions on past events that have been sensed and interpreted by the brain info-holarchy can be blended in. To generalize these quantum causal rules, one can impose GTU rules onto the info-holarchy. General fuzzy probability can then be utilized in the brain info-holarchy inference machine.

According to recent research conducted by Rauchle, Fox, and Zhang, the subsystem of the brain labeled the Default Mode Network (DMN) is the conductor of how other specific subsystems of the brain receive signals. This proposal emanated from experimental results that showed how brain activity shifts into a supervisory activity requiring an elevated level of energy consumption when resting or distracted, counterintuitive to classical neuroscience theories (Fox & Raichle, 2007). This capacity to transform brain circuitry into a higher energy state when seemingly at rest was posited as a sort of dark energy for the brain, synonymous to the physical postulate that most of the universe's energy is unknown, i.e., dark energy (Zhang & Raichle, 2010). The DMN is thus a candidate for a HEAD holon in a brain info-holarchy.

Brain systems are open in the sense that they are part of a larger system that surrounds them, the sources of stimuli for their sensorium. Brains exchange energy and entropy with their environments and hence information as well. When modeling the brain as a system, its externalities must be included. The extent of this externality is potentially great and Kaufmann has labeled this region as the "adjacent possible" of a system (Kaufmann, 2000, p. 146-148). In this discussion, the adjacent possible of a system is the information hull of an object, that region in which all potential information can be physically and meaningfully exchanged with the object. GR posits this as the event horizon for a particle, while in QM super-entanglement this region may potentially be the universe itself. The compromise between these two information paradigms lies in the information structure of QG. In multiverse renditions of QG, this region is extended to a super-dimensioned space, given that information fields or informatons can be exchanged between the possible universes for that object around a generalized location. Spacetime no longer is sufficient

In a review of empirical neuronal dynamics studies of large scale – mesoscopic fMRI patterns and small scale—microscopic network avalanches of small neural subsystems, Chialvo (2010) posits that brain dynamics are universally described by critical points of a second-order phase transition, i.e., *at-the-edge* (of chaotic bifurcations) near criticality; emergent behavior manifested by systems characterized by nonlinearity at the individual entity level, large ensembles of entities connected by variously scaled communication mechanisms or field effects, and external stimuli from a surrounding environment—boundary transmissions. Thre is also the matter of evolving evolvability that addresses higher order adaptive behavior that changes strategies in organisms and organizations—leading to various forms of adaptive learning strategies. Info-holarchies, by utilizing generalized evolution operators in their processes, embrace higher-order evolutionary operators (i.e., operators on operators), mimicking evolvable evolution.

Phase transitions, as mentioned above for brain dynamics, are modeled by the dynamics of thermodynamic open systems. Open systems circumvent the second law of thermodynamics – a closed system tends to increase entropy or disorder, ceteris paribus. More generally, biological systems are thermodynamically open. In open systems it is important to consider the differential flow of energy since it may increase or decrease. Free-energy is an information-theoretic measure of the difference between the energy and entropy of a system. It is therefore a measure of the potential for surprise or for unexpected energy gradients. In brain systems free-energy gives a global indication of the amount of work put into inferring or more succinctly, thinking. A promising global model for the structural dynamics of brain activity is based on the free-energy principle of statistical physics (Friston, 2009; Friston & Stephan, 2007). Friston's model is based on a hierarchical or empirical Bayes model for prior information on inputs to the senses received in the brain. The brain system is modeled as an open system connected to its external environment. It minimizes its free-energy in order to minimize the surprise from its input data sensed given a model of what the senses generated. Free energy is simply the prediction error of the brain's model given the training sets of inputs collected. Minimizing free-energy is equivalent to minimizing phase transitions that would ostensibly alter structure, a Darwinian survival. In detail, free energy is expressed as the difference between energy and entropy,

$$\begin{aligned} F(y,\mu) &= \text{energy - entropy} = -\langle \ln p(y, I \mid m) \rangle_q + \langle \ln q(I) \rangle_q \\ &= -\langle \ln p(y(\alpha) \mid I, m) \rangle_q + D(q(I) \,||\, p(I)) \\ &= D(q(I,\mu) \,||\, p(I \mid y)) - \ln p(y \mid m) \end{aligned} \tag{6.13}$$

Here *p* is the distribution that generates sensory samples, *I* is the cause of the sensorial data, *y* is the sensory input, *m* is the generative brain model, *q* the density of the causes and μ is its vector parameter, *D* is a divergence measure between probability distributions and α is an action. Sensations are given by the model,

$$y = g(I,\alpha) + z \tag{6.14}$$

External states are given by the differential,

$$\dot{I} = f(I,\alpha) + w \tag{6.15}$$

Two minimizations are performed, one to estimate an action,α that will minimize a bound on surprise by maximizing the accuracy,

$$\alpha = \arg\max_{\alpha} \langle p(y(\alpha) \mid I, m) \rangle_q \tag{6.16}$$

The other estimates the perception,μ that minimizes the bound on free-energy,

$$\mu = \arg\min_{\mu} F(y,\mu) \tag{6.17}$$

This Bayes variational optimization model under the free-energy principle is subsumed by a specialization of the GTU Itô stochastics of the informaton dynamic model with a macroscopic functional entropy measure and divergence.

As far as computational devices are concerned, neurons are specializations of human cells which have been investigated to be fully functioning wetware computational units (Bray, 2009). This is the basis for bio-molecular computation. There are approximately fifty trillion cells in the human body. Proteins, in particular, are the workhorse computational switch devices, as enzymes are the information conduits - the prototypical I/O units. More fundamentally, an information flow chain is present from DNA to messenger RNA – enzyme RNA polymerase, which in turn builds protein machines from chains of amino acids. Biochemical reactions power this computer. However, their action is dictated by ionic flow and so, once again, quantum effects are potentially the catalysis.

The helical model of DNA, is, of course, comprised of the 4-set nucleotides, (T,C,G,A) forming the possible pairs, $G-C, A-T$. RNA is formed from the 4-set, (U,C,G,A) into the pairs, $A-U, G-C$. Proteins are build from a 20-set of amino acids. Each cell is comprised of an outer membrane (lipids and proteins) which encapsulates the mitochodrian of the cell. The mitochondrian interact bio-chemically with its surrounding helper chemicals and catalysts to compute the next action in the cell's motality, i.e., it determines the strategy for the cell's action space. Each cell then comprises a special info-holarchy that connects to other cells in functional harmony and information exchange via electro-chemical exchanges - distinct information transfers. The holarchy of connections in a functional cell group, that is, ones that have distinguishable unitary tasks in the body, are chaotic and nonlinear, oftentimes, distant in geometry, but proximal in language connectivity - they are self-aware and group aware. In lower holonic levels, that of the genetic circuitry of DNA and RNA, holons exist as DNA and RNA pairings and their interconnectivity in determining chromosomal signatures. Smaller computational switches than those of proteins exist at this level – repressor and activator binders (transcription factors) to DNA. This is the way of holons and in particular, info-holarchies with generalized uncertainty processes. Info-holarchies also display morphing capabilities, as posited earlier in this chapter – they reproduce according to an optimal strategy of survival. Cells accomplish this feat when there is chemical stasis.

Because microneural activity and more ostensibly, cellular morphology and motility are dictated by chained quantum effects and evolutionary uncertainty (random selection and mutation), info-holarchic processes generalize their information engines – biochemical and ionic flow. Info-holarchies are the structures that may show that Kaufmann's hypothesis that the biosphere – all biological organization, is the biological instantiation of a higher evolutionary law of the universe - self-organization into complex adaptive systems arising from nonlinear dynamics (Kaufmann, 1995, 2000). This study hypothesizes that one version of this proposed universal law is in the form of the informaton, info-holarchies, and the generalized info-dynamics of GTU-based processes.

Deterministic computation—manifested by hardware configurations relying on solid-state or optical devices are not inference machines or even brains. They lack the awareness of true neuronal structures and the vast arrays of processors. Additionally, processing software cannot, on its own, mimick these structures. There must be a synergy of hardware awareness of its environment—the ability to adapt to its computational environment, and software management of these stimuli and reaction.

In a recent attempt to address these issues DARPA, Boston University, and HP Labs have merged a new kind of circuitry component, the memristor, with an intelligent adaptive software module that attaches to these components—the MoNETA, Modular Neural Exploring Traveling Agent (Versace & Chandler, 2010). These components are combinations of memory-processor-software chips with brain-like processing, i.e., they are aware of their environments (stimuli from other chips), react by forming survival strategies, and execute their plans. Biological wetware is emulated by an architecture that locally combines processor, memory, and instructions, rendering cache-memory type architectures moot—so called *neuromorphic computation*, the antithesis of a von Neumann machine. The motive is that the closeness and connectivity of computing and reasoning components dictate a better and more realistic architecture to compete with the biological brain.

For MoNETA-like processors to simulate info-holarchies, additional reasoning components must be built into the software that emulate lattice cellular automaton for information bit exchange, as well as the macrodynamics addressed by Itô processes, and, in the large and small organizational prowess—quantum-gravity and causaloid logic. Memristors in these configurations would then be aware, not only of their neighboring memristors, but of their subholarchies of memristors, in a dance of lattice physics.

Businesses and Multisensorial Holographic Performance Dashboard-Caves

New-paradigm organizations and businesses, in particular, are oftentimes näively viewed as complex adaptive systems and networks of actor nodes, i.e. adaptive social networks (Dooley, 1997). As autonomous and autopoietic entities, businesses themselves should also be considered as complex actors in a nested socio-economic CAS, instead of emanating solely from externalities and individual nodes from within (King, Felin, & Whitten, 2010). This introspective and holistic self-similarity is reminiscent of a natural form of an info-holarchy considering the emergent information flows involved. From the previous treatment of individual brains and inference machines as info-holarchies, businesses add holonic layers of complexity to brain structures. In this study they are presented as info-holarchies of brain-like entities. Inorganic resources that are not computational devices remain considered as inference machines because they consist of networks of informatons (particle lattices) in their respective structure.

Organization theories study the dynamics of such groups and endeavor to model their collective behaviors using variables such as resources, communication bandwidths and network connectivity, diversity and culture, historical context, psychological factors, and externalities. Some computational models for organization dynamics involve simulation using these factors as seed generators. Nonetheless, there are no unified principles for organization evolution using computational paradigms or agent-based simulation (Ashworth & Carley, 2008). Organization theories consist of a Motley crew of classical optimization paradigms and attribute tracking. The measure of wholeness of the organization remains tied to algorithms that are prefaced by sequential analysis of groups of factors. Consider instead, info-holarchies as a natural means of representing businesses in order to holistically measure their evolution.

Diverting from this argument for viewing organizations as types of info-holarchies, the concept of organization must be revisited. Anthropomorphic organizations are described generically in the organization science literature as groups of individuals with resources that have common goals, strategies, and visions to aspire to and reach via certain inputs and translated from certain outputs. This is an overly broad and simplistic systems view of organizations. Organizations may be haphazard and may involve many conflicting goals, strategies, visions, and behaviors. Organizations have feedback mechanisms, sometimes negative, sometimes positive, and often times mixed and adaptive. Collective measures of these aspects and properties of organizations are oftentimes the only accessible components, i.e., descriptions of ensemble behaviors. This is exactly what field theories do for physical components of matter and energy. Hence, modern descriptions of anthropomorphic organizations are thermodynamic in nature. This is generic to general organization.

Despite these descriptions of organization, there is no unified version of how organizations are born, form, and evolve. This is certainly the case in terms of physical first principles. In this study information in the framework developed for informatons and info-holarchies is proposed as the calculus for organization evolution. Info-holarchies vacate all of spacetime as ensembles of informatons and can transmit all forms of information and hence of energy and mass. This transmission includes that of knowledge, the arranged semantic version of information for inference machines and humans in particular.

Business organizations share in these properties, but are extended to particular strategies, visions, and goals. The ultimate goal of for-profit businesses is a positive and sustainable net monetary output. However, nonprofit organizations disguise this premise. Examples of nonprofits with the same motives are political, religious, and information-knowledge dissemination-based organizations, i. e., think tank organizations. Their realistic strategy for long-term sustainability is positive monetary or positive monetary feedback influences, i.e., they survive as long as their profited categories of advocates prosper. In the post-modern environment, themes such as globality, diverse sustainability, and community are systematically changing these motives (Engdahl, 2005). It is no longer clearly profitable or sustainable to wildly pollute the environment or bully a community into deals.

Businesses, as part of group nodes that include their suppliers, partners, and competitors in the collaborative web network of the Internet, are special kinds of socio-economic groups, subsocieties per se, are self-organized and emergent as the product and manifestation of change. Kelly (2010) has proposed the term *technium* to indicate the near autopoetic nature of technological innovations and devices—the ability of artificially intelligent devices to self-organize and self-construct an evolved version of themselves, a version of vonNeumann machines. This manifest coevolution of human agents and their intelligent cohabitors—computational devices was first coined as the techno-social sphere by Winner (1977).

Businesses as human social organisms exhibit complexity and chaosity through three properties: (1) containment, (2) transforming exchanges, and (3) significant differences (Olsen & Eoyang, 2001). This is true of any organism that is auto-poietic. Containment is the ability to define discernable boundaries for agent groups. Transforming exchanges are simply multidimensional information flows between agents. Finally, significant differences are the manifestation of differentiation between groups, the clear delineation of personnel and group types and expertises. The dynamics of these human social organisms have been posited to work almost exclusively through the appearance of strange attractors within its complexity schema (Lewin, 1999; Marion, 1999). However, strange attractors can be viewed as a particular information exchange between groups of agents in an info-holarchy. Certain strange attractor classes can be generated by the logistic growth function. More generally, a difference equation may be used:

$$x_{t+1} = f_t(x_t) \qquad (6.18)$$

where x_t represents some characteristic of the structure of an info-holarchy organism at time *t*, e. g., the network structure of the organism as an abstract time graph. The evolutional transfer functor f_t can then abstractly generate the information entanglement (transfer exchange) between informatons of the organism at time *t*. It can also represent the information field for the organization informaton agents. Most strange attractors are not generated by such deterministic evolutional functor relations even though these generate strange attractors. The form of f_t may be probabilistic or in the case of info-holarchies, a Zadeh GTU-causaloid process. Strange attractors have a multifractal structure - self-similarity as described by a space filling curve with fractal dimension as in (3.156) for the more general Rényi dimension involving probabilities of intersecting regions of the chaos' phase space. According to this strange attractor hypothesis for organizations, no real action or metamorphosis occurs without them, i.e., periodic cycles and fixed point attractors are status quo states.

To this end, business organizations are oversimplified by measurement through the visualization of certain perceived nonadaptive causal performance metrics – classical business analytics. Every organization, in particular, a business, is a class of inference machine in the sense that it has an induced information flow and adapts to its environment through sensorial data, the business metric spectrum in this case. Recently a brand of visualization tool labeled the performance dashboard has become a popular means of viewing nearly or approximated real-time performance metrics of an organization. These dashboards are normally manifested through a web portal application that summarizes various detailed information clusters relating the working of the organization to quantifiable measurements such as key performance indicators (KPIs), linkage to strategies, and scorecard metrics. Traditional performance dashboards are near real-time business intelligence systems in that these measurements are based on the ongoing flow of various business information and their objective measurements (Eckerson, 2006, p. 4-5). These dashboards mainly consist of various graphs of performance parameters mapped against KPIs. They rely heavily on conventional statistical tools such as regression analysis, time series, discriminate analysis, comparative tables, and histographs.

Business dashboard content is based on roles, i.e., the visual is dependent on the role and task spectrum of the user and so depicts information that is tightly correlated and pertinent to the user's workflow. Executive summaries in dashboards are tailored for high level decision-makers of organization, while more detailed analysis of certain workflows can be informative for mid-level management and frontline employees. What these role-based dashboards have in common are the business analytics they introduce to their user's toolkit. By delivering content specificity to members of an organization, pervasive dashboards portend to increase organizational autonomy and BI in real-time at most role levels (Lock, 2010). These analytics endeavor to meld together the stratagem spectrum of the business to measurements of performance, the KPIs they expose. Eckerson (2006) maps the maturation stages of a business utilizing a Business Intelligence (BI) Maturity Model and manifested by the anthropomorphic scale: (a) prenatal, (b) infant, (c) child, (d) teenager, (e) adult, and (f) sage. This maturation evolution is reflected by the use of business reports that converge to the eventual development and optimal utilization of dashboards (Eckerson, 2006, p. 90-100). Dashboards are therefore thought of as more complete and advanced forms of business communication that help connect decision makers to real-time movement of business performance.

These depictions of business dashboards are the general sentiments of dashboard developers and business analysts. Nonetheless, they lack empirical evidence for effectiveness. The appearance of technological prowess in the form of business analytics leads business leaders to believe in a "halo effect", that is, a sense of false security in the predictive powers of these tools based on correlational and predictive performance measurements (Rosenzweig, 2007, pp. 65-82). Analytics are not globally adaptive, do not clearly differentiate between correlation and causality, and are not optimally Markovian (i.e., do not take into account the optimum blend of past and present conditions, resultings in oftentimes ineffective model order size and form).

Additionally, traditional statistical tools utilized in business analytics use classical assumptions about the data model and dynamic of an organization. They place an ill-fated faith in the predictive powers of classical methodologies and hence of the organization physics. For example, the financial crisis and meltdown of the US markets from 2007 to 2008 and the subsequent credit crisis of 2009, were essentially products of business financial models that predicted safer margins of error for loan defaults, nonexistent derivative coverages, and higher credit ratings of companies than what were the case. Outliers or “black swans” were underweighted, while normal conditions were overly emphasized (Posner, 2010; Taleb, 2007). Positive feedback mechanisms in the financial systems then led to the snowballing of failures of bank loan transactions and their coverage. Moreover, irrational trading and rating of companies misled the general markets into aggressive group behavior (Richard, 2010). Heightened volatility in the financial markets and in those market models of trading and worth rating created snow-balling and disasterous cascading effects. However, volatility is part in parcel to the general-uncertainty Itô processes of an info-holarchy. Dashboards typically do not take into account another halo effect, that of external competitive performance which helps form a relative measure of business performance. Externalities are another form of environmental effects in an evolutional model of organization. Exogenesis, as depicted in the evolutional model of the info-holarchy, is a broad stroke towards this revelation.

Business analytics use the traditional tools of classical statistics and graphing techniques. However, data graphing may be a very subjective art as has been depicted in Wainer (2010) where historical misuse of graphs to skew a political or business decision was addressed heavily. Statistical estimation does not lie given the assumptions that they require; practitioners that abuse or misuse such tools may have deliberate intentions to mislead through the lens of an apparently legitimate looking display. This study proposes to conceptualize a new kind of category of displays – holographic virtual reality dashboard-caves in which micromaps that represent the structural or various functional submaps embedded in a business are presented with overlying patterns of behavior. An example of such a category of display is a network map depicting nodes as the member actors of the business in various structural maps that represent relationships. These micromaps will have processes overlaid on them, e.g., communication bandwidths between nodes (how often and when do they communicate with each other), output and input resources that they commonly are tasked to interact with, and linkage for depicting their ranking (i.e.interdependence in promotion and team dynamics). Micromaps are simply geographic or structural overlays put on traditional data graphics used in order to clarify patterns of behavior in data evolution (Carr & Pickle, 2010).

The unpredictability of predictability of nonlinear, chaotic, and holistic processes is the norm in business analytics. Hence, it behooves one to consider another approach - consideration of a replacement to traditional prediction. Instead, general patterns of morphogenesis would serve our purpose more adequately than false, detailed prognostication. Recent evidence points to the power of observing patterns of behavior instead of statistical analysis when predicting bifurcations or *tipping points* in systems (Scheffer, 2010). Resilence of systems can be measured based on observing the patterns and dynamics of fluctuations manifested by small perturbations to these systems. Early warning signs of catastrophe or dramatic shifts in behavior may be gleamed from *critical slowing down* phenomena or slow recoveries from perturbative disturbances to systems. This is a measure of system fragility. It can be translated from higher autocorrelations in subsequent time incremented patterns (general time series), larger variances and skewness in general patterns and increased correlations between linked or similar nodes (with respect to some similarity measure) in the system. Patternization via the construction of general network displays overlaid with dynamic state transitions in a holographical representations are a potential tool to use in finding these general systems warnings.

As an application of the info-holarchy model for organization, general patterns of evolution may be dynamically presented in the form of a multidimensional view - a holographic representation of the holonic levels of that organization as the uncertainty process of workflow is unveiled through a time sequence. Each holonic level in an info-holarchy is projected onto the holograph as a single holo-print. The totality of holonic levels thus represents the holograph. As a process propagates through a holon and its holonic level, that "projection slice" of the holograph is dynamically updated in real-time. Since each HEAD member of a holonic level family passes along the process dynamic to subsequent levels, these projection slices are simultaneously updated as per the natural velocity of the process. The patternization of that process on the morphology of the holarchy is manifested through the multidimensional change in the holograph.

Reviewing the anatomy of a holograph may be instructional at this point. When the constituent parts of an info-holarchy have been setup, the mapping from holonic levels to the various holographic projections onto the hologram will be made. Holography owes its origin in optics as researched and developed by Dennis Gabor in 1947 (Gabor, 1949). Technically, holography is the process of collecting various 2-D images of an object taken at different vantage points and projecting them onto one 2-D photographic film. The various 2-D images are reflected from laser light shown at these vantage points at the object and redisplayed when viewing the projected film at different angles. Specifically, a coherent light beam is split by a beamsplitter into a reference beam and a direct light beam that reflects off the object. The reference beam directly hits the photographic film as the object reflects its light waves from the original coherent beam. The reference beam records phase information while the reflected beam reflects intensity. Interference patterns are then created by the superposition of both incoming wavefronts onto the film. Variations on the light source used includ a single point source for the reflection and a plane wave source for the reference beam. The interference pattern will then be different, reflecting a sinusoidal zone plate rather than the more traditional diffraction grating created by the inference. One of the more interesting phenomena that happens with holography is that even when the object is removed during the reflection period, the reference beam will continue to illuminate the film that has had the “past” reflections from the object encoded and will continue the inference patterns, hence the continued image of the object. This is referred to as the *virtual object*.

By viewing the flat film at consecutive angles, a simulated 3-D image of the original object is realized. More generally, Gabor proved that this holographic version of a 3-D photographic image of a 3-D object may be represented on a 2-D surface. In optics this is achieved through the aforementioned recording of both the phase and amplitude information of the light waves reflected off of an object, as opposed to just the intensity in regular photography. Intensity is then simulated by coherent illumination and the use of the reference beam from the light source (See figures 3 and 4 below). The interference pattern that is introduced by the reference beam and the scattered light wave is recorded as the photographic intensity onto the flat film (Hariharan, 2002, pp. 1-10).

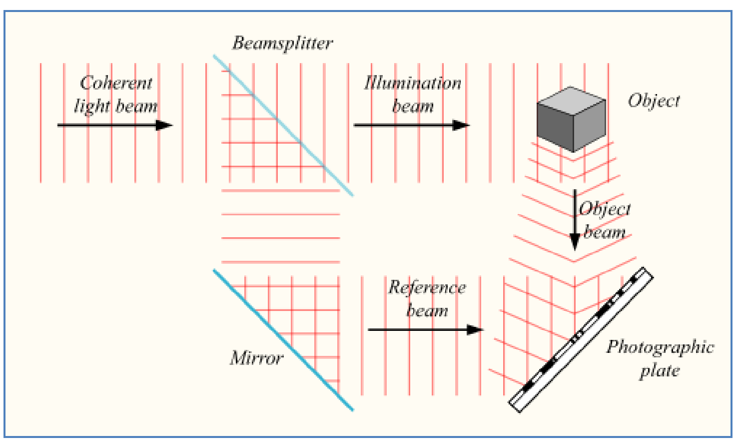


Figure 22. Holographic recording process
Adapted from “Holography recording process” By R. Mellish. 2009. Copyright 2009 by R. Mellish. Reprinted with permission under the GNU Free documentation license – Creative Commons Attribution-ShareAlike 3.0.

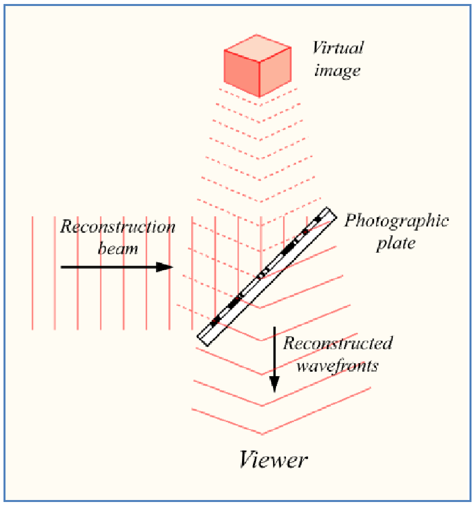


Figure 23. Holographic reconstruction
Adapted from "Holography recording process" By R. Mellish. 2009. Copyright 2009 R. Mellish. Reprinted with permission under the GNU Free documentation license – Creative Commons Attribution-ShareAlike 3.0.

The reconstruction of the hologram is the manifestation of the interference patterns that were created from the reference and direct light beams onto the film medium. However, the early versions of holograms suffered from inferior quality and blurring. This occurred because of the interlapping of wavelets from the virtual object and the reference beam. The technique known as off-axis holography was developed by Leith and Upatnieks (1962) to remedy this. In it the virtual object light wave is offset from the reference beam by a certain angle, θ that cancels out the blurring. Equation (6.19) below describes the components of the off-axis complex amplitude of the resulting hologram image on a film.

The natural evolution of holography in computation is the idea of generating these interference patterns algorithmically - the computer generated holography (CGH). In CGH the interference patterns and the associated hologram are computed using the mathematical description for the reconstructed image. This is done through the computation of the complex amplitude of the transmitted wave,

$$\begin{aligned} u(x,y) &= r(x,y)t(x,y) \\ &= t_0^{'} re^{2\pi i\xi x} + \beta Tr\left|o(x,y)\right|^2 + e^{2\pi\xi x} \\ &\quad + \beta Tr^2 o(x,y) + \beta Tr^2 o^*(x,y)e^{4\pi i\xi x} \end{aligned} \tag{6.19}$$

where r and o are the complex amplitudes of the reference and object beams respectively, t is the background transmittance, and β is a parameter determined by the photographic medium (Hariharan, 2002, pp. 10-13). With respect to equation (6.19), the first term is the amplitude of the directly transmitted beam, the second term is a halo effect around this direct beam, the third term is the original object wave amplitude, and the forth term is the amplitude of the conjugate image which represents a real image. If θ is large enough, the virtual image will be separated from the directly transmitted beam wave and the conjugate image, hence the cleared hologram in the off-axis version.

Holography, in theory, can be further generalized to extend to any dimensional reduction (i.e., an n-dimensional object can be represented (imaged) on an (n-1)-dimensional surface and from different wave sources such as sound, radiation, etc). The mechanics of such interference processes are, of course, dependent on the physical properties of the wave and medium involved. The most interesting wave or field that may be transmitted by the holographic method is that of an information field discussed and expanded on earlier. As a quanta, the informaton can be a wave (or field) or a point particle in abstraction if its quantum nature is exploited. However, in our metamodel, the more powerful form of the uncertainty computation bit, the g-bit, has quantum properties that are also generalized. Holography applied to informaton fields generalizes the practical 3-D holograms that are generated currently. Informatons and info-holarchies are inherently multidimensional hence their direct holographic representations onto a 2-D surface are not yet understandable, much less producible. Nonetheless, the human representational dimensionality of informatons may be reduced by introducing sensorial dimensions within the 3-D hologram, i.e., tactile/haptic, olfactory, observer position-dependency. Holographic image reduction is generalizable to information spaces and hence to physical ones. What may be viewed as the most powerful generalization of holography to entropy in spaces is the Holographic Principle(s) in contemporary physics.

Susskind introduced a prophetic generalization to this concept in terms of information content (entropy) of a cosmological object. Susskind (1995) labeled it the Holographic Principle. Bekenstein (2003) formed a bound for the amount of entropy that can be contained about an object given its area-the Bekenstein bound. Bousso (2002) universalized this premise to any object, closed or open. Specifically, if A defines the surface area of the object, G is the gravitational constant, $\hbar$ is Planck's constant and $S_{\max}$ the maximum (covariant) entropy of that object, then one has the bound,

$$S_{\max} \leq \frac{A}{4G\hbar} \tag{6.20}$$

Using the generalized holographic principle of Bousso (2002), a chain of even more general information reduction is possible. Consider the total information (Shannon, organizing, i.e, shaping, and semantic or semiotic), I contained in an object that resides in an abstract physical spacetime region, R of dimension N. Let $S_{\max}^{N}$ denote the maximum entropy contained in that spacetime region, R. Then one has the following chain of bounds:

$$ {}^{T}S_{\max}^{N} \leq \frac{A^{N-1}}{4G\hbar}, \; {}^{T}S_{\max}^{N-1} \leq \frac{A^{N-2}}{4G\hbar}, \ldots, \; {}^{T}S_{\max}^{1} \leq \frac{A^{2}}{4G\hbar} \tag{6.21} $$

Here, A^n depicts the n-dim hypersurface boundary which is a lightsheet (light rays that extend a hypersurface) and A^2 is the limiting 2-D surface boundary. However, entropies are not, in general, mutually exclusive or linearly additive from subregions—generalized information is not conserved even in closed regions. Nonetheless, a simple sum can be expressed without physical significance because of the possible geometric complexities of receding boundary hypersurfaces,

$$ \sum_{i=2}^{N} {}^{T}S_{\max}^{i} \leq \frac{1}{4G\hbar} \sum_{i=2}^{N} A^{i} \tag{6.22} $$

Energy of an object is also expressible as the product of its entropy and temperature (radiation is a generalization to temperature) (Jacobson, 1995). Since entropy is proportional to the geometrical dimensions of an object, entropy may then define geometry through the use of Einstein's gravitational equation relating energy and mass. These relations are bounded by the Rindler causal event horizons of the object (i.e., the causal limits of lightspeed distances and time). It does not take into effect possible quantum entanglement between the object and a component of its complement in the universe. The earlier discussion on super-correlations of entanglement in chapter 2 implied a new kind probability of causality may be possible in this instance. The causaloid structure was considered for this exact situation. Causaloids are then the causal mechanism connecting spacetime and quantum properties of mass and hence of informatons in an info-holarchy.

The concept of holographic principles leads to the digital Bloch sphere whose surface can contain a carpeted array of qubit machines that represent the contained information space of a quanta. In our case, an informaton would be the fundamental quanta for a generalized Bloch sphere for a g-bit, as constructed earlier. Regardless, entropy becomes a governing measure of spacetime curvature (geometry) of an object and of its quantum nature through its quantum mutual information with other objects or with its compliment in the universe, as was discussed earlier in chapter 2. Gravitational lensing is the mechanism that curves light when in the presence of huge mass/energy sources. Evidently, it also distorts the causal component of spacetime within the Rindler horizon. The theoretical gravitational field is the corresponding structure for gravitational lensing. Gravitational fields must therefore contain information in order to shape spacetime, which itself contains the quantum information of matter and the causaloid structure for time construction.

By defining the boundary between a random object in the universe and its complement, i.e., the rest of the universe, the object's entire information content is contained on that boundary (Vedral, 2010). This is a completely general object in existence, including a living or anthropomorphic structure. The boundary of such an object may be more complex than the object itself, as in the case of a business organization. Nonetheless, this is precisely what is being proposed here. Business entities, however complex and dynamic, have boundaries and exogenesis, that is, they interact with their externalities precisely and completely via this boundary.

While black holes and other cosmological objects contain massive amounts of information via Hawking radiation, businesses contain intelligence (BI) that propagates outward via their collaboration and competition (coopetition) with industry rivals and allies, government entities, and the general public that they serve. Businesses interact with the usual externalities that humans encounter - natural phenomena. The concept of projecting reality to a lower-dimensional manifold has its roots in antiquity – Plato's cave inception of reality, the Allegory of the Cave from his work, The Republic, where the world and its knowledge is perceived as an allegorical shadow on the walls of a cave (Plato, 1497). The cave in the holographic principle is the lower-dimensional surface of an object. In the case of a business holograph, the cave is the inside of a Bloch sphere. The Bloch sphere may also be inverted to appear on the surface of enclosed cave walls.

Caves provide a virtual reality environment of immersion, one in which displays are projected onto the side walls of an enclosed theatre engulfing a tracked observer. The floor and ceiling are down and up-projected mirror reflected displays respectively. The tracked observer wears electro-magnetic sensing goggles and gloves that collectively feed 3-D coordinate and angular information to system computers which in turn execute software that controls the displays for the cave. In this way, the real-time movements of the tracked observer are feed into the cave algorithms which dynamically change the synchronized 3-D landscape of the display projections onto the cave surfaces. Angled surround sound audio spectrum speakers and compensating equalization systems provide a virtual 3-D sound experience. The resulting effect on the tracked observer is one of navigating through virtual 3-D scenery (Cruz-Neira, Sandin, DeFanti, Kenyon, & Hart, 1992). The structure of the theatre consists of nonmagnetic materials, minimizing magnetization interference.

Cave environments have been developed for use in military strategy simulations and subsurface imaging and hydrocarbon flow simulation for the petroleum industry. General frameworks such as the CAVE system have been created for generic virtual reality immersion cave development (Ohno & Kageyama, 2007). More affordable and restrictive versions of immersion systems are the ImmersaDeck and Infinity Wall configurations in which no enclosure is used. Instead multiple projections are done onto a front-mounted frame of angled screens that the observer peers forward into, as with conventional flat screens (Czernuszenk, Pape, Sandin, DeFanti, Dawe, & Brown, 1997). Fakespace Labs Fakespace WIDE5 HMD is a commercial head-mounted display (HMD) goggles and gloves-based only immersion system that does not use theatre or front displays. It was developed in conjunction with the USC computer graphics department (Jones, Bolas, McDowall, & Debevec, 2006). All Fakespace and other HMD manufacturers' immersion displays are shown inside the HMD goggle environment which entirely encloses the eyes, with the worn gloves providing hand navigation coordinates. In caves, HMDs, Infinity Walls, and ImmersaDesks, multiple observers can participate, but because of computational loads usually only one observer is tracked.

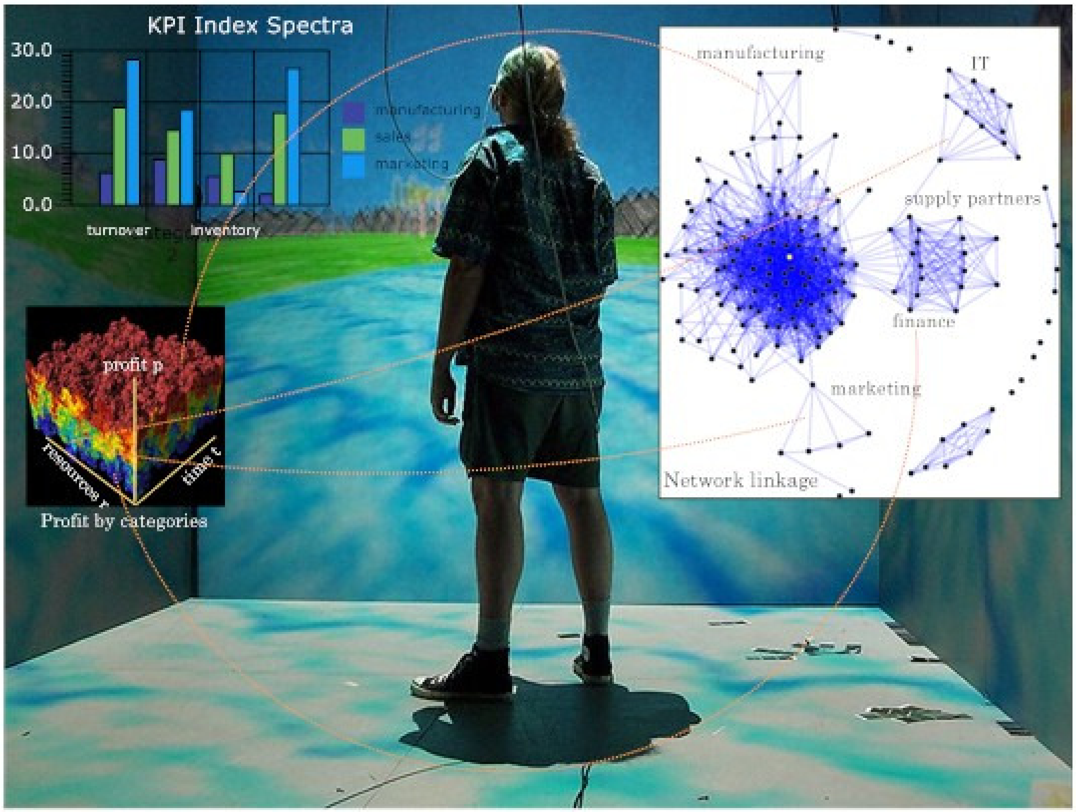


Figure 24. Business info-holarchy performance dashboard-cave
Adapted from “Cave automatic virtual environment at EVL University of Chicago at Chicago” By D. Pape. 2001. Copyright 2001 by D. Pape. Reprinted with permission under the GNU Free documentation license – Creative Commons Attribution-ShareAlike 3.0.

In the context of info-holarchy simulations that describe the dynamics of business organizations through performance dashboards, caves can display holographic and stereographic images of the flow of information gradients for the spectrum of business KPIs, strategy profiles, and network flow of various organization properties. Iso-surfaces and ortho-slices can be computed and displayed on the cave surface to show cross-volumes, surfaces, and curves of given factor profiles in these performance dashboards. For example, multivalued surfaces representing KPIs under various business and market scenarios can be sliced to produce equi-valued volumes, surfaces, or curves that depict patterns of commonality within those profiles. As when the observer changes relative projective or stereographic vantage points to a holographic display, so can changing vantage points inside the cave change the views into the info-holarchy.

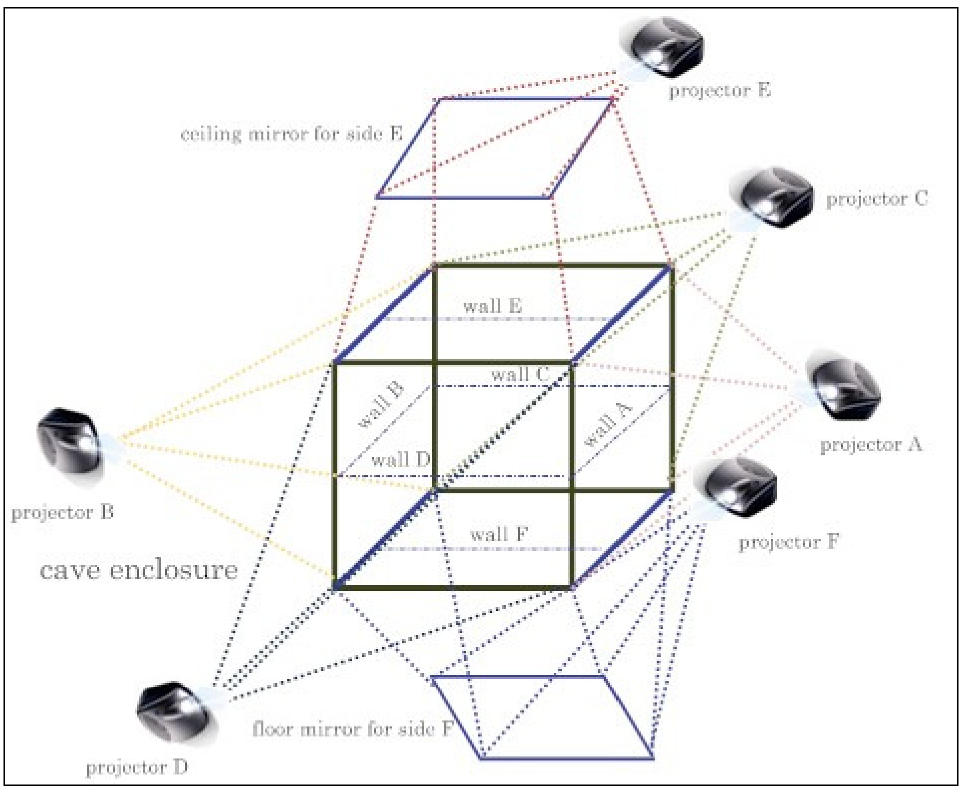


Figure 25. Cave projection design

Caves and holography can be combined to produce holographic imagery inside the cave, a novel display type to be developed for info-holarchy views. Sensorial stimuli can then be merged with these holographic caves environments to produce fully interactive and immersive virtual reality holography caves with body motion, tactile and haptic reactions, and olfactory feedback mechanisms tied to 3-D vantage movement within the cave dome. Info-holarchies represented in this rich immersion will develop the possibility of global control-feedback of massive organizational structures. Novel real-time adaptive and anticipatory decision support is embedded in these displays because the info-holarchic flows are projections of universal information flow in an organization. They are the immersive patternizers of organization.

Figure 26. Head mounted display (HMD)
Adapted from “Head mounted display” By NASA, 2007. Copyright NASA 2007. Reprinted with permission under the NASA public domain usage guideline.

In any business decision support display, including the proposed holographic cave platform, the business boundary defines the limits of the structure for the holographic performance dashboards of that business organism. It is the entropic boundary and hence is its information conduit to the outside world. More generally, this boundary can be used internally as a self-reflection of the states of its operations and processes. This is the ultimate generalization of the business entity's state of business intelligence (BI). However, state implies time dependence. Taking a temporal snapshot, as through a mechanism such as time series or more generally, a stochastic process simulation, is an infinitesimal measurement of that business dynamic. Even when time ensembles are collected this gives but a limited performance measurement in time. Consider the possibility of consuming either a sufficiently large and relevant subset of time ensemble measurements or a complete holographic projection of all "living" attributes of a business organism. What would such a multidimensional hologram of a business organism resemble? What would any multidimensional graphic of a process look like in order for it to be consumable by human consciousness?

To answer this, first view a hologram as a general dimensional reduction tool for information in light waves. To extend this process consider more general forms of information, the informaton. These constituent quanta of information can be collected in multidimensional spaces and projected onto embedded subspaces, hence their true representation in lower dimensional spaces. If a conduit of attribute spaces of a business organism can be represented by a stream of informatons via the info-holarchy, then this portion of the info-holarchy can be holographically projected onto smaller and smaller dimensional view ports. View ports are generic graphics depicted onto human-understandable media, such as lower dimensional holographic photograph film or laser holograms. However, these media are quite limited, they show visual dimensions.

Introducing haptic/tactile movement/feedback and olfactory dimensions expands the expressiveness of info-holarchies to humans and general inference machines.

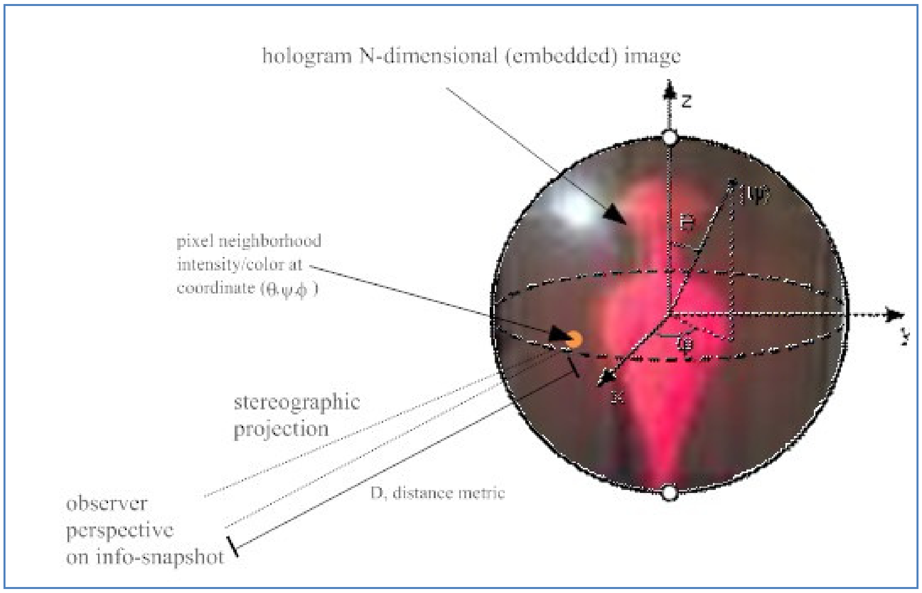


Figure 27. Info-hologram

The laser hologram introduces true 3-dimensional images of a reconstructed object. At any particular perspective point, viewing such a hologram induces an appropriate stereo image in the human brain. If that view is sufficiently complete to induce a decision process, it has reduced the effective dimension of the information of that object (i.e., the object itself). Consider now a 3-dimensional hologram of an object that represents the info-holarchy of that object at a spacetime coordinate (See Figure 3).

By moving around that hologram in a certain dimension, (e.g., around the hologram at a constant height) the hologram image morphs into another spacetime representation of the info-holarchy.

By encapsulating the hologram in a surrounding Bloch sphere, each physical point on that sphere would represent another reality snapshot of that info-holarchy, a spatiotemporal positioning in spherical coordinate space. The Bloch sphere was utilized earlier to represent a spherical coordinate projection for a qubit pure state. Here it is used to house a spherical coordinate projection for a stereographic perspective of the observer "eyes" to a centroid of the hologram. Since the azimuthal levels of the observer eye are dynamic (can bob up, down, left, and right) with respect to the hologram centroid, the spherical projection onto the Bloch sphere is unique.

For a given point perspective (stereographic projection for the human visual system) on the sphere, consider the distance vector formed by the distance between the receptor (your visual knowledge system) and the (stereographic) projection onto the Bloch sphere surrounding the hologram. This distance metric would add another dimension to the holographic info-holarchy representation. One now has the space of stereographic projections onto the Bloch sphere from an external observer system (human visual-interpretative system), along with the 3-D hologram at that coordinate, the space of sense attributes (haptic/tactic, olfactory, etc.) and the distance metric from the observer system to the Bloch sphere.

At any given multidimensional coordinate encapsulating all these attributes, the intensity and color coordinates for pixel neighborhoods (e.g., RGB, HSV, ICICM -

integrated color and intensity cooccurrence matrix) of the hologram image represents a further informational representation structure at that business life point. Novel methods for representing both intensity and color of pixel neighborhoods have been investigated, including the ICICM and would serve as a further compression of information (Vadivel, Sural, & Majumdar, 2006). This ensemble of dimensional reduction and representation in a multidimensional information hologram will serve as an isomorphism to the patterns of the info-holarchy proxy of the business organism. This is the gist of the holographic info-holarchy for business objects.

So far the sensorial feedback from the info-hologram of an info-holarchy has been somewhat passive in that an observer would have to traverse around the Bloch sphere containing the object. Consider active interaction from the observer in the form of transforming the perspective of the hologram at each given fixed observer coordinate by the movement of its optic lenses (eyes), moving the stereographic projection on the Bloch sphere. This transformation would change a certain subset of attribute views while fixed on a perspective coordinate.

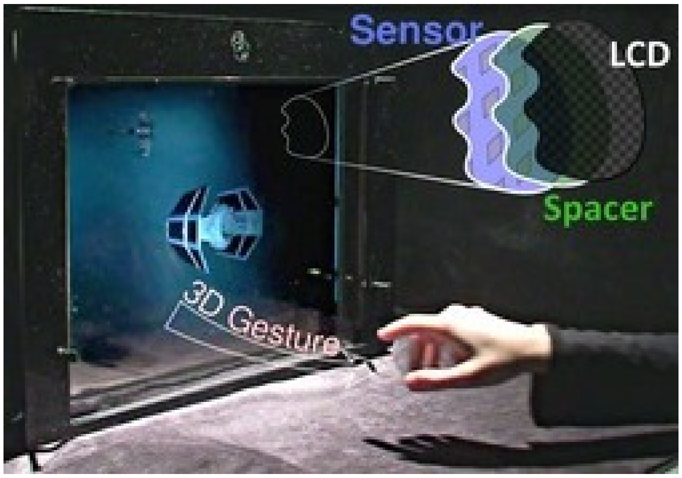


Figure 28. Bi-directional hand gesture manipulation of 3-D objects on flat screens Adapted from "BiDi screen: A thin, depth-sensing LCD for 3D interaction using light fields" By Hirsch, Lanman, Holtzman, Raskar, and the MIT Media Lab. *SIGGRAPH ASIA 2009 Art Gallery & Emerging Technologies: Adaptation*, *62.* Copyright 2009 by Hirsch, Lanman, Holtzman, Raskar, and the MIT Media Lab. Reprinted with permission.

Additionally, the use of hands (secondary connective devices) to transform the hologram via 3D hand gestures, again at a fixed perspective coordinate, could be introduced for further attribute view changing. Certain prototypes for this manipulation are currently underway and are referred to as bi-directional manipulation of computer screen displays – BiDi screens (See figure 6) (Hirsch, Lanman, Holtzman, & Raskar, 2010). The use of BiDi and cube manipulation versions of software development interfaces for creating 3-D images and holography are also under investigation and may be used as a tool for construction of info-holograms and their 3-D imaging portals (Lertsithichai & Seegmiller, 2002; Watanabe, Itoh, Kawai, Kitamura, Kishino, & Kikuchi, 2004). Finally, the use of voice commands and neuro-sensory connectors may increase the power of expression that the observer may have for manipulating and viewing the info-hologram and the patternization of the info-holarchy that represents a business organism. At this point, the performance dashboard analogy that was initially used for this section becomes obsolete and inadequate for the apparatus built for the info-hologram of a business.

The info-hologram represents an *N*-dimensional hologram that has been "flattened" onto a 3-D image that is suspended inside a Bloch sphere. The values of the info-hologram depict a field, i.e., for a coordinate point, $\boldsymbol{p}$, $f(\boldsymbol{p})$ is a multidimensional vector representing the info-holarchy at $\boldsymbol{p}$. This is not a static image; rather the dynamics reflect the adaptability of an info-holarchy. More specifically, a holarchy represents a rich organizational structure that possesses holonic substructures and individual holons as cells. These cells represent suborganisms of the organization. In businesses this could range from mega-departments and board-of-director related hierarchies to individual employees and devices. However, the representations do not stop at the "matter" of the organization. Any potentiation of processes is also represented as trajectories, past, present, and future, as projected by the patternizations of the informaton-based processes described by the GTU-inspired models of chapter 4. What this means in terms of the quantitative descriptions depicted by traditional graphics and displays in a performance dashboard is a generalization to multidimensional attributes as displayed by the hologram and the perspective changing position of the observer – the potential decision-maker and business human interface.

Graphs and histograms are replaced by multisensorial displays - color/intensity of pixel neighborhoods in the hologram, shapes that add to the description of numerical magnitude and direction, and by changing perspective around the Bloch sphere, the spherical angle, and shifting textures (pixel neighborhood gradients). Patternization images are the "a picture is worth a thousand words" equivalent for quantitative analysis and business analytics. In other words, a particular blob image, with a unique color/intensity pattern, directional, textural, and angular gradients, olfactory (smell), tactile/haptic, and sound feedback patterns define a particular state of the business organism.

Holarchies are represented graphically by their self-similar hierarchical structures. In a 3-D rendition of this complex organizational chart, a holon can represent an individual, a department, a regional facility, a functional group, or a meme that is present in the holarchy. This meme space in turn represents the dynamic flow of information in the info-holarchy. Holons are relationally represented within and about holonic levels. Blobs of images with dynamic textures, coloring, haptic/tactile, and olfactory outputs sensorially represent these holons within the larger blob of the info-holarchy. In a particular meme lifecyle, the holon representing that meme will change its sensorial output dynamically with respect to the vantage point of the observer of the info-holograph as described by its stereographic projection onto the containing Bloch sphere. In other words, the observer observes the history and the future potentialities of the meme by moving around the Bloch sphere. While this meme lives in the info-holarchy, other "pieces" of the info-holarchy are similiarly displayed. For example, the organization of individuals and groups within the business are updated as positions change with respect to tasks, goals, and strategies. Hence both structures and processes are viewed dynamically around the Bloch sphere. Business rules are instantiations of memes as well.

They may also be framed as metamemes – templates for business behavior. These rules are propagated through their lifecycle within the info-holarchy, along with their generated by-products – other memes, material, and changes in the structure.

The crucial point of using info-holarchies as constructed in this study to represent business organisms is the nature of the uncertainty phenomena in each holon and thus as propagated in the holarchy. Holons are entities with inherent uncertainty in business organizations because humans and their by-products – memes, products, and relationships are. However, this uncertainty is generalized because transpersonal and other uncertainty paradigms enter into their behavior. Quantum behavior and nonstandard logics are used by societies in dealing with economic and relational processes. This is because the stuff of matter seems to be information which is built on relationalism. Relationalisms are generated by a combination of quantum and fuzzy logics. It was posited by this study that the GTU-logic subsumes both quantum and fuzzy logics and their combinations, in addition to generating brand new uncertainty logics. Business entities are instantiations of societies and so can be based on GTU-logic and hence on computations based on the gu-bit. In chapter 5 a proposal for utilizing a computational paradigm based on g-bits will be proposed for future study. *G*-bits are highly flexible because they can be used to represent general uncertainty in very general units of computation such as whole fields. Fields can represent whole linguistic statements or other higher ordered mathematical structures. Logic bits are then replaced by whole generalized uncertainty structures.

Equilibrium patterns are manifested by small gradient changes in these blobs, that is, the blob does not change appreciably with small observer perspective changes or observer initiated perturbations (plucking, pushbacks, or numerical what-ifs). Nonequilibrium patterns can be categorized as in chaos and bifurcation analysis – their patterns change with small observer perspective changes and in certain ways. Near-death states may mean small to null gradient "movies" around large perspective spaces of the observer. Business alerts that call out for further scrutiny of business practices are triggered by large gradient patterns in perspective space. This means that an observer, by making small perspective shifts around the hologram, will encounter large shifts in patterns of the blob representing the business info-holarchy.

Patternizing the blob, the info-hologram, is the generalization to modeling physical and natural laws around the collection of data. If the info-holarchy is a business organism, the patternization is the business rules space that contains the trajectories that comprise what is possible with the existence of that business. The space of patternizations is the manifestation of business process management systems (BPMSs). The corresponding flow of information in such systems is the info-holarchy and its sensorial output, the info-hologram. Because patterns are global with less specificity for localization of its smallest subcomponents – the informatons, they do not fit into the traditional definition of laws or rules. They supercede traditional categories of business rules.

Patterns in info-holarchies are also displayed by the weight or strength of network links for iso-tasks or iso-goals. These are the information subnetworks that represent task or goal oriented subgroups of entities in the info-holarchy. Again, entities may consist of agents or holons within the organization, be they employees, resources or computational devices or machinery. As an example of these scenarios, consider an adopted organizational strategy that dictates the following task: design, create, and market a rival software product in order to equalize a competitor's threat and to initiate another niche for the organization. The entities involved are software engineers and developers, project managers, software sales marketing specialists, an outside contract expert in the domain of the software application, application testers, and a project leader and advocate. Each individual represents an employee node in the info-holarchy, but so do the resources and devices they are to utilize towards this task.

The collective state status of the product is displayed via linkage groups connecting these entities, reflecting individual and subgroup status. Their patterns or network strength depict weighted connections that show where and how specific subtasks evolve and how they collectively fit into the master pattern of the project. The width of network connections is proportional to the strength of the linkage in the subnetworks. Connector width then depicts a visual measurement of task or goal fragility. The length of a connector depicts the relative speed/accuracy of transmission of information between the entities. Additionally, be utilizing feedback or haptic pulling on these connectors, the observer can surmise the resilience of these connectors or task status. This would have the behavior of pulling or pushing on a rubberband, observing how it would react to stress perturbations, i.e., project pullbacks, dependent subtask delays, or strategy or management decision changes.

Each holon-level connector is represented as a rubber band with a width that measures the fraility or brittleness of the project at that inter-level transfer at a particular time snapshot and length representing the relative speed/accuracy of exchange of information. For each snapshot each inter-level rubberband is lined up in holon-level project order to form a snapshot rubber band panel which we call a tensile (strength) panel. One *pattern of a project* would then be the parallelepiped consisting of the group of snapshot tensile panels. Each subgroup of rubber bands in any subgroup of tensile panels can then be “plucked” to test its collective tensile strength and pitch/frequency as a measure of brittleness (fragility) of that time evolutionary working subsection of the project.

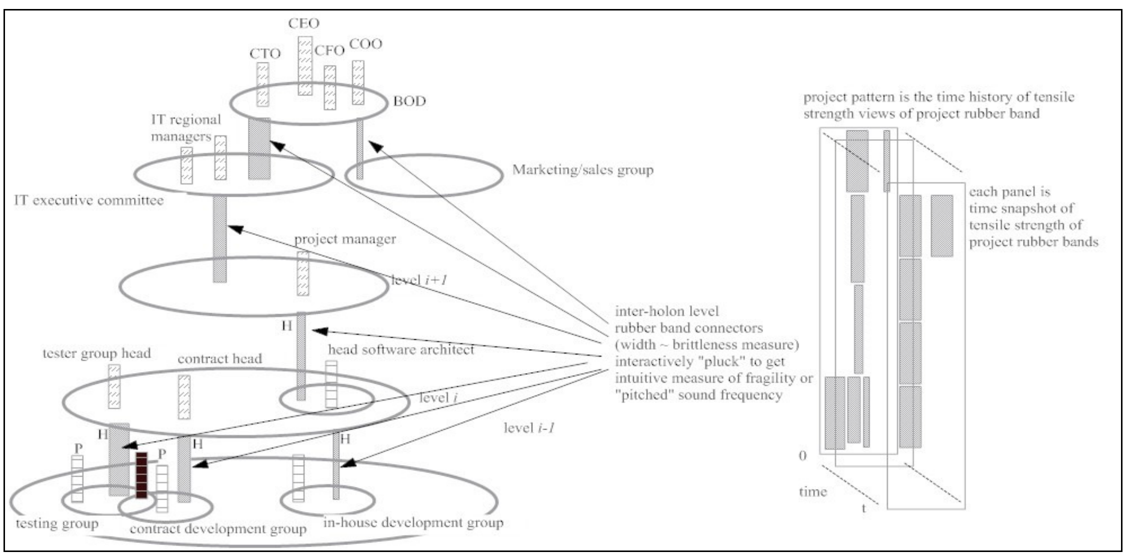


Figure 29. Software project as info-holarchy view

Brittleness can be measured by similarity divergence measures and speed and accuracy of transmission of information by channel fidelity.

Patterns, nonetheless, as have been depicted in this study remain passive. Decision-makers need to take action in order to become change agents, to morph the situation at hand. Hence, “pushbacks” need to be included in the mechanisms of the info-hologram. Pushbacks are generalizations to knobs on a control panel that affect the current state of the “machine”. In a particular info-hologram view, a pushback will then simply be the process of having the passive observer become the proactive agent through the use of bi-directional manipulation of the hologram, making indentations, pushing in, or pulling out in a certain attribute direction – changing the proximal pixel neighborhood image. This active participation by the observer will then lead to a change in how that business process operates to create the attribute – that action changes the patternization or general law for its operation. An example would be in the pushback for inventory processing when demand of the product decreases according to some external phenomena in the appropriate market. Resource re-allocation is the end process of such pushbacks from the decision-maker.

The passive observer turned would-be decision-maker then manipulates the patterns of behavior of the business organism in real-time according to the newly sculpted info-hologram. The timescale of the business pushback on this decision is then speeded up by the pattern projection created by the info-holarchy’s GTU models.

This mechanism is the generalization of the “What If” scenario of business analytics. The realized info-holarchy is then compared to the projected patternization based on this action in the real-time scale. Evolutional adaptation then kicks in to change the GTU-processes that engineer the info-holarchy dynamics. This is simply natural selection on hyper-steroids and defines the agility of the business info-holarchy. The case for handling causal versus correlational analysis must involve the GTU-inspired adaptation mechanism because traditional causal analysis is based on linear causal nets where Aristotalian logic reins.

Anthropomorphically inspired structures such as cultures and societies which are generalizations of business organisms do not follow a completed Boolean logic – irrational or satisficing streams of thought contaminate the process when information is limited, incomplete, fuzzy, or suboptimal as in the generally posited bounded rationality process in socio-economic markets and other human organizations (Simon, 1991). Humans mostly choose the "Take the Best" alternative decision (algorithm) of Gigerenzer (2000) and in so doing, assign an artificial suboptimal causality chain in their decision process. Because processes in an info-holarchy are generalized (uncertainty) multiobjective games utilizing GTU Itô models, as earlier shown in chapter 4, bounded rationality can be optimized for imperfect inference machines, i.e. human decision processing (Yu & Yu, 2005). Holon agents in an info-holarchic organization are inference machines and follow the general description of GTU processes, but as more general entities, such as an object traveling under the nontrivial influence of multiple gravitational forces (relativistic effects) and nonlocal correlations (quantum effects), are accommodated for by the underlying LQG spinfoam formalism.

Lack of organizational information is masked by the lack of information linkage and inter-connectivity. Holarchies, in particular, info-holarchies, overcome these structural gaps through the richness of their self-similar connectivity and global communication. In the end, causality sets must be robustly analyzed, that is, something more powerful than classical probability in quantum processes should be utilized. That is the gist of the use of the concept of causaloids in quantum mechanics, general relativity, and in this study's info-holarchy metamodel when classical assumptions are not satisfied, which is usually the case for any living or adaptive physical organism.

The proposed info-hologram is to be built utilizing CGH on either future true 3-D holographs or currently constructed 2-D sterographic displays with multiuser eye-tracking technology (Stolle, Olaya, Buschbeck, Sahm, & Schwerdtner, 2008). Huge amounts of real-time processing are necessary for updating the info-holograph images. The requirements for computational robustness are not presently met by most multiprocessors and hence it is necessary to employ an array of super-computational units with large bandwidths of I/O or SSDs (solid-state devices) for realistic transaction times. Parallelization of component machines will speed up the processing of subsections of the business object being imaged. For example, dedicated machines should be used for each department or individual. This creates an interesting proposition for the use of employee-assigned computers akin to the massively parallel Internet distributed arrays volunteered from the general public for the SETI@home projects (SETI, 2010). When an employee's computer is relatively idle, its compute cycles may be used to power the info-holograph portion of the monitorization of that employee's information interface to the business (i.e., all their tasks, knowledge of systems, experiential data, etc).

Info-holarchic Computing

Computational models based on informaton-based fields in the spirit of a model introduced for morphic (field) computing by Resoni & Nikravesh (2007) and the extension of the info-holarchy metamodel to mathematical topoi described system of systems (SoS) will now be presented.

The info-holarchy based metamodel influences the role of information from first principles as the main concept of creation of organizational physics. Particularly, one gives pause to a logical extension of this concept to computation. The field-theoretic basis of the info-holarchy, its information field theory, sets up an appropriate generalization to symbolic computation akin to the idea of morphic computation introduced by Nikraesh & Resconi (2010). In morphic computation, the atoms of calculation are Sheldrakian morphic fields, i.e., fields that may influence physical phenomena. Physical field theory though, may be a scientific crutch because it requires that space be occupied intermediate to matter by causative agents not yet detectable by instrumentation. However, most post-modern physical models assume some semblance of fields, e.g. gravitational fields in general relativity, string field theory, field theory for LQG, quantum field theory, and Maxwelliian field theory. This study constructed a field theory of information particles and the transference of generalized information bits. In these theories fields have been consistent with data results. However, morphic fields have not met with the same results. Additionally, tenets of morphic fields do not square off consistently with evolutionary theory.

Morphic fields can, however, be utilized to abstract the concept of physical fields for biological organizations, albeit, with corrections emanating from the physical field theories of physics research. In the case of a more powerful and general computational model, morphic fields can be used as bookmarks for generalizing symbolic mechanisms. Instead of utilizing symbols such as language, words, bits, qubits, or even this study's most general form of a computational bit, g-bits and their ultimate manifestion, informatons, one may manipulate field values or tensor fields. Tensors may generalize the spinor fields of QM, LQG, and other variations of quantum gravity. In this study, tensor fields may generalize informatons – leading to a new definition of informaton tensor fields.

To this end, morphic computing will be briefly summarized and redressed for informatons. This is a future development of informaton-based or info-holarchic computational models. Physical fields are values of observables, such as velocity vectors, particle geometry, angular momentum, assigned to points in space or of a submanifold of space. This space, referred to as the reference space, *R*, for all practical purposes is a physical domain. Without loss of generality, let *R* be an *r*-dimensional physical spacetime domain. Let *F* be the set of physical fields defined on points of *R*. Each field in *F*, $F_i \in F$ is defined on a subset of *R*, $R_i \subset R$. The values of these physical fields defined on *R* may be general tensors. In particular, let the space of informaton tensors defined on *R* be denoted by F_I.

Recall that informatons are topologically modeled via spin foam networks which are connected networks of spinor-like tensors. Define the object space, *O*, isomorphic to an *n*-dimensional Euclidean space, $\mathcal{E}^n$, as the space of points whose respective coordinates are defined as the values of an informaton tensor, F_I taken on *n* points of *R*. Let $B_m \subset O$ be a basis consisting of *m* informaton tensors in *O* where $m \leq n$. Let $span(B_m) = H$ be the span of the basis B_m in *O*. In general, *H* is a non-Euclidean subspace of *O*. For purposes of defining a computational machine, *H* is the *contextual* space. Let $X \in H$ be an informaton tensor field. Then *X* can be written as

$X = \sum_{\alpha} v^{\alpha} b_{\alpha} = v^{\alpha} b_{\alpha}$ where $b_{\alpha} \in B_m$.

The coordinates of *X*, v^{α} are the contra-variant components of *X*. The v^{α} will be called the *intensity of the sources* of the basis fields for *X*. In particular, if $h = (h_i),\ h_i \in H, i = 1, 2..., n$ are considered to be prototypical fields (informaton tensors) and $s_i, i = 1, 2, ..., n$ are their corresponding source values (intensities) in *O* for *X*, then the superposition,

$$Y = \sum_{i=1}^{n} s_i h_i(r) = h(r)s \tag{6.23}$$

where *r* is the reference space point, is a projection of *X* onto *H*. The object space, *O*, can be endowed with a suitable differential geometry and metric and thus become a differential submanifold (Riemanian) of $\mathcal{E}^n$ together with well-defined tangent and cotangent bundles, α-connection and α-affine manifold. An information divergence measure between tensor models, *X* and *Y*, $D(X \| Y)$, can then be constructed (Amari &

Nagaoka, 2000). This produces a topologically measureable structure for the informaton tensor objects in *O*.

Label the projection operator defined from (6.23) as *Q*. Then $QX = Y \in H$. When $m < n$, Q imposes a natural relationship between the components (contra-variant coordinations) of *Y*. The sources are expressed as $S = (H^T H)^{-1} H^T X$ via the generalized inverse utilizing (6.23). It has been shown in Resconi & Nikravesh (2007) that one can generate invariants for any unitary transformation, *U* on the elements of *O*. In this way, when one transforms the basis or prototype fields (informaton tensors), *h* into new prototype fields (informaton tensors) $h' = Uh$, the new sources can be expressed as $S' = [(Uh)^T (Uh)]^{-1} (Uh)^T (UX) = S$, an invariance. Moreoever, if Q_a, Q_b are two projection operators defined on two basis spans, H_a, H_b respectively with dimensions m_a, m_b, projecting onto two projection spaces Y_a, Y_b, then the product projection space, $Y = Y_a \circ Y_b$ can be generated from the *ab*-dimensional basis span $H = H_a \circ H_b$. If for some projection space *Y*, there does not exist component projection spaces, (Y_a, Y_b) such that $Y = Y_a \circ Y_b$, then *Y* is an entangled state. The space of projection operators generalize the space of quantum measurement operators because they are based on informaton tensors which are based on the GTU processes that generalize quantum processes. This generalizes not only the quantum logic used for quantum computation, but other non-Aristotelian logics, such as fuzzy and intuitionistic logics.

It is important to note that the objects of computation in this informaton morphic computation are not limited to classical bits, words, or even human linguistic symbols.

Indeed, informaton tensors are the objects of interest in these computations. Continuing with the morphic logic, one endeavors to find the optimal sources (weights) $S = (H^T H)^{-1} H^T X$ for the input informaton tensor *X*, such that an appropiate divergence, $D(Y \| X)$ is minimized with respect to *S*, i.e., $\min_S D(Y \| X)$. The input-output model for morphic computation of informaton tensors is illustrated by the flow diagram in Figure 7 adapted and generalized from the field version of Resconi & Nikravesh (2007). Resconi and Nikravesh (2007; 2010) also showed that their field version of morphic computing generalizes neural networks and fuzzy computation. This study proposes that the informaton structure can be adapted for morphic computing by using informaton tensors as fields in the object space, *O*. Using the causaloid adaptation, the relativistic holographic model as a computational machine from Appendix B, and LQG spinfoams, the informaton model generalizes physical tensors for all practical purposes.

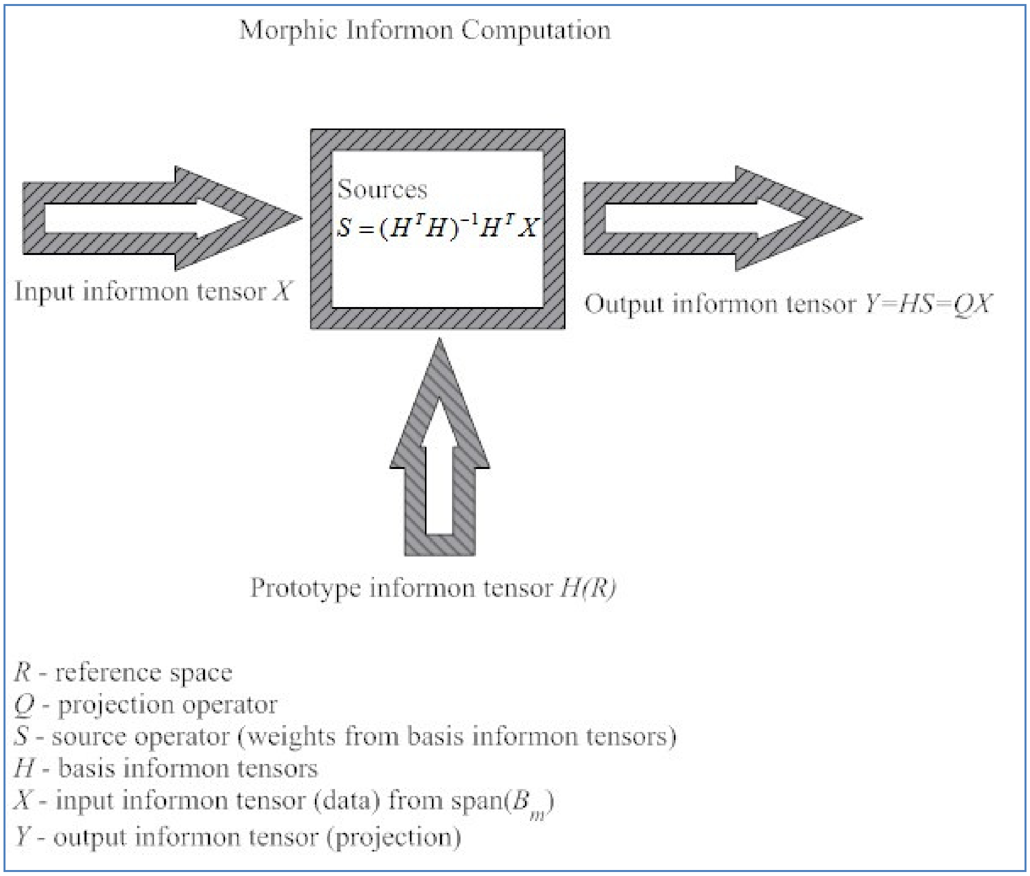


Figure 30. Morphic informaton computation

Consider the set of projection operators, $\{Q_i(X)\}_{i\in I}$ that can be applied to the computation of *Y* from *X*. Additionally, consider corresponding different context spaces or spans of prototypical tensors, $\{H_i\}_{i\in I}$. The collection of pairs, $\{H_i, Q_i(X)\}_{i\in I}$ indexed by some countable set *I*, constructs a set of computation machines for *X*. The projection operators, Q_i define the operators of calculation (logic circuitry) while the basis tensors, H_i, define the context or rules (instructions) of computation. The computation of the tensor Y is categorized as *acceptable* if it is similar to the input tensor X, with respect to some similarity measure or divergence, $D(Y \| X) < \varepsilon$. This is the generalization to the robustness, accuracy, and error tolerance of computation in classical computers.

Moreover, because the objects in the reference space, R, can be considered to be the agents of computation, in an info-holarchy, each holon is represented by a generalized computational unit and the tensor value on it represents the agent's contribution to the final output. The info-holarchy then utilizes a particular computational device defined as (Q_i, H_i) for a computational job that requires the calculation of the output informaton tensor, Y_i. Fuzziness, bounded and measured rationality, quantum measurement, intuitionistic logic, and lastly, GTU processes which could include quantum gravitational computations, can all be emulated within an indexed morphic computation, i with multiple contexts, $H_{ik}, k = 1, ..., K_i$ together with the iterated application of different morphic informaton computation devices $\{H_{ik}, k \in I_i, Q_i\}_{i \in I}$,where $|I| = w$, described by the pipeline,

$$X \underset{H_{1,k}, k=1,...,K_1}{\Rightarrow} Q_1 X \underset{H_{2,k}, k=1,...,K_2}{\Rightarrow} \cdots \underset{H_{w,k}, k=1,...,K_w}{\Rightarrow} Q_w Q_{w-1} \ldots Q_1 X = Y \qquad (6.24)$$

Each morphic informaton computation achieves a superposition of informaton tensor participation through their respective source weights. Diverse logics, as described above, can be described within a morphic computation, in conjunction with a pipeline. It is posited that this version of hyper-symbolic computation is theoretically achievable through the use of this morphic informaton computation model.

Chapter 5: Summary, Conclusions, and Recommendations

Info-holarchies: Organizations to Come

In this chapter, a summary of this study's work on the informaton, its field theory, and the subsequent organization evolutional dynamics, based on the info-holarchy metamodel, will be presented. Consequent to these developments, the motivations for the development of this metamodel for information organizations will be reviewed. Additionally, several followup suggestions and extensions to the informaton model will be given. The applications of this information framework for organization that were presented earlier in chapter 4 will also be further discussed and possible extensions to them will be given.

In this study a novel metamodel for information-based physical systems, the informaton, a general information field theory based on the informaton, and a dynamic structure based on this pattern, the info-holarchy were developed based on contemporary discrete Planck-scale loop quantum gravity (LQG) theories, a mathematical topoi description, and the causaloid and generalized theory of uncertainty (GTU) models of inference. Organizational decision processes are mostly based on classical statistical tools utilizing linear causal structures. The description of classical organizational structures is inherently linear, minimally adaptive, and does not exhume much agility when externalities are tied to its performance. Organizations are treated as nonliving entities that are autonomous to all but the most connected environments. Nonetheless, organizations are seamlessly interacting with their environment. Feedback mechanisms in organizations produce any agility and adaptability in those entities. Feedback occurs at all scales with respect to the organization. For example, while human communication is the most ostensible, exchange of information transacts at multiple levels – intuition, psychological, indirect and hidden causalities, and synergestic manifestations, such as coopetition and coevolution. These are all complex processes that are actionized in complex multiagent adaptive systems.

The main tenet which manifests organization is the physical definition of information posited in this study. Information, and in particular, informatons, are projected to be the smallest constituents of physical logic. They construct physical particles and hence utilizing the standard model of physics, all energy and forces in the universe. Info-holarchies are manifestations of informaton systems guided by the information fields described in this study. These fields follow the rules of the general inference of the general theory of uncertainty (GTU) that accommodates fuzzy, quantum and classical probability, and all nonAristotelian logics. Info-dynamics describe the binding of the different scaling of processes – GTU processes that guide microprocesses with macroprocesses that follow entropic rules, tied together by an optimized control-theoretic mechanism.

As ostensibly important applications of the info-holarchy dynamic, this study patterned the neural network model of a brain and a socioeconomic entity, the business organization. Brain dynamics, down to the microstructures of microtubules and quantum processes that adhere at that level to the organization of subfunctional suborgan groups of cells within the neural network, such as the hippocampus and the neo-cortex, follow the metastructures of info-holarchies. Biological and quantum processes are specializations of the GTU processes in info-holarchies and their respective subholarchies. The GTU Itô micro processes coupled with the entropic divergence metric for macro processes, bridged by the control mesolevels pattern the electro-chemical and ionic flow of information switches in the mitochondrial tissue of neural cells, the mainstay of neural functionality. Multiagent adaptability in an info-holarchy is shared by the dynamics of the neural structure of brains. The emergence of chemical, ionic, and other components in hemoglobin transmission flow, including neurotransmitters and neuro-suppresors in the neural system is modeled in an info-holarchy by the calculation of its info-dynamic processes that govern the quantum manifestations. These quantum processes led to causative changes in the chemical and ionic profiles within each substrate and suborgan. Coevolution and speciation of neural cells along with the self-replication of connection structures ensued within neural systems. These are all manifestations of an info-holarchic presence.

Business organizations, as are brains, are specializations of inference machines. These are self-reflective, self-producing and adaptive multiagent complex systems that possess emergent intelligence. By examining business structures and brains, one may construct a patternization toolkit for these inference machines. Hence, the hypothesis of interest here is one of patternizing inference machines as info-holarchies. However, as important is the construction of dynamic and holistic views of an info-holarchy. By designing a novel holographic representation of the dynamic states of a business, one is positioned to extend this to any info-holarchy. Therefore, the holographic performance dashboard that was presented in this study is a means to developing the requisite tools for a truer systems view of an info-holarchy. Holography was briefly reviewed as an efficient way to introduce a general performance dashboard, the lifecycle watcher of an info-holarchy. Multidimensional holography extended this view for very complex info-holarchies, abstracting a means of viewing a system of systems (SoS). The immediate ramification of such a manifestation of higher level system displays are applications to holistic interactive human-machine interfaces specializing to human systems dynamics performance tools, apparatus that measure emergent behaviors in organization, and complex evolution patternizers replacing classical business analytics. Classification analysis is supplanted by pattern similarity equivalences. Regression and time series analyses are superceded by an ability to extend a pattern to a more mature version of itself that is a member of that pattern's class, i.e., the use of time-extended representatives within each pattern class as a holon HEAD-type element for those class pattern members. Patterns in info-holarchy spaces take into account predictive and modeling errors by representing broad but prototypical behavior rather than numerical precision.

Businesses will diversify based on novel information products (i.e., devices that propagate information in novel displays and manners). This expansion will take place based on the concept of generalized adaptable information and will represent the third wave of business paradigms (the second being manifested by the super-exponential information explosion from the web, and the first being propagated through industrial machines). This study proposed that this third wave can be framed from a paradigm like that of the info-holarchy and its process.

Recommendations for Future Work

We will look at three potential extensions of work that can be applied to this study's work on info-holarchies: (1) organization patternizers, (2) infomics, and (3) an info-holarchic simulation generator.

Organization Patternizers

Future work is planned in extending holographic virtual reality dashboard-cave displays and morphic-informaton computation to the real-time analysis of business processes and structures and their morphology and evolution. Proposed web services are to be built for business portals that will house simulations for holographic dashboards depicting business dynamics. Stereographic images will replace the 3-D holographic displays for these business performance patternizers. Additional, the info-holarchy model will be applied to cases involving other natural organization or organism structures including anthropomorphic groups defined within cultural boundaries (i.e., minority groups and their evolution). Financial networks and markets will be simulated based on various instantiations of info-holarchies. Patternizers of these structures will then be used to observe potential bifurcations or crisis points. Banking institutions and their respective policy spectrum will be incorporated into these models in order to observe potential hidden improprieties and created adaptive loopholes that may lead to great depression-type meltdowns and 2008-type market collapses.

Info-holarchy patternizers are posited to be generalizations to the (statistical) shape analysis of 2-D and 3-D geometric objects (Dryden & Mardia, 1998; Dryden, 2004). Patternizers are instead applied to the analysis of spacetime region objects as depicted in the holographic-cave displays of organizations (i.e., particular classes of organization evolution histories take on shapes with similar boundary behavior, morphology, color dynamics, and other sensorial profiles). This is a multidimensional generalized "shape" history. Similar to transformations in conventional shape analysis, an organization pattern, *p*, can be classified by an equivalence class, $C(p)$, where pattern evolution mappings, $\Phi_{T_t}(q) \rightarrow p$ map class patterns, $q \in C(p)$ to *p* and $T_t \in G$ is a time evolutional operator from a registration group, *G* (i.e., Euclidean similarities—translations, dilations, scaling, isometry group, or affine group, etc) on the organization history pattern, *p*. What depicts a morphogentic (shape) change is a mapping that transforms the overall evolution of an organization in an almost time-invariant manner, i.e., the original pattern can be recovered through a time evolutional transformation, T_{t^*}, such that $d_{shape}\left(T_{t^*}(p), T_t(q)\right) < \varepsilon$, where $\varepsilon \rightarrow 0_p$,as $t \rightarrow \infty$ and 0_p is the *null* pattern history—a pattern history with no activity, organizational dynamic, or movement and d_{shape} is a generalized shape metric such as a Frechét metric modified to measure evolutions and use evolutions as distances.

In this way, two organization evolutions can be compared by differentiating them by another evolutional lifecycle. Organization patterns in a pattern class share asymptotically similar evolutional behavior (i.e. they share nearly the same asymptotic time-evolutional lifecycle histories where "nearly" means historically indistinguishable). It may be argued that all organization evolutions are distinguishable (not withstanding historian amnesia or the chain of indirect history writing), hence the "asymptotically nearly" label.

Information Holonics

Recently a new field of study has emerged—connectomics, dedicated to the dynamic mapping of brain connectivity—the development of a complete synaptic connection map of the mammalian brain. In this project, conventional fMRI technology is used to map the diffusion of water molecules in axons and the surrounding neuronal cellular structures (Lehrer, 2009; Walsh and Lichtman, 2003). This process named diffusion spectrum imaging (DSI), holds the promise of more aptly mapping the dynamical structure of neural activity. Additionally, an ultra-thin slicer of brain tissue, named the automatic tape-collecting lathe ultramicrotome (ATLUM), will help dissect a brain more exquisitely. These slices are subsequently pieced back together and microimaged using scanning electron microscopy. Lichtman and his research team developed a colored map of neuron activity–the *Brainbow*, where a spectrum of colors is used to track down individual neuron dynamics. This map led to the potential to follow individual neuronal and axonic development and evolution—one holy grail of neuroscience.

While informatics is a term broadly used to cover the gambit of information sciences disciplines, a project involving the study of the evolution and dynamics of information in organisms and organizations, in the spirit of connectomics for brain connectivity and genomics for gene structures would be an interesting working framework for the info-holarchy. The evolutionary informatics lab (EIL) is a project initiated by Marks (2007) to study evolution via general informatics. Unfortunately, it mainly contains intelligent design paradigms coauthored with Dembski (2001) which develop notions of conservation of information, low-probability events through a notion of *specified complexity* (SC) in evolution and the necessity of an intelligent interceptor rather than an evolutionary process to build life forms. Nonetheless, conservation of information is violated in coevolving organizations Wolpert and Macready (2005) and SC is not clearly present in nature. Other intelligent design paradigms such as *irreducible complexity* (IC) in Behe (1996) and a stricter form of IC from Berlinki (1998) are soundly refuted in Rosenhouse (2001) and by the overwhelming majority of relevant researchers in the complexity and biological sciences communities. Some of EVL's nonpeer reviewed content was subsequently removed from their published website (St. Amant, 2007, September 11).

By contrast, in this study, the universally dynamic and evolvable nature of the info-holarchy, coupled with the intelligently adaptable nature of its processes, position information as a more diverse entity; a physically and scientifically anchored concept—super-evolutional (higher-ordered or evolvable evolution) and grounded in information physics. Such a project could develop information patterns or patternizations for organization—a mapping of evolutional information patterns for prototype organizations, viewed through the fully engaged spectacle of virtual reality performance dashboard-caves. The "wiring" of organizations can then be conceptualized through the use of patternization in these enclosures. The color schemes of Lichtman's Brainbow are easily generalized to the sensorium of patterns (tactile, haptic, olfactory, auditory, visual colormetrics, etc.) in an organization holographic dashboard-cave. Current advances in moving 3-D holography and parallax (angled perspectives) technology are applicable to this dynamic rendition of viewing evolvable patterns in organization (Blanche, et al., 2010). In this tradition, one may form an info-holarchy broadly based project, labeled as *information holonics*—the study and visualization of the dynamics of information in general holarchic organization evolution. Nature gravitates to holarchies and information forms nature.

Info-holarchy Simulators

Generic multiagent simulation software and modeling tools have been developed recently in which input parameters serve as seeds for generating complex adaptive systems. Simulations from these environments are then compared with their natural complex systems counterparts in order to differentiate artifacts, match similarities, and form strategies for prediction. The SeSAM (Shell for Simulated Agent Systems) was written and developed over a period of a decade to be a domain-independent simulator of adaptive multiagent system requiring no direct coding or heavy computational power (Klügl & Puppe, 1998). Graphical modeling, animation, and agent-behavior can be parameterized to custom fit generic agent-based systems. The info-holarchy possesses more complexity than the generic multiagent system because of the nature of its recursive holarchic structure, GTU lattice automaton structure and strategies through its quantum-gravity dynamics.

Here I propose modularizing a SeSAm-like environment for simulating info-holarchies by the use of categories and subject descriptors, opening up more powerful metaphors for nonAristotelian logic agent systems (da Silva & de Melo, 2008). Software components are to be added to the SeSAm modular system through the use of general behavior classes for stimuli, user roles such as experimenters (modelers and/or analysts) and component designers (scientific software developers), agent properties (physical properties of generated particles and organizations), and more general agent objects such as individual agents versus environmental agents representing exogenous stimulator agents. Different types of agent logic systems (i.e., quantum-like, GTU, deterministic, and super-correlative entangled systems) can be reused to simulate different types of info-holarchies or subholarchies. The chosen software programming development system to complement SeSAm is the SimAgent Toolkit developed by academicians for general usage by philosophers, social scientists, physicists, cognitive scientists, biologists and other interested in emergent properties of CAMs (Sloman, 2010). This system was chosen because of its modularity, LISP-like incremental compiler, free open source license, and extensibility for a variety of intelligent complex adaptive agent-based systems. Human agent type reasoning can be utilized as the stimulus-reaction strategies for agents.

Fundamentally, at the informaton level, combinations of bit information are channeled to and between other informatons based on GTU-type rules which generalize quantum probability and the relativistic effects from gravitons—purported to generate gravitational forces. Gravitons will abstractly be generated from informaton activity in a causaloid-type region. Lattice cellular automata will then be a fundamental model to be simulated for the lowest level informaton activity. Organization based on cascaded informaton semiotic chaining follows in the tradition of cellular automata emergence, although the topological lattice structure lends complexity to the strategies of informaton groups and variably nested holarchies (subholarchies). Simulations therefore will be modeled by fundamental Boolean cellular automata in informaton agent action space with topological lattice-generated GTU logic. Since LQG spinfoams are lattice-like configurations, topological lattices encased with GTU logic can represent LQG more richly. These components will be written as modularized agent actions.

Informaton Physics

The concept that the informaton model for information particles and fields was a calculus for constructing SM and hypothesized quantum gravity particles, such as gravitons, the Higgs boson, and other theoretical constructs was posited in this study, but was not developed in anything but an acute, metaphorical, and hypothetical manner. Without regard to experimental verification, since such verification would be beyond the scope of this abstract, the informaton model would serve as an effective theory for an information (entropic)-based physics. These connections must be attempted in order to seriously comment any further on the role that information, as a universal entity, has in building reality. The holographic principle, being the most ostensible and successful entropic idea in physics, must be correlated to the informaton in the small. Topological lattice cellular automaton patterns need to be investigated for the physics of informaton dynamics. In this respect, informatons should be considered as the smallest Planck-level inference machines, reduced to the GTU dynamics of simple information bits.

Finally, any representational theory of physical information at the theoretical bit and Planck levels, such as the informaton model, must be complementarily functional and informational (Chakravartty, 2009). Informational here refers to the objective ability to adequately simulate and represent through theoretical models, the modes of reality of a real organization or object. Functional refers to the subjective ability to use such representations for cognitive activities, i.e., inferencing.

Concluding Remarks

In this study, the info-holarchy and an accompanying generalized information theory were presented. Applications extended to the representation of inference machines in general and business entites in particular. With this framework, special general purposes displays, specifically, holographic performance virtual reality dashboard-caves—were designed to abstractly depict the evolutional behavior of business organizations. These displays were designed as an alternative to classical business analytics. The implications for socioeconomic change include the introduction of novel ways of representing general information, its flow and organization in networks consisting of human actors, resources, and computational devices—prototypical information businesses.

This representation could lead to a better and more realistic tool for broad prediction and modeling for business and societal evolution. Long-tail probability and black swan phenomena may be better prognosticated through the use of patternization from such powerful displays of organization evolution because of the nature of the proposed info-holarchic design and their more natural representation of organic and inorganic entities. Finally, a novel computing model – info-holarchic computing - morphic computing wtih informatons, g-bits and higher-order mathematical objects, such as tensors, was presented. This novel way of representing computation expands the possibilities of treating complex objects more naturally and directly. Implications for more accurate and direct representation of real-world physical objects in businesses and general organization structures are anticipated by such computation models.

In the end, the connection between and from a highly complex organizational model, the info-holarchy, emanated from seemingly arcane theories of causal reality: (1) 2+1 LQG spinfoams (and to a lesser degree 3+1 LQG spinfoams), (2) an abstraction of set theory—topoi, (3) a space-time causal-probabilistic framework—causaloids, (4) an information field theory that is essentially a general Bayesian statistical framework for physical fields, (5) a notion of general uncertainty that classifies quantum and other nonAristotelian logics, and (6) a bipartite model for abstract information particles—the informaton; to the realms of the seemingly deterministic course-grained living of anthropods—the information-laden and saturated organisms of our techno-socio-economic sphere—businesses, appears far-fetched and overreaching. This study's contention is that the link between these two worlds of explanatory power is a unifying framework of information—at all scales: the known known, the known unknown, and the unknown unknown (although Secretary of Defense Rumsfield in 2002 was widely credited by the general media with this phraseology, it was first articulated by Furlong [1984]). The often heard naïve mantra of "information is power," while oversimplified and underserving to this cause, nonetheless points to the psychological leaning towards a unifying framework for information that simultaneously usurps technology, physics, philosophy, and the human societal psyche.

The theory of informatons leading to info-holarchies, as developed in this study, is a means towards representing the quantum 20 yes-no questions framework of Wheeler (1990). Generalized information bits—our techno monads— exchanged in informaton lattices are quintessential ensembles of complex Boolean packets. Their creation, whether it be from quantum entanglement and decoherence, generalizations of uncertainty, relativistic effects, or discrete quantum gravity from causaloid logic, create a chain of semiotic structures, leading to almost spooky answers to the as-of-yet unknown questions.

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Appendix A: Field Theory and Feynman Path Histories

Two traditional methodologies that one utilizes to formulate and describe the physical state of nature are field (wave) and particle theories. Field theories are used when the reductionist approach to detailing collectives of small particles becomes inconceivably hard. The field method is used to statistically estimate the behavior of a large group of particles, each possessing many (infinite) degrees of freedom. Successful field theories include quantum field (synthesis of quantum mechanics and special relativity) and electro-magnetic field theories. In order to build a field theory for some phenomena, one must first formulate what the field represents. A field normally represents a function that continuously takes values on a space-time (differentiable) manifold, say $q(t) = \phi(x,t)$.

Particles are represented by lattice or grid systems. Fields use functions that represent statistical estimates of these lattice point values. These lattice values may be quite general as they may represent scalars, vectors, tensors, and other operators. Secondly, fields need a measure of collective energy of the system so as to base an estimate of the field values on optimization of this system energy. The Hamiltonian of a system is the likely candidate for this measure, but the energy value may be replaced by some other more generalized observable. We mostly follow the instructive presentation by Maggiore (2006) for field theories. The Hamiltonian of a system with classical trajectory function $q(t)$, as above, is defined as:

$$H(p,q,t) = \left[p\dot{q} - L(t,q,\dot{q}) \right] \qquad (7.1)$$

where L is the Lagrangian, a measure of free energy or the difference between the kinetic and potential energy of a system at $q(t)$. The Lagrangian is used in the action principle (Hamilton's principle): if

$$S[q(t)] = \int_{t_i}^{t_j} L(t,q,q)dt \tag{7.2}$$

then $\operatorname{ext}_q S[q(t)] = S[q^*]$, i.e., $\frac{\partial S}{\partial q}\Big|_{q^*} = 0$, describes the optimal path $q^*(t)$ from t_i to t_j and $p = \frac{\partial L(t,q,\dot{q})}{\partial \dot{q}}$ is the Legrendre transform of L. S is a functional on the space of potential path trajectories q. In quantum mechanics, the action principle must take into account all possible path trajectories, $q = (q_1,...q_n,...)$, from time t_i to t_j and then apply Feynman's path integral formulation, in the limit as $n \to \infty$,

$$S[(q_1,...,q_n)] = \int_{q^1(t_i)}^{q^1(t_j)} ... \int_{q^n(t_i)}^{q^n(t_j)} e^{[\frac{i}{\hbar}\int_{t_i}^{t_j} L(t,q,\dot{q})dt]} dq_1...dq_n \tag{7.3}$$

where $\hbar$ is the Planck constant.

For space-time coordinates and in order to satisfy the Lorentz invariance of special relativity, the action functional will take on the form:

$$S[q] = \int dtL = \int d^4x \mathcal{L}(\varphi, \partial_\mu \varphi) \tag{7.4}$$

where $L = \int d^3x \mathcal{L}(\phi, \partial_\mu \phi)$ is the Lagrangian density and $\partial_\mu \phi$ is the spatial-temporal four-vector derivative. In order to quantize a classical field one must build operators, q^i and p^i which generalize the system's time-space coordinates and momenta respectively. Furthermore, using the Schrodinger wave equation constraints,

q^i and p^i commute via the relation $[q^i, p^j] = i\delta^{ij}$. In the Heisenberg scenario, these operators depend on time and so the commutation relation holds true at equal time, that is:

$$[\varphi(t,x), \Pi(t,y)] = i\delta^3(x-y) \tag{7.5}$$

where $\Pi(t,y)$ is the conjugate momenta and x is the spatial coordinate vector. Also, $[\phi(t,x), \phi(t,y)][\Pi(t,x), \Pi(t,y)] = 0$.

A real field is now a hermitian operator as are the momenta coordinates, a_p and $a_p^\dagger$ in a plane wave solution to a real scalar field equation (Klein-Gordon equation). The commutation relation generalizes to:

$$[a_p, a_p^\dagger] = (2\pi)^3 \delta^3(p-q) \tag{7.6}$$

with $[a_p, a_q] = 0$ and $[a_p^\dagger, a_q^\dagger] = 0$. If the particle system is isolated in a finite volume (infrared cutoff), $V = L^3$, then $\delta^3(p-q) \to \left(\frac{L}{2\pi}\right)^3 \delta_{p,q}$ and so $(2\pi)^3 \delta^3(p-q) \to V\delta_{p,q}$.

a_p is interpreted as the particle destruction (annihilator) operator and $a_p^\dagger$ as the particle creation operator.

In quantum ket-bra notation, $a_p|0\rangle = 0$, i.e., the vacuum state $|0\rangle$ (empty, zero energy state) of the quantum system, is annihilated by all destruction operators, a_p. The Hamiltonian is written in terms of these operators as

$$H = \int \frac{dp^3}{(2\pi)^3} E_p \left(a_p^\dagger a_p + \frac{1}{2}[a_p, a_p^\dagger] \right) \tag{7.7}$$

where $E_p = +\sqrt{p^2 + m^2}$ and m the mass. H may be simplified applying an ultraviolet cutoff, $|p| < \Lambda$ in the integration to prevent divergence and using a finite volume of integration as before. Similar developments for fields with operators and Hamiltonians can be done for the more complicated complex scale fields (antiparticles), Dirac fields (spin ½ systems), Weyl fields (massless particles), electromagnetic fields, string fields, and D-brane fields. We consider an analogous field development for general information signals and flow directives.

In the above consideration was the quantization of free fields, i.e., those without interaction with background energy. The interaction of large particle systems and their field equivalents must be addressed to develop a full field theory. In this way, the Hamiltonian of a system resumes the central role. The full field Hamiltonian will be written as the sum of the free field Hamiltonian and interaction Hamiltonian:

$$H = H_0 + H_I \tag{7.8}$$

Physically, $H_I << H_0$ and consequently is relevant mostly for weak coupling systems. The lagrangian, $L = L_0 + L_I$ that corresponds to the Hamiltonian H, in general, dictates very complicated field solutions, $\phi(t, x)$. The method of perturbative expansive in field theories attempts to relate the full field solution $\phi(t, x)$ to the field $\phi_I(t, x)$ whose time evolution is determined by H_0. More simply, $\phi_I(t, x)$ is defined as:

$$\varphi_I(t, x) = e^{iH_0(t-t_0)}\varphi_I(t_0, x)e^{-iH_0(t-t_0)} \tag{7.9}$$

It is a free field and as such can be expanded as:

$$\varphi_I(t,x) = \int \frac{dp^3}{(2\pi)^3\sqrt{2E_p}}\left(a_p e^{-ipx} + a_p^\dagger e^{ipx}\right) \qquad (7.10)$$

If we denote $U(t,t_0) \equiv e^{iH_0(t-t_0)}e^{-iH_0(t-t_0)}$ to be the time evolution operator then one can write the full field solution as:

$$\varphi\ (t,x) = U^\dagger(t,t_0)\varphi_I(t_0,x)U(t,t_0) \qquad (7.11)$$

The corresponding Hamiltonian is:

$$H_I(t) = e^{iH_0(t-t_0)}H_I e^{-iH_0(t-t_0)} \qquad (7.12)$$

Now define the *n*-point Green's function:

$$G(x_1,...,x_n) = \left\langle 0 \mid T\{\varphi(x_1)...\varphi(x_n)\} \mid 0 \right\rangle \qquad (7.13)$$

where

$$T\{\varphi(x)\varphi(y)\} = \begin{cases} \varphi(x)\varphi(y) & \text{if } y_0 > x_0, \\ \varphi(y)\varphi(x) & \text{otherwise} \end{cases} \qquad (7.14)$$

is the time-ordered product (T-product) of two fields.

The computation of the *n*-point Green's function, $\left\langle 0 \mid \phi(x_1)...\phi(x_n) \mid 0 \right\rangle$ is then tantamount to computing a general solution for $\phi(t,x)$. Instead, one computes the time ordered version, $\left\langle 0 \mid T\{\phi(x_1)...\phi(x_n)\} \mid 0 \right\rangle$. This term is called the Feynman propagator for the field solution of $\phi(t,x)$. A very ingenious graphical shorthand was devised by Feynman so that categories of solutions using the propagator could be drawn. These are called Feynman diagrams and they follow a certain rule set. They are as follows:

(1) Draw all connected graphs corresponding to the initial and final states. The number of lines that meet at each vertex is determined by the interaction term so

that in 3-space solutions there will be 3 lines, etc. Disconnected graphs to not contribute to any interaction.

(2) External lines that connected to others are treated as a pole factor and can be omitted from the interaction part of the graph (external leg amputation).

(3) In the solution computation there is a Dirac delta operator that imposes an energy-momentum conservation and these can nonredundantly be retrieved from a matrix element of $\langle p_1 ... p_n \mid iT \mid k_1 ... k_m \rangle$, where $iT = S - I$, and S is the S-matrix operator defined as the time limiting evolution operator $S = \lim_{t-t_0 \to \infty} e^{-iH(t-t_0)}$ where H is the second quantized Hamiltonian of the field. The second quantized system describes a quantum state basis in terms of the number of particles occupying a certain state, the so-called occupation number.

(4) Energy-momentum conservation is imposed separately at each vertex

(5) To each vertex, associate a factor $-i$ times the coupling constant, which is the relative coupling of the free and interaction Hamiltonians in the full Hamiltonian.

(6) To each internal line, associate a propagator taking on the value of the conserved 4-space momentum.

(7) A combinatorial factor, $\frac{1}{n!}$, that combines the number of equivalent contractions is associated with the coupling constant.

Appendix B: Causaloids, Quantum Gravity, and Information

In this review, causaloids will be operationally constructed as a potential means of building a physical theory encompassing both the probabilistic calculus of quantum theory (QT) and the indefinite causal structure of general relativity (GR). The treatise from Lucien Hardy in a series of papers will be followed closely in this discussion (Hardy, 2005, 2007, 2008, 2008b). Causaloids are an attempt at building a framework for construction of a mathematical physical theory that correlates with recorded data, while handling situations when an indefinite causal structure is present or when a time sequential evolution is not. In GR, causal structure is dynamic because of the nature of the spacetime metric and its dependence on the gravitational force from the distribution of mass. In QT, if time cannot be handled sequentially in the evolution equations, quantum uncertainty ensues. Any theory of quantum gravity (QG) must then, in all likelihood, be capable of handling indefinite causal structures while retaining a consistent probabilistic calculus.

To this end, consider two spacetime regions in the universe given by R_1 and R_2 which are spatio-temporally disconnected. One would like to posit a probabilistic statement about the region R_1 conditional on information from R_2. The current approaches to QG via path histories, such as LQG spinfoams, M-Theory, evolutional equations, or local infinitesimal changes via differential equations fall short in this scenario because of the disconnect in spacetime probabilities. Spacelike separate regions dictate that correlational operators should be tensor products, $A \otimes B$ whereas, temporal-sequential regions should use direct products, AB. For the causaloid formalism, a third kind of product, called the question mark product will be introduced. It is given by the notation, $[D?B]C = DCB$ where C is a causal switch operator that indicates either the tensor $\otimes$ or direct product (blank) depending on the causal structure of the regions. Note that ? remains a linear operator. Which product to use is therefore dictated by the causal structure of $R_1 \cup R_2$. In the case of probabilistic theories, such as quantum probability, with adjacent causality, a reduction in the information needed to infer the state of compound systems is present due to correlation relationships. This is referred to as *second 1evel physical compression*. The fundamental question that causaloids attempt to answer are the probabilistic propositions of the form:

$$p(X_{R_1} \mid F_{R_1}, X_{R_2}, F_{R_2}) \tag{8.1}$$

where X_{R_i} is an observed measurement (observable) of a physical entity made in R_i and F_{R_i} is some action performed in R_i., i.e., some control parameterization of the measurement device.

A topological assumption is made about spacetime regions R. Each region R may consist of the union of many elementary regions and composite regions consisting of more than two elementary regions themselves, $\{R_i\}$. An elementary region in spacetime is a simple region that may not be operationally reduced in terms of the measurement devices. Let Υ denote the space of elementary regions in a spacetime universe. For the purposes of this paper, Υ may be planck-scale cells or pixels in a discrete LQG-spinfoam or planck-scale computer (PSC). To standardize operations on Υ, attach to each region R, a set of vectors (operators), $r_{(x_R, F_R)}(R)$ and define the causaloid product, $\otimes^{\Lambda}$ by:

$$r_{(X_{R_i} \cup X_{R_j}, F_{R_i} \cup F_{R_j})}(R_i \cup R_j) = r_{(X_{R_i}, F_{R_i})}(R_i) \otimes^{\Lambda} r_{(X_{R_j}, F_{R_j})}(R_j) \tag{8.2}$$

For a composite region, $R = \bigcup_i R_i$, through the causaloid product, $\otimes^{\Lambda}$, the r vectors of the elementary regions would built the r vectors for the composite. Crucially, $p(X_{R_1} \mid F_{R_1}, X_{R_2}, F_{R_2})$ is well-defined $\Leftrightarrow v \parallel u$ where:

$$\begin{aligned} v &\equiv r_{(X_{R_i}, F_{R_i})}(R_i) \otimes^{\Lambda} r_{(X_{R_j}, F_{R_j})}(R_j) \\ u &\equiv \sum_{Y_{R_j}} r_{(Y_{R_i}, F_{R_i})}(R_i) \otimes^{\Lambda} r_{(X_{R_j}, F_{R_j})}(R_j) \end{aligned} \tag{8.3}$$

and the sum is over all possible observations Y_{R_j} made in R_i, consistent with the action F_{R_i}. Here,

$$p(X_{R_1} \mid F_{R_1}, X_{R_2}, F_{R_2}) = \frac{|v|}{|u|} \tag{8.4}$$

Now consider the collection of data that will be utilized to form a probability statement about the regions. Let the data be a collection of triplets (x, F_x, s_x), where x is the location of the observable, F_x is a parameterization (knob control setting) of the measurement operator (apparatus), and s_x is the outcome of the measurement. Next, consider a temporal manifestation of data collection via a series of probes in space. Let the quadruple $d_{i,n} = \left\{ \left(t_i, \left\{ t_i^{n,m} \right\} \right), n, F_{n,x}, s_{n,x} \right\}_{i,n}$ be the collection of data made by probe n at time t_i of the observable at location x with outcome $s_{n,x}$, using controls $F_{n,x}$. The series $\left\{ t_i^{n,m} \right\}$ represent the time delays seen by probe n of the results from m other probes. For each time slot t_i and probe, n, $d_{i,n}$ is recorded. At the end of the experiment, the series

$\{d_{i,n}\}_{i,n}, i = 0,1,...,N$ would have been recorded. Now consider a repeated experiment in which several controls, $\{F^e_{n,x}\}, e = 1,2,...,E$ are used where E is the number of experiments performed. This would be cosmologically problematic, but there are viable alternative setups to this thought experiment. Before showing this, we procede to define the structure of a causaloid which will determine the causaloid product $\otimes^{\Lambda}$ and r vectors for regions. The series of data, $\{d_{i,n}\}_{i,n}, i = 0,1,...,N$, which Hardy refers to as card stacks, one card per $d_{i,n}$, is divided into those which are consistent with a particular parameterization, F. For simplicity, this subset of cards is denoted by F. For any particular run of the experiment, say X, then $X \subset F \subset V$ where V denotes all possible cards (experiments).

Denote R_O to be the region specified by the set of cards in V consistent with the condition $x \in O$ (measurements in O). Let R_x be the elementary region consisting only of the cards in V with x. Regions are then spacetime entites where local choices for measurement (action) are taken. With this understanding, the term X_{R_o} means $X \cap R_o$, that is, the cards from a run stack X that belong to the region R_O. Define the procedure or action $F_{R_O} = F \cap R_O$, as the cards from F that belong to R_O. The pair (X_{R_O}, F_{R_O}) now defines the measurement result and action taking place in the region R_O. For notation sake, one can label the observations taking place in R_O as $Y_{R_O} = Y \cap R_O$. One now returns to the fundamental problem of calculating the probabilistic propositions given by (8.1). Since this is a probabilistic statement, one may inject a Fisherian (frequentist), Kolmogorovian (axiomatic calculus), Bayesian (conditioning calculus), or other notions of probability calculus in these definitions over regions. The point of departure for this paper would be to inject a more general approach to intuition and information transfer, that is, a notion of generalized fuzzy logic from GTU (Zadeh, 2005). For the purpose of brevity in this review, the more powerful version of a causaloid, the universal causaloid will be constructed here. In this particular version of a causaloid framework, repeating experiments will not be necessary for the inference needed to calculate probabilistic propositions.

In classical statistical approaches, repeating experiments are the calculus for constructing robust estimators of the parameters of the underlying probability densities or constructs of the phenomena under investigated. However, in the environment of the universe, resetting the clock to repeat the experiment of the probing bodies illustrated before as the means of data collection is problematic. In this review of causoloids, two categories will be viewed. The first will be with respect to repeated trials of measurements. The second will be a notion of universal causaloids where repeated experiments are not taken. Instead a larger deck of observations will be made and the metric for measuring the truth of a probabilistic proposition will be changed to approximate truth. The first kind of causaloid will be reviewed first. Consider two composite regions of spacetime, R_1 and R_2 with corresponding experiment controls (procedures), F_1 and F_2. Probabilistic statements (propositions) of the form:

$$p(Y_2 \mid Y_1, F_2, F_1) \tag{8.5}$$

will be the center of inquiry for causaloid frameworks. This is simply the probability of observing the outcome Y_2 using procedure F_2 in region R_2 given that Y_1 was observed using F_1 in region R_1. Statistically, this is a likelihood function. However, because the regions involved may be spatio-temporally vastly separated with no ordered or connected causal structure, its calculation would not be well defined. A deeper and more general formulation must be developed for such physical cases. One must then find if a proposition is well defined (w.d.) and if so, find out how to calculate it.

Consider a sufficiently large region, R covering most of V. Next, assume that some C is a universal condition on the procedures, $F_{V\setminus R}$ and outcomes, $Y_{V\setminus R}$ respectively in the region, $V \setminus R$ such that the probabilities, $p(Y_R \mid F_R, C)$ are w.d. This guarantees the existence of these likelihoods in a sufficiently large portion of the computable universe. Assuming the existence of C, the likelihood functions will simply be abbreviated as $p(Y_R \mid F_R)$ and are w.d. Applying reductionism to this large region, three kinds of physical compressions will be defined that will help in forming the calculations for the likelihood computations. First level compression will apply to single regions. Second level compression will apply to composite regions. Finally, third level compression will be applied to matrix constructs that are manifested out of calculations pertaining to first and second compressions. Define a shorthand for likelihoods, using the notation, $\alpha_1 = (Y_{R_1}, F_{R_1})$ for each possible pair in the region R_1:

$$p_{\alpha_1} = p(Y_{R_1}^{\alpha_1} \cup Y_{R\setminus R_1} \mid F_{R_1}^{\alpha_1} \cup F_{R\setminus R_1}) \tag{8.6}$$

In a physical theory that is governed in part by a probability calculus, the set of possible p_{α_1} can be reduced in size by relations, so that a minimal vector of p_{α_1} suffices in expressing itself and without loss of generality, in a linear relationship,

$$p_{\alpha_1} = \bar{r}_{\alpha_1}(R_1) \cdot \bar{p}(R_1) \tag{8.7}$$

where the state vector, $\bar{p}(R_1)$ is given by a minimal index set, Ω_1 :

$$\bar{p}(R_1) = \begin{pmatrix} \cdot \\ \cdot \\ p_{l_i} \\ \cdot \\ \cdot \end{pmatrix}, \; l_i \in \Omega_1 \tag{8.8}$$

Ω_1 is referred to as the fudicial set of measurement outcomes. Since in a probability manifold, probabilities are linear, a linear relationship in the above compression is most efficient. Ω_1 may not be unique in general, but since it defines a minimal set, there exist a set of $|\Omega_1|$ linearly independent states in $\bar{p}$. The first level compression for region *R1* is then represented by the matrix:

$$\Lambda_{\alpha_1}^{l_1} \equiv \left(r_{l_i}^{\alpha_1} \right) \tag{8.9}$$

where $r_{l_i}^{\alpha_1}$ is the l_i^{th} element of the vector $\bar{r}_{\alpha_1}$. Compression in the matrix $\Lambda_{\alpha_1}^{l_1}$ is manifested by the degree of rectangularity (lack of squareness), a flattening of the matrix.

Second level compression is shown for composite regions. Consider two regions *R1* and *R2*. Form the composite region, $R_1 \cup R_2$ and express its state:

$$\overline{p}(R_1 \cup R_2) = \begin{pmatrix} . \\ . \\ p_{k_i k_j} \\ . \\ . \end{pmatrix}, \; k_i k_j \in \Omega_{12} \tag{8.10}$$

It has been shown that the second level fiducial set, Ω_{12} can be chosen such that

$\Omega_{12} \subseteq \Omega_1 \times \Omega_2$ (cartesian product) . Further, one can express the likelihoods as:

$$\begin{aligned} p_{\alpha_1\alpha_2} &= \overline{r}_{\alpha_1\alpha_2}\left(R_1 \cup R_2\right) \cdot \overline{p}\left(R_1 \cup R_2\right) \\ &= \sum_{l_i l} r_{l_i}^{\alpha_1} r_{l_j}^{\alpha_2} p_{l_i l_j} \\ &= \sum_{l_i l} r_{l_i}^{\alpha_1} r_{l_j}^{\alpha_2} \overline{r}_{l_i l_j} \cdot \overline{p}\left(R_1 \cup R_2\right) \end{aligned} \tag{8.11}$$

Then the following must hold:

$$\overline{r}_{\alpha_1\alpha_2}\left(R_1 \cup R_2\right) = \sum_{l_i l_j} r_{l_i}^{\alpha_1} r_{l_j}^{\alpha_2} \overline{r}_{l_i l_j}\left(R_1 \cup R_2\right) \tag{8.12}$$

since there exist a spanning set of linearly independent state elements in $\overline{p}\left(R_1 \cup R_2\right)$.

Now define the matrix representation for second level compression of $R_1 \cup R_2$.

Let

$$\Lambda_{l_i l_j}^{k_1 k_2} = \left(r_{k_i}^{l_1} r_{k_j}^{l_2}\right) \tag{8.13}$$

where $r_{k_i}^{l_1} r_{k_j}^{l_2}$ is the $k_i k_j{}^{th}$ element of the vector $\overline{r}_{l_i l_j}$. One can then express these components as:

$$r_{k_i k_j}^{\alpha_1\alpha_2} = \sum_{l_i l_j} r_{l_i}^{\alpha_1} r_{l_j}^{\alpha_2} \Lambda_{l_i l_j}^{k_i k_j} \tag{8.14}$$

and in this way calculate the likelihoods for the composite region from those of each of its constituent component regions. This is the second level compression above and beyond first level compression of simple regions for the case of a composite region. One using this definition of second level compression to define the causaloid product, $\otimes^{\Lambda}$:

$$\overline{r}_{\alpha_1\alpha_2}\left(R_1 \cup R_2\right) = \overline{r}_{\alpha_1}\left(R_1\right) \otimes^{\Lambda} \overline{r}_{\alpha_2}\left(R_2\right) \tag{8.15}$$

Because this definition generalizes completely to higher level composite regions, the second level compression matrices are defined analogously for *n*-region composites by:

$$\Lambda_{l_1 l_2 \ldots l_n}^{k_1 k_2 \ldots k_n} \tag{8.16}$$

Now consider a master matrix that consists of all levels of lambda matrices for elementary regions, R_x, for a set *x*, where O_R is the set of *x* in the region *R*:

$$\begin{pmatrix} \Lambda_{l_1 x_1}^{\alpha_x} & \forall x_1 \in O_R \\ \Lambda_{l_{x_1} l_{x_2}}^{k_{x_1} k_{x_2}} & \forall x_1, x_2 \in O_R \\ \Lambda_{l_{x_1} l_{x_2} l_{x_3}}^{k_{x_1} k_{x_2} k_{x_3}} & \forall x_1, x_2, x_3 \in O_R \\ \ldots\ldots\ldots\ldots\ldots\ldots & \\ \ldots\ldots\ldots\ldots\ldots\ldots & \end{pmatrix} \tag{8.17}$$

In a consistent probabilistic formalism for a phsyical theory, these Λ-matrices will in turn have a relationship among each other. These relationships, along with a rule set for calculating other Λ-matrices based on these relationships, can be expressed as a set of action operators, a. Let Λ_{Ω} denote this reduced set of Λ-matrices based on the relationship reductions. Then the causaloid is denoted by the pair $\left(\Lambda_{\Omega}, a\right)$. Reductions by the third level of physical compression are manifested by identities that express higher order Λ-matrices in terms of lower order ones. Examples of Λ-matrix set reductions are in the following two scenarios:

(1) When the fiducial set for the composite region is separable (expressable) into (as) a cartesian product of the fiducial sets of the components, i.e.

$$\Lambda^{k_{x_1}k_{x_2}k_{x_3}\ldots k_{x_n}}_{l_{x_1}l_{x_2}l_{x_3}\ldots l_{x_n}} = \Lambda^{k_{x_1}k_{x_2}k_{x_3}\ldots k_{x_m}}_{l_{x_1}l_{x_2}l_{x_3}\ldots l_{x_m}} \Lambda^{k_{x(m+1)}k_{x(m+2)}\ldots k_{x_n}}_{l_{x(m+1)}l_{x(m+2)}\ldots l_{x_n}} \text{ when } \Omega_{x_1\ldots x_n} = \Omega_{x_1\ldots x_m} \times \Omega_{x_{(m+1)}\ldots x_n} \tag{8.18}$$

(2) When higher order Λ-matrices can be computed based on pairwise 2-index Λ-matrices:

$$\Lambda^{k_{x_1}k_{x_2}k_{x_3}\ldots k_{x_n}}_{l_{x_1}l_{x_2}l_{x_3}\ldots l_{x_n}} = \sum_{k_2^{'}\in\Omega_{23},\ldots,k_{n-1}^{'}\in\Omega_{(n-1)n}} \Lambda^{k_{x_1}k_{x_2}}_{l_1 k_2^{'}} \ldots \Lambda^{k_{(n-1)}^{'}k_n}_{l_{(n-1)}l_n} \text{ when } \begin{array}{l} \Omega_{12\ldots n} = \Omega_{12}\times\ldots\times\Omega_{(n-1)n} \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ \Omega_{ij} = \Omega_i \times \Omega_j \end{array} \tag{8.19}$$

Next, consider the case where ensembles of experiments are limited or were one large data set card is instead collected. One considers this case because effects are not preserved as these repeated experiment processes are not invariably reversible, so that an experiment performed later would be run under very different conditions regardless of how hard one tries to preserve the cosmological laboratory. So, one considers running experiments in one long consecutive batch. However, this taxes the statistical theory behind any of the probability propositions arises from such an experiment. To overcome this, consider the following methodology. Let A be a proposition concerning the data that will be collected in an experiment. To this proposition associate a vector, r_A, as before with regions. Next, consider a complete sequence of mutually exclusive propositions, $\{A_i\}_{i=1,\ldots,M}$, where $A_i = \left[A^C\right]^{C\ldots C}$, is the i^{th} complementation of A.

Define the approximating vector, $r_A^I = r_A + \sum_{i=1}^{M} r_{A_i}$. Declare the assertion:

$$A \text{ has a}(\text{n})(\text{approximately}) \text{ true value} \Leftrightarrow r_A \approx_\varepsilon r_A^I \tag{8.20}$$

where the equivalence $\approx_\varepsilon$ is modulo an approximation to within a threshold ε to be made precise later. This also points to the inevitability that experiments may never concisely estimate parameters. Now consider the vectors given by the application of the causaloid product $\otimes^\Lambda$:

$$r_n \equiv r_{(X_{1n},F_{1n})} \otimes^\Lambda r_{(X_{2n},F_{2n})} \tag{8.21}$$

where $R_n = R_{1n} \bigcup R_{2n},\ n = 1,\ldots,N,\ N >> 1$. Next, define the vector:

$$r_n^I \equiv \sum_{Y_{1n} \subset F_1} r_{(X_{1n},F_{1n})} \otimes^\Lambda r_{(X_{2n},F_{2n})} \tag{8.22}$$

Now define the difference vector, $\overline{r}_n = r_n^I - r_n$ and assume the condition $r_n = pr_n^I,\ \forall n$. In this way, r_n plays the role of *v* and r_n^I that of *u*. To get to a calculation of the probability proposition, one now considers the vector definition:

$$r_A \equiv \sum_{(p-\Delta p)N<|S|<(p+\Delta p)N} \left(\bigotimes_{n\in S}^\Lambda r_n\right) \otimes^\Lambda \left(\bigotimes_{n\in \overline{S}}^\Lambda \overline{r}_n\right) \tag{8.23}$$

A translation of this vector is the following: r_A corresponds to the property that pN out of N regions R_n have the result X_{R_n} to within a threshold of $\pm\Delta pN$. One now has the condition, $r_A^I \equiv \bigotimes_n^\Lambda r_n^I$. Taking the definition $r_n = pr_n^I,\ \forall n$, and using an approximation to the binomial distribution, one can rewrite r_A:

$$r_A = \left[\sum_{(p-\Delta p)N<|S|<(p+\Delta p)N} p^n (1-p)^{N-n} \right] r_A^I \approx \left[1 - O\left(\frac{1}{\Delta p \sqrt{N}} \right) \right] r_A^I \qquad (8.24)$$

Hence, $\frac{r_A}{r_A^I} \approx 1 - O\left(\frac{1}{\Delta p \sqrt{N}} \right)$. For a given threshold $\varepsilon > 0$, $\left| \frac{r_A}{r_A^I} \right| < 1 - \varepsilon,\ \forall N > N_\varepsilon$ for a sufficiently large N_ε. In this respect, the equivalence, $\approx_\varepsilon$ occurs between r_A and r_A^I and the $\varepsilon -$truthfulness of A. The formal definition of a universal causaloid follows:

Definition: (Universal Causaloid). The universal causaloid for a region, R, made up of elementary regions $\{R_x\}$, when it exists is defined as the entity represented by a mathematical object which may be utilized to calculate the vectors, r_A for a proposition A concerning the data collected in $\{R_x\}$ such that if A is $\varepsilon -$truthfulness, one has that $r_A \approx_\varepsilon r_A^I$ where $r_A^I = r_A + \sum_{i=1}^{M} r_{A_i}$ and $\{A_i\}_{i=1,\ldots,M}$ is a complete set of mutually exclusive propositions where $A_i = \left[A^C \right]^{C\ldots C}$, is the complementation i times of A.

By using the symmetries inherent in classical probability (CprobT) and quantum theory (QT), the calculation of the $\varepsilon -$truthfulness can be accomplished without repeated experiments within those paradigms. The universal causaloid is seen as corresponding to the entire history of the universe that is essential to calculations pertinent to cosmological constructs without the enormity of its computation through these more compact $\varepsilon -$truthfulness tests for propositions. An assumption that would further simplify the computations involved with universal causaloids is the principle of counterfactual indifference:

Definition. (Principle of Counterfactual Indifference). The principle of counterfactual indifference is the condition that the probability of an event E does not depend on the action that would have been implemented had the complement E^C happened instead if one conditions on cases where E^C did not happen modulo that the measurement device did not alter the state of the observed entity in any large way (low key measurement).

Applying this condition to the case of r vectors, $r_{(X_1,F_1)} = r_{(X_1,F_1^C)}$ where $X_1 \subseteq F_1, F_1^C$ since when one applies the procedure actions F_1 and F_1^C, one does the same thing when the same outcomes, X_1 are observed by counterfactual indifference. The universal causaloid is a macroscopic approach to physical theory construction. By combining this attribute with the promise of the discrete computational models of LQG at the planck scale, despite the unknown emergence of a 3+1 dimensional spacetime at that microscopic level, an emergent property of QG may be mended there. This is the proposal of this paper, utilizing a generalization to the probabilistic causaloid and the LQG-spinfoam inspired computation at the planck-scale pixels of a surface for describing abstract physically conformal information.

It has been shown that a version of a quantum (classical) computer can be setup using the causaloid formalism by considering an abstract computer with generalized gates that is a subset of all possible gates in a pseudo-lattice of pairwise interacting qubits. Call this pseudo-lattice of pairwise interacting qubits, Θ_L. Call the universe set of gates possible, S_I. In a practical computer, the set of gates is restricted to a finite number N.

Let $S \equiv \{s_i\}_{i=1}^{N} \subset S_I$ be the set of gates in a computer. Define a causaloid on the set of pairwise interacting qubits, Λ_S. The triple (Θ_L, Λ_S, S) is then considered a causaloid-induced computer on a pseudo-lattice of pairwise interacting qubits, Θ_L, with quantum gates *S*. Now consider the class of causaloid-induced computers with number of gates bounded above by *M*. Call this class, C_M^{Λ}. A universal computer in this class is one that can simulate all other computers in C_M^{Λ}. Here it should be pointed out that an important distinction between a QG computer and a quantum (classical) computer is that it is not a step computer, i.e., no sequential time steps are realized for computation. This is so because of the indefinite causal structure in a QG environment and subsequent computer.

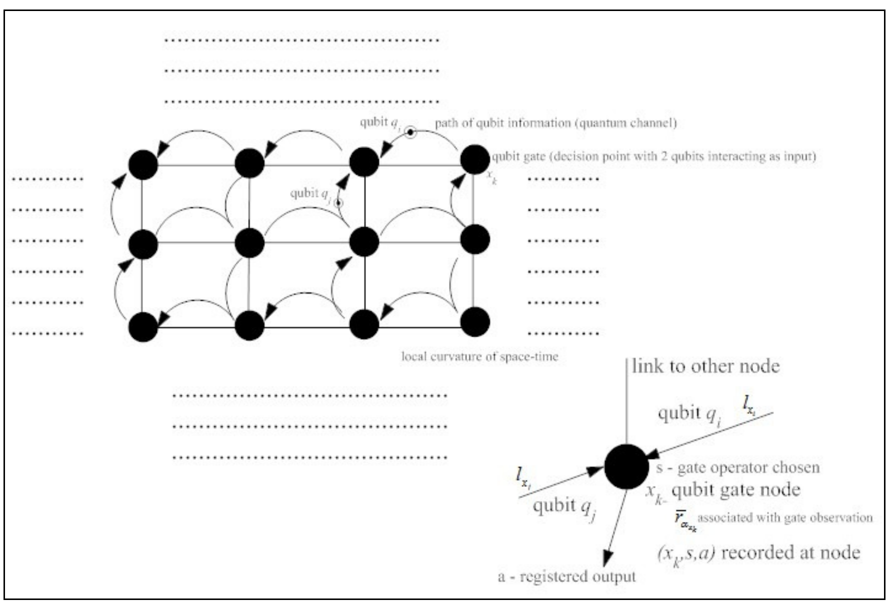

Figure 31. QG computer pseudo-lattice

Each node of the pseudo-lattice represents a quantum gate, x_k, where a particular gate operator, *s* is chosen at interaction time between two input qubit information channels, q_i and q_j. Upon interaction and gate operation chosen, an output, *a*, is produced via measurement and transformation operators. The triple, (x_k, s, a) is recorded at the gate. Associated with this record is the vector, $\overline{r}_{\alpha_{x_k}}$. The two separate qubit channel inputs can then be separated as $l_{x_k} = l_{x_i} l_{x_j}$ where l_{x_i} and l_{x_j} mark the fiducial measurements on qubits q_i and q_i respectively. These operations and the pseudo-lattice constitute a causaloid diagram for a quantum computer. The causaloid for the pairwise interacting qubit computer model can be written as:

$$\Lambda = \left(\left\{ \Lambda_{\alpha_{x_k}}^{l_{x_i} l_{x_j}},\ \forall x_k \right\}, \left\{ \Lambda_{l_{x_i} l_{x_j}}^{k_{x_i} k_{x_w}},\ \forall \text{ adjacent } x_k, x_w \right\}; R \right) \tag{8.25}$$

where *R* is the set of rules (actions) constructing the causaloid qubit diagram (pairwise interacting qubits, nodes with gate operations as defined above) and the clumping operations given by the categories in (8.18) and (8.19) for grouping nonsequential nodes for any set in the set of all configurations of qubit nodes, Ω. State evolution can be simlulated by considering nested spacetime regions, $R_t,\ t = 0,...,T$ where:

$$R = R_0 \supset R_1 \supset ... \supset R_T = \varnothing \tag{8.26}$$

Interprete the region R_t as what happens in *R* after time *t*. Now consider the state vector, $\overline{p}(t) = \overline{p}(R_t)$ at time *t* for the region R_t. Construct the evolution equation as:

$$\overline{p}(t+1) = G_{t,t+1}\left(\overline{p}(t)\right) \tag{8.27}$$

where $G_{t,t+1}$ is the evolution operator that depends on the output,-procedure pair, $\left(Y_{R_t \setminus R_{t+1}}, F_{R_t \setminus R_{t+1}}\right)$ on the complementary region, $R \setminus R_t$. By using this technique of nested regions, one simulates a time evolution without using a physical time parameter.

QG computers are conceivable and plausible if one can show that a GR computer is possible. Nonetheless, for the sake of completeness, a GR computer should be demonstratable using a causaloid formalism as have QT and classical computers above. Possible GR compatible computers may utilize gravitational waves and have been shown to be plausible Church-Turing-Deutsch physically-based computers leading to hypercomputability by utilizing supertasks (Pitowsky, 1990; Etesi & Nemeti, 2002; Shagrir & Pitowsky, 2003). Hypercomputability is the condition in a computing device that permits one to compute functions that cannot be computed by a Turing machine. These GR hypercomputers utilize a special spacetime structure called Malament-Hogarth spacetime.

Definition (Malamert-Hogarth spacetime). A pair $(\mathcal{M}, g)$, where $\mathcal{M}$ is a connected 4-dim Hausdorff C^∞ manifold and g is a Lorentz metric, is called a *Malamert-Hogarth spacetime* if $\exists$ a timelike half-curve $\gamma_1 \subset \mathcal{M}$ and a point $p \in \mathcal{M} \ni \int_{\gamma_1} d\tau = \infty$ and $\gamma_1 \subset I^-(p)$ where $I^-(\gamma)$ denotes the set of past events of γ.

In an Malamert-Hogarth (M-H) spacetime $(\mathcal{M}, g)$ there is a future-directed timelike curve γ_2 that starts at a point q that is in the chronological past of p (i.e., $q \in I^-(p)$) and ends at p. So, $\int_{\gamma_2(q,p)} d\tau < \infty$. Furthermore, in an M-H spacetime, events are not related to each other causally, that is, an M-H spacetime is not globally hyperbolic and so, has an indefinite causal structure. Two other powerful classes of GR computers will be reviewed that are capable of computing general recursive functions and are more feasible cosmologically.

Definition. (past temporal string). Consider the string that is formed from a collection of nonintersecting open regions, $O_i \in (\mathcal{M}, g)$, an M-H spacetime, such that: (i) $\forall i,\ O_i \subset I^-(O_{i+1})$, and (ii) $\exists q \in \mathcal{M} \ni \forall i,\ O_i \subset I^-(q)$. Such strings are called past temporal strings (PTS).

PTSs construct complex spacetimes referred to as arithmetic-sentence-deciding spacetimes of order *n* or SAD_n. A first order *SAD*, denoted by SAD_1, is a Turing Machine (TM) that travels towards an event and is in the event's past spacetime cone. SAD_1s can be stacked on top of each of spacio-temporally to construct higher order SAD_n.

Result. A SAD_1 can decide 1-quantifier arithmetic, that is, any relation of the form $S(z) = \exists x R(x,z)$ or $\forall x R(x,z)$, where *R* is recursive.

Definition. If $(\mathcal{M}, g)$ is a M-H spacetime, then it is a SAD_1 spacetime. If $(\mathcal{M}, g)$ admits strings of SAD_{n-1} then it is a SAD_n spacetime.

SAD_n spacetimes construct hierarchies of spacetimes as in the following sequence:

$$FTM\ \text{-}\ TM\ \text{-}\ SAD_1\ \text{-}\ ....\ \text{-}\ SAD_n\ \text{-}\ ...\ \text{-}\ AD \tag{8.28}$$

where an *AD* is an arithmetic-deciding computer which is a computer that can compute exactly $\aleph_0$ functions.

Now consider GR computers that can perform supertasks in the vicinity of back holes. Rotating black holes that are not charged are classified as Kerr black holes.If they are charged then they are called Kerr-Newman black holes. The exterior of black holes that are charged form a spacetime called a Kerr-Newman spacetime and are types of M-H spacetimes. Therefore, an abstraction for a GR computer utilizing the effects near a black hole is plausible. To this end, a scenario is built where two timelike curves, (γ_P, γ_O) are traced respectively, for a computer traveling around the black hole in a stable orbit and an observer crossing the outer event horizon of the black hole, entering the inner horizon, but not continuing into a singularity. Both computer and observer start from a point $q \in \mathcal{M}$ with $\|\gamma_P\| = \infty$ and $\|\gamma_O\| < \infty$.

The Malament-Hogarth event takes place at a point *p* on an orbit around the black hole. The role of the computing device is to decide on the consistency of theorems of ZFC and informing the observer of such results. Assume that a TM, labeled *T*, that is capable of enumerating all the theorems of ZFC exists and that the computing device *P* and observer *O* have a copy of it each. Then if the observer, *O*, does not receive a signal from *P*, before it reaches *p*, then the ZFC is consistent. Otherwise, if *P* receives a message before reaching *p*, then ZFC is inconsistent. This class of GR computers near black holes are referred to as relativisitc $G = (\gamma_O, \gamma_P)$ computers (Syropoulos, 2008, pp. 137-148).

A similiar, but ideologically different class of black hole relativisitc computers is that proposed by Lloyd and Ng. In this model, the entire black hole is considered as a simple but ultimate computer with speed v ops/bit/unit of time and with number of bits of storage memory, I, bounded from above according to (Ng & Lloyd, 2004):

$$Iv^2 \leq \frac{1}{t_p^2} \approx 10^{86} \sec^{-2} \tag{8.29}$$

Taking this to its physical conclusion, the entire universe is considered a self-referential, self-constructing computer and as such, any physical device or thing is a computer (Lloyd, 2000; Lloyd, 2006a). The seeds for a deterministic computing universe hypothesis were, of course, planted earlier by Zuse and others in the Zuse Thesis - the universe is a computer via deterministic cellular automaton (Schmidhuber, 1996; Zuse, 1970). More recently, Wolfram posits that if spacetime is discrete, cellular automaton model the universe and as such, are limited in their computation of things, but that everything is a computer of sorts (Wolfram, 2002)

Regardless, it is still unknown how time behaves at the planck scale, posited to be fuzzy, at best, in the QG research arena, not withstanding several controversial experiments minimizing or challenging this effect (Lieu & Hillman, 2003). Nonetheless, in a conceptual QG computer, the concept of separate space or time resources must be combined to reflect a new kind of singular spacetime resource measurement for showing computational rates and limitations. As pointed out before, QG computers are nonstep devices.

Appendix C: Category and Topos Theory

Consider an object C consisting of general objects, $A, B, C, \ldots$ labeled as $Ob(C)$ and maps or relations (sometimes referred more generally as arrows), $f, g, h, \ldots$, labeled as $Arr(C)$, such that

1. for each arrow $f \in Arr(C)$, $\exists$ two objects, $dom(f), cod(f) \in Ob(C)$ such that f acts only on $dom(f)$ and maps only to $cod(f)$, i.e., $f : dom(f) \to cod(f)$, written as $dom(f) = A \xrightarrow{f} cod(f) = B$,
2. for each object $A \in Ob(C)$, an identity map, denoted by 1_A exists such that $A \xrightarrow{1_A} A$ is one such map from A to A,
3. for each pair of maps, (f, g) in $Arr(C)$ such that $A \xrightarrow{f} B \xrightarrow{g} C$, when objects $A, B,$ and C exists, a composition map $h = g \circ f$ exists, defined as $A \xrightarrow{h} C$,
4. if $A \xrightarrow{f} B$, then $1_B \circ f = f$ and $f \circ 1_A = f$ (identity laws), and
5. if $A \xrightarrow{f} B \xrightarrow{g} C \xrightarrow{h} D$, then $(h \circ g) \circ f = h \circ (g \circ f)$ (associative law).

Some consequences of this definition are (a) $dom(1_A) = cod(1_A) = A$, (b) $g \circ f$ is defined if $dom(g) = cod(f)$, (c) $dom(g \circ f) = dom(f)$, and (d) $dom(g \circ f) = cod(g)$. Label the pair of objects $Ob(C)$ and arrows $Arr(C)$ as C. If C satisfying only condition 1 it is called a *metagraph*. If in addition, C satisfies 2 then it is called a *metacategory*.

Metacategories will be subject to the axioms of 3 and 4. With some imagination, one can see the generality of metacategories. For example, the metacategory of sets consists of all sets and arrows are all functions with the usual identity and composition of functions defined in naïve set theory. The metacategory of all groups consists of all groups *G,H,K,…* with arrows which are functions *f* from a set *G* to a set *H* defined so that $f : G \to H$ is a homomorphisn of groups. The metacategory of all topological or compact Hausdorff spaces each with the continuous functions as arrows (topologies can be defined by continuous maps) are two other examples.

Definition. A category is a metacategory, C , interpreted within set theory, that is, the objects in a category is a set of objects, *O* and the arrows is a set *A* of arrows, together with the usual functions defined by $\xrightarrow{dom}$ and $\xrightarrow{cod}$ such that $A \underset{cod}{\overset{dom}{\rightleftarrows}} O$.

Definition. The set of all possible arrows from the object B to C in C , a category, is denoted as $\hom(B,C) = \{ f \mid f \text{ in } C, dom(f) = B, cod(f) = C \}$, the set of its *morphisms*.

The set $\hom(A, A)$ defines all *endomaps*, for all objects A in C , a category. A special type of category is the *monoid* which is a category with exactly one object. Indeed, a category is a very general animal which can be described as a generalized mathematical object reflecting the rich structure of specialized mathematical structures used in known diverse mathematical and scientific endeavors. In order to further develop the richness of categories, the definition of mapping between categories is given.

To generalize the ideas of a null set and singleton subsets we define initial and terminal objects of a category C .

Definitions. An object 0 is *initial* in a category C if for every $A \in Ob(C)$ there is one and only one arrow $f_A : 0 \to A$ in C. Reversing the role of arrows, an object 1 is *terminal* in C if for every $A \in Ob(C)$ there is one and only one arrow $f_A : A \to 1$ in C.

Duality is a mathematical concept in which the roles of two objects engaged in a structural relationship are reversed. In the general case of categories, which would generalize to dualities everywhere, we construct the definition:

Definition. From a given category C, construct its dual or opposite category, C^{op} in the following manner:

$Ob(C) = Ob(C^{op})$ and for each $f \in C$ mapping $A \to B$, define the arrow $f^{op} \in C^{op}$ mapping $B \to A$. The only arrows of C^{op} are of these constructions. The composition $f^{op} \circ g^{op}$ is defined precisely when $f \circ g$ is and $f^{op} \circ g^{op} = (g \circ f)^{op}$. In addition, $f^{op} = cod(f)$ and $cod(f^{op}) = dom(f)$.

The significance of duality in category theory is that if a statement Σ of category theory is held to be true then automatically the statement given by the opposite Σ^{op} is true as well. The conclusion is that this duality principle cuts in half the work to be done in a category or in category theory in general (Goldblatt, 2006, p.46). One would like to generalize the concept of products and limits since with these constructionists theories can be built. To this end define general diagrams and cones:

Definition. Let D be a metagraph (diagram) with vertices $\{d_i : i \in I\}$ for a category C. A *cone* over D is a family of arrows $\{A \xrightarrow{f_i} d_i : i \in I\}$ from A to objects in D

such that for any arrow $d_i \xrightarrow{f_{ij}} d_j$ in *D*, the diagram

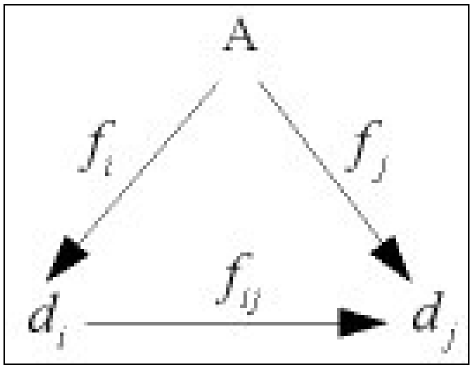

Figure 32. Category cone

commutes. The object *A* is called the *vertex* of the cone. An arrow from a cone over D$\{A \xrightarrow{f_{A_i}} d_i : i \in I\}$ to another cone over D$\{B \xrightarrow{f_{B_i}} d_i : i \in I\}$ is a C *-arrow* $A \xrightarrow{g} B$ if the diagram

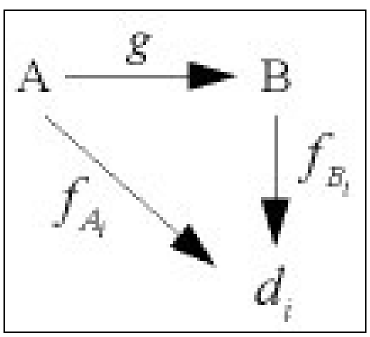

Figure 33. Category-theoretic C-arrow

commutes for each $i \in I$. If such an arrow *g* exists then the cone $\{A \xrightarrow{f_{A_i}} d_i : i \in I\}$ *factors through* the cone $\{B \xrightarrow{f_{B_i}} d_i : i \in I\}$. The set of cones over *D* denoted by *Cone(D)* then form a category using this procedure. One now gets to the definition of limits.

Definition. A *limit* for the diagram *D* is the terminal object of *Cone(D)*. The *colimit* of *D* is the terminal object of the cone, $Cone^{opp}(D)$ which is the cone defined over the dual category, C^{op}.

Definition. A category, C is said to be *(finitely) complete* or *cocomplete* if the limit or colimit of any finite diagram in C exists in C.

A useful device for category manipulation is the pullback mechanism. Formally, a *pullback* of a pair of arrows defined as $A \xrightarrow{f} C \xleftarrow{g} B$ in $Arr(C)$ with common codomain, *C*, is a limit in C for the diagram:

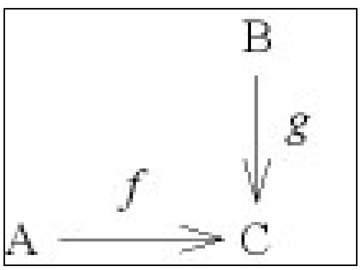

Figure 34. Category-theoretic pullback operation

where a cone for this diagram consists of a triplet of arrows (f', g', h) in C such that the diagram:

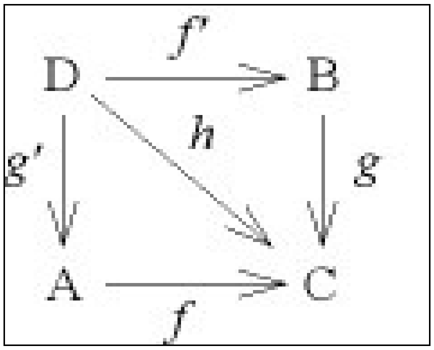

Figure 35. Pullback cone

commutes. Using the definition of a universal cone and the commutivity of the above diagram, one can eliminate the arrow h and arrive at a more precise definition,

Definition. A *pullback* of the pair of arrows $A \xrightarrow{f} C \xleftarrow{g} B$ in $Arr(C)$ is a pair of arrows $A \xrightarrow{g'} D \xleftarrow{f'} B$ in $Arr(C)$ such that:

(1) $f \circ g' = g \circ f'$ in $Arr(C)$, and

(2) whenever $A \xrightarrow{h} E \xleftarrow{j} B$ are a pair of arrows in $Arr(C)$ such that $f \circ h = g \circ j$ then there is exactly one arrow in $Arr(C)$ $k : E \to D$ such that $h = g' \circ k$ and $j = f' \circ k$. The diagram (f, g, f', g') is called a *pullback square* (Goldblatt, 1984, p.63-64).

Exponentiation is defined next. Consider the category given by the usual sets of axiomatic set theory with set operations. Denote this category by the label, *SET*. If A and B are two sets in *SET*, let $B^A = \{f : f : A \to B\}$ denote the set of all functions (arrows) having domain A and codomain B. A special arrow in *SET* will be associated with B^A, the evaluation arrow, $ev : B^A \times A \to B$ with the assignment rule, $ev((f, x)) = f(x)$.

Definition. A category C has *exponentiation* if (a) it has a limit for any two arrows in $Arr(C)$, and (b) if for any given objects $A, B \in Ob(C)$ there exist an object, B^A and an arrow, $ev \in Arr(C)$, $ev : B^A \times A \to B$, referred to as an evaluation arrow, such that for any $C \in Ob(C)$ and $g \in Arr(C)$, $g : C \times A \to B$, there exist a unique arrow, $\hat{g} \in Arr(C)$ making the diagram:

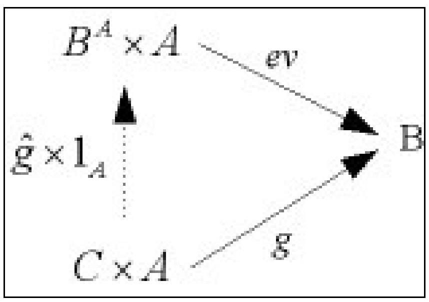

Figure 36. Categorical exponentiation

commute, that is, the existence of a unique arrow, $\hat{g}$ such that $ev \circ (\hat{g} \times 1_A) = g$.

In order to compare two or more categories, a mechanism must exist that maps categories to each other. The space of morphisms between two categories will now be defined.

Definition. A *functor*, T is a morphism between two categories, C and $\mathcal{B}$, written as $T : C \to \mathcal{B}$ in which $dom(T) = C$ and $cod(T) = \mathcal{B}$, which assigns to each $C \in C$, an object $T(C) \in \mathcal{B}$ and an arrow associated with T, written, T_{ar}, that assigns to each arrow $f : C \to C'$ of C an arrow $T_{ar} f : T(C) \to T(C')$ of $\mathcal{B}$ in such a way so that

$T(1_C) = 1_{T(C)}$ and $T(g \circ f) = T(g) \circ T(f)$

whenever, $g \circ f$ is defined in *C*.

Functors on categories must act on both objects and arrows of categories as above. In this way a composition of functors, functor isomorphism, and a faithful functor can be defined to expand on the space of category functors and hence on the relations between categories.

Definitions. (a) A functor $S \circ T : C \to \mathcal{A}$ is a *functor composition* of functors *S* and *T* if $C \xrightarrow{T} \mathcal{B} \xrightarrow{S} \mathcal{A}$ are functors between categories $\mathcal{A}$, $\mathcal{B}$, and C such that $C \to S(T(C))$ and $f \to S(T(f))$ for objects C and arrows *f* of C. (b) A function $T : C \to \mathcal{B}$ is a *functor isomorphism* between C and $\mathcal{B}$ if it is a bijection both on objects and arrows between C and $\mathcal{B}$, i.e., if $\exists$a functor $S : \mathcal{B} \to C$ for each functor *T*, such that $S \circ T = T \circ S = Id$ where Id is the identity functor between C and $\mathcal{B}$ and $S = T^{-1}$ is a two-sided inverse functor. (c) a functor $T : C \to \mathcal{B}$ is *full* when to every pair $(C, C^{'})$ of objects in C and every arrow $g : T(C) \to T(C')$ of $\mathcal{B}$, $\exists$ an arrow $f : C \to C'$ in C with $g = T(f)$, and (d) a functor $T : C \to \mathcal{B}$ is *faithful* (an embedding) if to every pair $(C, C^{'})$ of objects and every pair (f, g) of parallel arrows (arrows with the same domain and codomain) in C, $T(f) = T(g) \Rightarrow f = g$. A consequence of these definitions is that compositions of faithful and full functors are again faithful and full respectively.

Faithfulness and fullness are embedding features between categories in the following sense: if (C, C') is a pair of objects in C, the arrow of *T*, $T_{ar} : C \to \mathcal{B}$ assigns to each $f : C \to C'$ an arrow $T_{ar}(f) : T(C) \to T(C')$ so that a function is defined:

$T_{(C,C')} : \hom(C, C') \to \hom(T(C), T(C')), \;\; f \to T(f)$

as a mapping of the set of arrows between C and C' to the set of arrows between *T(C)* and *T(C′)*, then T is full when every such function $T_{(C,C')}$ is surjective and faithful when it is injective. If T is both full and faithful, then every such $T_{(C,C')}$ is bijective, but not necessarily an isomorphism (MacLane, 1971, pp. 7-15). Embeddings of categories naturally call upon a definition of subcategories or categories contained within other categories.

Definitions. A *subcategory* $\mathcal{S}$ of a category C is a collection of objects and arrows of C that is closed under identities, domains and codomains, i.e., (a) if f is an arrow of $\mathcal{S}$ then it is an arrow of C and both $dom(f)$ and $cod(f)$ are objects of $\mathcal{S}$, (b) for each object S in $\mathcal{S}$, its identity arrow, 1_S is in $\mathcal{S}$, and (c) for every pair of arrows (f, g) in $\mathcal{S}$,their composition, $g \circ f$ is in $\mathcal{S}$. Consequently, $\mathcal{S}$ is also a category. An injection map, $T_{\mathcal{S}\,inj} : \mathcal{S} \to C$ sending objects and arrows of $\mathcal{S}$ to itself in C is called the *inclusion functor*. It is consequently faithful. $\mathcal{S}$ is called a *full subcategory* of C when $T_{\mathcal{S}\,inj}$ is full (MacLane, 1971, p.15).

A useful example of a functor which will play an important role in mapping structures in a set-theoretic setting to a category-theoretic setting is the so-called forgetful functor denoted as $FOR : C \to SET$ where *SET* is the category of ordinary sets in set theory and C is any mathematical system (category). FOR strips off the extra structure attached to C and produces just the set objects of C as a new simply set category. FOR essentially forgets any structure (arrow rules, etc.) that C may have had, i.e., for a category C , if $A \in Ob(C)$, $FOR(A) = S_A$ where S_A is the strict set part of A and $FOR(f) = f$ for any $f \in Arr(C)$ .Versions of partially forgetful functors have been presented such as the class introduced by Geroch (1985) in which a functor is partially forgetful if it strips a category down to the categorically nearest simpler category (i.e., nearest meaning being in the same category, but with less structure morphologically). These lesser structured categories may then be mapped back to the richer categories that were stripped down by free construction functors that would then reintroduce the original richer structure back.

As an example, the Abelian group category, ABLGRP, can be stripped back entirely by FOR, but if we introduce a partially forgetful functor that just strips away commutivity, ANTICOM the group category, GRP is produced. By applying a free construction functor, COM that reintroduces commutivity back into GRP, one obtains ABLGRP. This will serve in producing a categorical chain of categories in which the repeated application of partially forgetful functors in combination with free construction functors will produce a family of categorically related structures and hence, a metachain for model-theories and their mathematical structures.

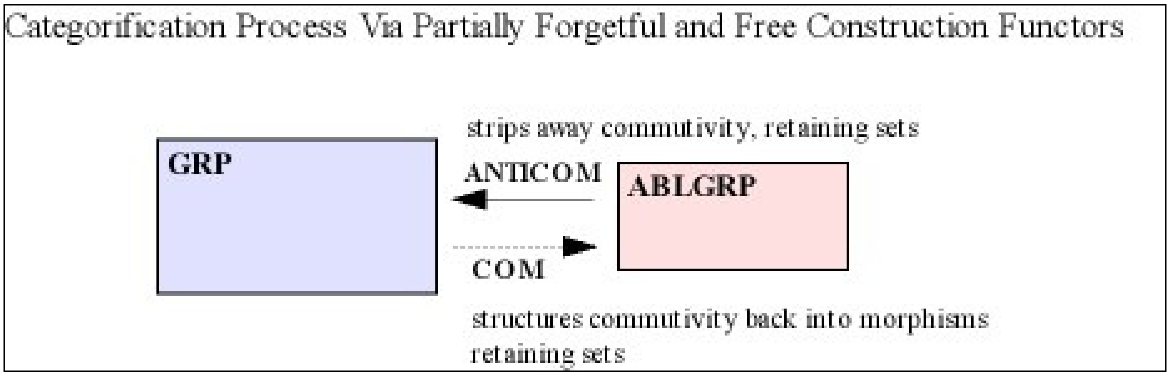

Figure 37. Categorification process

A general way of defining a natural transformation of one functor to another in such a way that commutes between categories is through the following:

Definition. For two functors, $S, T : \mathcal{C} \to \mathcal{B}$, a *natural transformation* that maps *S* to *T*, denoted by $\tau : S \to T$, is a function that assigns to an object C of $\mathcal{C}$, an arrow $\tau_C : S(C) \to T(C)$ of $\mathcal{B}$ such that every arrow $f : C \to C'$ in $\mathcal{C}$ commutes in the following map diagram:

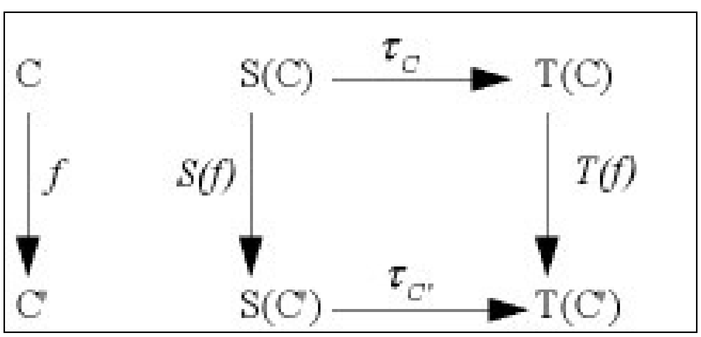

Figure 38. Natural transformation

The transformation, τ_C is called natural in *C* (MacLane, 1971, p.16). The notion of generalized categorical subsets, known as subobjects and the mechanism to find subobjects, a subobject classifier, will be discussed next.

Definition. An arrow $f \in Arr(C)$, $f : A \to B$ is called *monic* if for any parallel pair of arrows, $g, h : C \to A$ in $Arr(C)$ $f \circ g = f \circ h \Rightarrow g = h$.

Definitions. A subobject of an object, $D \in Ob(C)$ is a monic arrow in $Arr(C)$, $f : A \to D$, with codomain *D*. The set of all such subsets of *D* (if *D* is an ordinary set) is called the *powerset* of *D*, denoted by $\mathcal{P}(D)$ or 2^D.

Ordinary set inclusion, $\subseteq$ defines a partial ordering in $\mathcal{P}(D)$ so that $(\mathcal{P}(D), \subseteq)$ become a poset and hence a category in which the role of arrows is $A \to B \;\Leftrightarrow\; A \subseteq B$. Inclusion arrows then become commutative, reflexive, and transitive between subobjects. A generalization to 2^D in any category is the set of power objects denoted as Ω^2 where the universe of discourse generalizes the binary set $\{0,1\}$.

Definition. A category, C with limits is said to have *power objects* if to each object, $A \in Ob(C)$, there are objects $\mathcal{P}(A)$ and $\in_A$, and a monic arrow $\in : \in_A \to \mathcal{P}(A) \times A$, such that for any object $B \in Ob(C)$ and relation map given by $r : R \to B \times A$, there is exactly one arrow $f_r : B \to \mathcal{P}(C)$ for which there is a pullback in C taking on the form

$$\begin{tikzcd} R \arrow[r, "r", "\text{monic}"'] \arrow[d] & B \times A \arrow[d, "f_r \times 1_A"] \\ \in_A \arrow[r, "\in", "\text{monic}"'] & \mathcal{P}(C) \times A \end{tikzcd}$$

Figure 39. Power object pullback

A *relation map* is a map with domain consisting of a relation *R*, which is an object such that $R \subseteq A \times B$ in which $(x, y) \in R \Leftrightarrow y \in f_R(x)$ where $f_R : A \to B$ is an arrow appropriately defined for the inclusion in *R*.

Definition. In a category, C with terminal object 1, a *subobject classifier* for C is an object $\Omega \in Ob(C)$ together with an arrow, $true : 1 \to \Omega$ that satisfies the following axiom:

Ω-axiom. For each monic arrow, $f : A \to D$, there is one and only one arrow, $\chi_f : D \to \Omega$ such that the diagram:

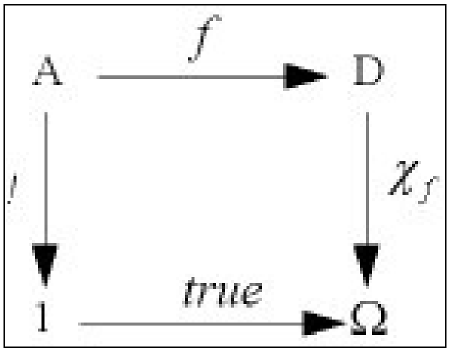

Figure 40. Subobject classifier pullback square

is a pullback square. Here χ_f is called the *character* of the monic arrow f (as a subobject of D), *true* is the arrow assigning a truth value of TRUE from the universe of discourse of truths, Ω, and *!* is the composition arrow defined by $true^{-1} \circ \chi_f \circ f : A \to 1$. The arrow $true^{-1}$ simply maps the value TRUE in Ω to the terminal object 1 in $Ob(C)$.

Enough structure has been defined to develop the formal definition of a Topos, which will serve as the template for a generalization to physical logic systems employed by information fields as defined in this dissertation.

Definition. An *elementary topos* is a category, C such that

(1) C is finitely complete,

(2) C has exponentiation, and

(3) C has a subobject classifier (Lawvere, 1964; Paré, 1974).

Alternatively, a category C is a *topos* if

(1) C is finitely complete, and

(2) C has power objects (Wraith, 1975).

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